# ALGORITHMS FOR AI LOGIC OF DECISIONS IN MULTI-AGENT ENVIRONMENT

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Abstract: This paper[1] suggests a temporal multi-agent logic $LD_{\mathcal{MA}}$ (with interacting agents), to imitate decision-making of independent agents, supported by access to knowledge through interaction with other agents. The interaction is modeled by considering all possible communication paths between agents in temporal Kripke/Hintikka like models. The logic $LD_{\mathcal{MA}}$ distinguishes local and global decision-making and is based on temporal Kripke/Hintikka models with agents accessibility relations defined between the states of time clusters. The main result provides a decision algorithm for $LD_{\mathcal{MA}}$ (so, we prove that the set of theorems of $LD_{\mathcal{MA}}$ is decidable), which also solves the satisfiability problem for $LD_{\mathcal{MA}}$.

## 1 INTRODUCTION

Applications of multi-modal and temporal logic to AI and CS is a popular area of research. In particular, they can be seen (based on the formalism of multi-agent logics) as a part (or implementation) of epistemic logic. Among epistemic logics to model knowledge a range from *S*4 to *S*5 has been investigated (Hintikka (1962) — logic S4, Kutschera (1976) argued for S4.4, Lenzen (1978) suggested S4.2, van der Hoek (1996) had proposed to strengthen knowledge according to system S4.3, van Ditmarsch, van der Hoek and Kooi together with Fagin, Halpern, Moses and Vardi (Fagin et al., 1995) and others assume knowledge to be S5 valid, see also (Halpern and Shore, 2004)). The approach, developed to model multi-agent environment in AI, often combines not only modal operations for agents‘ knowledge and Boolean logical operations, but also some other — e.g. operations for time — temporal operations, dynamic logic operations (cf. (Schmidt and Tishkovsky, 2004)). Through the prism of multi-agent approach we may view the logic of discovery, which has a solid prehistory, starting possibly from the monograph “Logic of Discovery and Logic of Discourse” by Jaakko Hintikka and Fernand Vandamme (Hintikka and Vandamme, 1986).

The Decision Logics apparently have interdisciplinary origin and they were influenced by ideas coming from researchers of widely varying background (cf. (Ohsawa and McBurney, 2003)). In particular, the modeling of environmental decision-support systems has been undertaken (Cortés et al., 2000; Avouris, 1995), tools involved in sematic web and multi-agent systems had been developed (Harmelem and Horrocks, 2000; Hendler, 2001; Arisha et al., 1999). Instruments for decision procedures in equational causal logic were created in (Peltier, 2003). Regarding multi-agent logics, many developed tools were inspired by the techniques of modal and temporal logic through mathematical semantics of Kripke/Hintikka models (Goldblatt, 2003; van Benthem, 1983).

In our paper we study a temporal multi-agent logic $LD_{\mathcal{MA}}$ with *interacting agents* with the purpose of finding a decision algorithm for this logic. The main idea is to study ways of passing knowledge between agents via possible communication paths, then model them in the temporal Kripke/Hintikka-like models by modal-like operation $D_l$ (locally taken decision), and extend the method to the global decision. We build our logic in a language, which considers and distinguishes *local decision* $D_l$ and *global decision* $D_g$ operators applied to formulas. An approach, which we use, is based on the research (Rybakov, 1997; Rybakov, 2005b; Rybakov, 2005a; Rybakov, 2006; Rybakov, 2007) on the representations of knowledge by

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inference rules in AI logics. The final result of the paper is a decision algorithm, which recognizes theorems of $LD_{\mathcal{MA}}$ (i.e., we prove that $LD_{\mathcal{MA}}$ is decidable). The algorithm works by reducing a formula to a logical consecution — inference rule, and checking validity of this rule in special Kripke-Hintikka models of size double exponential in the size of the original formula.

## 2 NOTATION, PRELIMINARIES

In the sequel we use the standard notation and well-known facts concerning modal, multi-modal and temporal logics, hence, some familiarity with the area is assumed. To formulate the logic $LD_{\mathcal{MA}}$, we proceed by introducing a semantic motivation for the choice of its language and interpretation. The Kripe/Hintikka models, upon which we base our logical language, are the following tuples — linear-time frames with time clusters to model agents‘ accessibility relations to information:

$$\mathcal{N}_C := \langle \bigcup_{i \in N} C(i), R, R_1, \ldots, R_m \rangle,$$

where $N$ is the set of natural numbers, $C(i)$ are some nonempty sets, $R, R_1, \ldots R_m$ are binary accessibility relations. For all elements $a$ and $b$ from $\bigcup_{i \in N} C(i)$,

$$aRb \iff \exists i, j \in N : i \leqslant j \ \&\ a \in C(i) \ \&\ b \in C(j);$$

any $R_j$ is a reflexive, transitive and symmetric relation, and

$$\forall a, b \in \bigcup_{i \in N} C(i) : aR_j b \implies \exists k \in N : a, b \in C(k).$$

The models based on these frames are intended to represent the reasoning (computation) in discrete time, so each $i \in N$ (any natural number $i$) is the time index for a cluster of states arising at a step in computation. Any $C(i)$ is a finite set of all possible states in the time point $i$, and the relation $R$ represents discrete current of time. Relations $R_j$ are intended to model the access to *knowledge (information)* for *agents* at any state of the cluster of states $C(i)$. As usual, any $R_j$ is supposed to be $S5$-like relation — a binary symmetric, reflexive and transitive relation.

The relation $R$ models the discrete flow of time, so, $aRb$ means that $a$ and $b$ are some states at the same time point or the state $b$ will be achieved at some point in the future, after the time point with the state $a$. The flow of time is supposed to be linear, which reflects well the human perception. We suppose the reasonong/computation to be concurrent — after a step in computation a new cluster of possible states appears, and agents will be given new access rules to the information in this time cluster. However, the agents cannot predict, which access rules they will have (that is why, in particular, we do not use *nominals*).

What is new in our approach is that we intend to make decision-making being based on non-deterministic agents‘ accessibility to knowledge (with the aim of finding algorithms recognizing logical laws of the proposed logic). To handle the problem we use a special logical language. It combines an extended language for agents‘ knowledge logic and a non-standard modal language for modeling time. The foundation of the language is the set of propositional letters $P$, the logical operations include usual Boolean operations, usual unary agent knowledge operations $K_i$, $1 \leq i \leq m$ and the modal operation $\Diamond$ for time. We also augment the language by taking weak necessity operation $\Box_w$ and decision operations $D_l$ and $D_g$, for local and global decision-making. Formation rules for formulas are as usual, and

- $K_i\varphi$ can be read: *an agent i knows* $\varphi$ *at the current state (informational node) of the current time cluster;*
- $\Diamond\varphi$ means: $\varphi$ *is possible in a future time cluster accessible from the current state;*
- $\Box_w\varphi$ stands for: *in any future time cluster there is a state where* $\varphi$ *holds* (so to say, $\varphi$ *is weakly necessary*);
- $D_l\varphi$ has the meaning : *it is decided that* $\varphi$ *holds locally;*
- $D_g\varphi$ has the meaning: *it is decided that* $\varphi$ *holds globally.*

A concrete knowledge situation is modeled by distribution of truth values of propositions from $P$ at the elements of the frame $\mathcal{N}_C$. More formally, we consider valuations $V$ of $P$, which are mappings of $P$ into the set of all subsets of the set $\bigcup_{i \in N} C(i)$, so in symbols,

$$\forall p \in P : V(p) \subseteq \bigcup_{i \in N} C(i).$$

If, for an element $a \in \bigcup_{i \in N} C(i)$, $a \in V(p)$, we say that *the fact p is true at the state a.*

Validness of formulas is defined as follows (below, $\mathcal{N}_C, a \Vdash_V \varphi$ is meant to say that *the formula* $\varphi$ *is true at the state a in the model* $\mathcal{N}_C$ *w.r.t. the valuation* $V$):

$$\forall p \in P,\ \forall a \in \mathcal{N}_C : \mathcal{N}_C, a \Vdash_V p \iff a \in V(p);$$

$$\mathcal{N}_C, a \Vdash_V \varphi \wedge \psi \iff [\mathcal{N}_C, a \Vdash_V \varphi \text{ and } \mathcal{N}_C, a \Vdash_V \psi];$$

$$\mathcal{N}_C, a \Vdash_V \varphi \vee \psi \iff [\mathcal{N}_C, a \Vdash_V \varphi \text{ or } \mathcal{N}_C, a \Vdash_V \psi];$$

$$\mathcal{N}_C, a \Vdash_V \varphi \to \psi \iff [\mathcal{N}_C, a \nVdash_V \varphi \text{ or } \mathcal{N}_C, a \Vdash_V \psi];$$

$$\mathcal{N}_C, a \Vdash_V \neg\varphi \iff \mathcal{N}_C, a \nVdash_V \varphi;$$

$$\mathcal{N}_C, a \Vdash_V \Diamond\varphi \iff \exists b \in \mathcal{N}_C : aRb \text{ and } \mathcal{N}_C, b \Vdash_V \varphi;$$

$$\mathcal{N}_C, a \Vdash_V \Box_w\varphi \iff$$
$$(a \in C(i) \implies \forall j \geq i\ \exists b \in C(j) : \mathcal{N}_C, b \Vdash_V \varphi);$$

$$\mathcal{N}_C, a \Vdash_V K_i\varphi \iff \forall b \in \mathcal{N}_C : aR_i b \implies \mathcal{N}_C, b \Vdash_V \varphi;$$

$$\mathcal{N}_C, a \Vdash_V D_l\varphi \iff$$
$$\exists a_{i_1}, \ldots, a_{i_n} \in \mathcal{N}_C : aR_{i_1}a_{i_1}\ldots R_{i_n}a_{i_n}\ \ \&\ \ \mathcal{N}_C, a_{i_n} \Vdash_V \varphi;$$

$$\mathcal{N}_C, a \Vdash_V D_g\varphi \iff \forall b \in \mathcal{N}_C : aRb \Longrightarrow \mathcal{N}_C, b \Vdash_V D_l\varphi.$$

From the rules above, we immediately see that introduced logical operations are not independent from the semantical viewpoint and that

$$D_g\varphi \equiv \neg\Diamond\neg D_l\varphi.$$

Therefore we omit $D_g$ from the further consideration.

**Definition 1.** *The logic* $LD_{\mathcal{MA}}$ *is the set of all formulas which are true at any state of any frame* $\mathcal{N}_C$ *w.r.t. any valuation.*

To compare the logical laws of $LD_{\mathcal{MA}}$ with the laws of the standard multi-modal logic, note that the following holds:

$$\Box_w p \to p,\ \Box_w p \equiv \neg\Diamond\neg p,\ \Diamond p \equiv \neg\Box_w\neg p \notin LD_{\mathcal{MA}}.$$

This can be derived immediately from the definitions, using only simple frames. Moreover,

$$\Box_w(p \to q) \to (\Box_w p \to \Box_w q) \notin LD_{\mathcal{MA}},$$

therefore, $\Box_w$-fragment of $LD_{\mathcal{MA}}$ is not a fragment of any reflexive modal logic, and also $\Box_w$ and $\Diamond$ are not mutual counterparts of each other. Thus $LD_{\mathcal{MA}}$ differs from any standard normal or not-normal multimodal logic. It is interesting, whether $\Box_w$ and $\Diamond$ may be mutually expressed by other non-standard ways.

At the same time (where $\Box := \neg\Diamond\neg$)

**Lemma 1.** *The following holds*

- $\Box(p \to q) \to \Box(\Box_w p \to \Box_w q) \in LD_{\mathcal{MA}}$,
- $\Box_w p \to \Box_w\Box_w p \in LD_{\mathcal{MA}}$,
- $\Box(\Box_w p \to \Box_w q) \vee \Box(\Box_w q \to \Box_w p) \in LD_{\mathcal{MA}}$,
- $\varphi \in LD_{\mathcal{MA}} \implies \Box_w\varphi \in LD_{\mathcal{MA}}$.

Thus $\Box_w$-fragment of $LD_{\mathcal{MA}}$ has some similarity with modal logics extending $S4.3$. The primal questions for any logic are the decidability problem and satisfiability problem (for a logic $\mathcal{L}$ it is of vital importance to recognize correct logical law of this logic, and solving the decidability problem means constructing an algorithm, which could recognize the logical laws). We address the decidability problem for $LD_{\mathcal{MA}}$ in the next section.

## 3 DECIDING ALGORITHMS

The decidability problem for $LD_{\mathcal{MA}}$, which we will be dealing with in the sequel, is how, for any given formula $\varphi$, to determine whether $\varphi$ is a theorem of $LD_{\mathcal{MA}}$ or not, in other words whether $\varphi \in LD_{\mathcal{MA}}$ or $\varphi \notin LD_{\mathcal{MA}}$ (if there is an algorithm for solving this task for a logic $\mathcal{L}$, then the logic $\mathcal{L}$ is said to be decidable). The logic $LD_{\mathcal{MA}}$ is an extension of a special many-modal logic, and we could try to use some well-known techniques to tackle the problem at hand. However the standard ways meet the obstacle of the operator $D_l$ used in our approach, mainly because it fails such techniques as filtration and dropping points. Several ways to approach the problem are nevertheless possible, and we will apply a technique based on our own approach with employing logical consecutions and validity of inference rules, which was tested for several logics earlier (Rybakov, 1997; Rybakov, 2005a; Rybakov, 2006; Rybakov, 2007). This tools use a representation of formulas by rules, and an algorithmic translations of rules into some *normal reduced forms*. Recall that a (sequential) rule is an expression of the form

$$r := \frac{\varphi_1(x_1,\ldots,x_n),\ldots,\varphi_m(x_1,\ldots,x_n)}{\psi(x_1,\ldots,x_n)},$$

where $\varphi_1(x_1,\ldots,x_n),\ldots,\varphi_m(x_1,\ldots,x_n),\psi(x_1,\ldots,x_n)$ are some formulas constructed out of letters $x_1,\ldots,x_n$. Letters $x_1,\ldots,x_n$ are called variables of $r$, symbolically

$$Var(r) = \{x_1,\ldots,x_n\}.$$

A formula $\varphi$ is valid in a frame $\mathcal{N}_C$ (notation $\mathcal{N}_C \Vdash \varphi$) if, for any valuation $V$ of $Var(\varphi)$ and for any element $a$ of $\mathcal{N}_C$, $\mathcal{N}_C, a \Vdash_V \varphi$.

**Definition 2.** *A rule r is said to be* valid *in the Kripke model* $\langle\mathcal{N}_C, V\rangle$ *with the valuation V (we will use notation* $\mathcal{N}_C \Vdash_V r$*) if*

$$\forall a : \mathcal{N}_C, a \Vdash_V \bigwedge_{1\leq i\leq m} \varphi_i \implies \forall a : \mathcal{N}_C, a \Vdash_V \psi.$$

*Otherwise we say that r is* refuted *in $\mathcal{N}_C$, or* refuted in $\mathcal{N}_C$ by $V$, *and write* $\mathcal{N}_C \not\Vdash_V r$.

A rule $r$ is *valid* in a frame $\mathcal{N}_C$ (notation $\mathcal{N}_C \Vdash r$) if, for any valuation $V$ of $\mathrm{Var}(r)$, $\mathcal{N}_C \Vdash_V r$. A rule $r$ is said to be in the *reduced normal form* if $r = \varepsilon_r / x_1$ where

$$\varepsilon_r = \bigvee_{1\le j\le m}\left(\bigwedge_{1\le i\le n}\left[x_i^{t(j,i,0)} \wedge (\Diamond x_i)^{t(j,i,1)} \wedge (\Box_w x_i)^{t(j,i,2)} \wedge (D_l x_i)^{t(j,i,3)} \wedge \bigwedge_{1\le s\le m}(\neg K_s \neg x_i)^{t(j,i,4,s)}\right]\right),$$

where all $x_l$ are variables, $t(j,i,z), t(j,i,k,z) \in \{0,1\}$ and, for any formula $\alpha$ above, $\alpha^0 := \alpha$, $\alpha^1 := \neg\alpha$.

For any formula $\varphi$ we can convert it into the rule $x \to x/\varphi$ and employ technique of reduced normal forms as explained below.

**Definition 3.** *Given a rule $r_{\mathrm{nf}}$ in the reduced normal form, $r_{\mathrm{nf}}$ is said to be a* normal reduced form for a rule *r iff, for any frame $\mathcal{N}_C$,*

$$\mathcal{N}_C \Vdash r \iff \mathcal{N}_C \Vdash r_{\mathrm{nf}}.$$

Based on proofs of Lemma 3.1.3 and Theorem 3.1.11 from (Rybakov, 1997), by similar technique, we obtain

**Theorem 1.** *There exists an algorithm running in (single) exponential time, which, for any given rule r, constructs its normal reduced form $r_{\mathrm{nf}}$.*

It is immediate to see that a formula $\varphi$ is valid in a frame $\mathcal{N}_C$ iff the rule $x \to x/\varphi$ is valid in $\mathcal{N}_C$, so from Theorem 1 we obtain

**Lemma 2.** *A formula $\varphi$ is a theorem of $\mathrm{LD}_{\mathcal{MA}}$ iff the rule $(x \to x/\varphi)_{\mathrm{nf}}$ is valid in any frame $\mathcal{N}_C$.*

Thus, to solve the question about decidability of $\mathrm{LD}_{\mathcal{MA}}$ it is sufficient to find an algorithm recognizing rules in reduced normal form which are valid in all frames $\mathcal{N}_C$. To begin with, we first will bound effectively the number of states in clusters on frames refuting the consecution. This step deals only with problems arising from interaction of agents. In the following lemma the representation of formulas by normal reduced forms of rules is essential.

**Lemma 3.** *If a consecution $r = \varepsilon_r/x_1$ where $\varepsilon_r := \bigvee_{1\le j\le m}(\bigwedge_{1\le i\le n}[x_i^{t(j,i,0)} \wedge (\Diamond x_i)^{t(j,i,1)} \wedge (\Box_w x_i)^{t(j,i,2)} \wedge (D_l x_i)^{t(j,i,3)} \wedge \bigwedge_{1\le s\le m}(\neg K_s \neg x_i)^{t(j,i,4,s)}])$, is refuted in a model $\mathcal{N}_C$, then r is refuted in a such model with clusters $C(i)$ linear in the size of r.*

In the proof of this lemma we cannot just use the standard filtration technique, because paths of interchanging knowledge-accessibility relations cannot be bounded by any filtration, decision formulas $D_l x_j$ pose the problem. But a refined technique based on normal forms of the rules works.

To further describe our algorithm we need the following special finite Kripke models. Take the frame $\mathcal{N}_C$ and some numbers $n, m$, where $m > n > 1$. The frame $\mathcal{N}_C(n,m)$ has the following structure: $\mathcal{N}_C(n,m) := \langle \bigcup_{1\le i\le m} C(i), R, R_1, \ldots, R_m\rangle$, where $R$ is the accessibility relation from $\mathcal{N}_C$ extended by pairs $(x,y)$, where $x,y \in [n,m]$, so $xRy$ holds for all such pairs, and relations $R_1,\ldots,R_m$ are inherited from $\mathcal{N}_C$. Given a valuation $V$ of letters from a formula $\varphi$ in $\mathcal{N}_C(n,m)$, the truth values of $\varphi$ can be defined on elements of $\mathcal{N}_C(n,m)$ by the same rules as for frames $\mathcal{N}_C$ above (actually, in accordance with the standard definitions of truth values for modalities). For illustration, we describe below basic steps for modalities.

$$\mathcal{N}_C(n,m), a \Vdash_V \Diamond\varphi \iff \exists b \in \mathcal{N}_C : aRb \ \&\ \mathcal{N}_C(n,m), b \Vdash_V \varphi;$$

$$\begin{gathered}\mathcal{N}_C(n,m), a \Vdash_V \Box_w\varphi \iff \\ [a \in C(i) \ \&\ i \le n \ \&\\ \forall j(m \ge j \ge i)\exists b \in C(j) : \mathcal{N}_C(n,m), b \Vdash_V \varphi]\vee \\ [a \in C(i) \ \&\ i > n \ \&\\ \forall j(n \le j \le m)\exists c \in C(j) : \mathcal{N}_C(n,m), b \Vdash_V \varphi].\end{gathered}$$

For $K_j\varphi$ and $D_l\varphi$ steps are exactly the same as for the models based on frames $\mathcal{N}_C$. Using this modified Kripke structures $\mathcal{N}_C(n,m)$ we derive

**Lemma 4.** *A rule $r_{\mathrm{nf}}$ in the reduced normal form is refuted in a certain frame $\mathcal{N}_C$ w.r.t. a valuation V if and only if $r_{\mathrm{nf}}$ can be refuted in a special model, based on a frame $\mathcal{N}_C(n,m)$, by a valuation $V_1$, where*

*(i) The size of any cluster $C(i)$ in $\mathcal{N}_C(n,m)$ is linear in the size of $r_{\mathrm{nf}}$;*

*(ii) n and m are exponential in the size of $r_{\mathrm{nf}}$;*

*(iii) The size of the frame $\mathcal{N}_C(n,m)$ is double exponential in the size of $r_{\mathrm{nf}}$.*

We did not specify the details of the "special model" and the valuation $V_1$, due to restriction on the size of the paper, but those conditions may be effectively verified. Therefore based on Theorem 1, Lemma 2 and Lemma 4 we conclude

**Theorem 2.** *The logic $\mathrm{LD}_{\mathcal{MA}}$ is decidable.*

The verification of the fact that a formula $\varphi$ is a theorem of $\mathrm{LD}_{\mathcal{MA}}$ consists in verifying of validity of the rule $(x \to x/\varphi)_{\mathrm{nf}}$ in Kripke/Hintikka frames $\mathcal{N}_C(n,m)$ of size double exponential in the size of reduced normal forms. The overall complexity includes also the complexity of reducing a rule to the normal reduced form, which is single exponential.

## 4 CONCLUSIONS

We propose the logic $LD_{\mathcal{MA}}$, which combines linear discrete time and agents' accessibility relations inside time clusters with interaction of agents via arbitrary paths of individual accessibility relations. This logic seems to be new and interesting, because it is able to model situations resistant to description by the standard modal language. We propose an algorithm for recognizing theorems of $LD_{\mathcal{MA}}$ (so, we prove that $LD_{\mathcal{MA}}$ is decidable), the same algorithm solves the satisfiability problem for $LD_{\mathcal{MA}}$. Future research might include, for instance, the case of non-linear time, i.e., with the time flow modeled by arbitrary reflexive and transitive binary relation, with no further restrictions. Another open and interesting problem is an explicit axiomatization of $LD_{\mathcal{MA}}$ and its above-mentioned extensions.

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