Adaptive Electrical Signal Post-Processing in Optical Communication Systems

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Abstract. Improving bit error rates in optical communication systems is a difficult and important problem. The error correction must take place at high speed and be extremely accurate. We show the feasibility of using hardware implementable machine learning techniques. This may enable some error correction at the speed required.

1 Introduction

Performance of a fibre-optic communication link is typically affected by a complex combination of random processes (such as amplified spontaneous emission noise, polarization mode dispersion and so on) and deterministic or quasi-deterministic effects (e.g. nonlinear inter- and intra-channel signal interactions, dispersive signal broadening, various cross-talks and so on) that result from particular system design and operational regimes. Any installed fibre link has its specific transmission impairments, its own signature of how the transmitted signal is corrupted and distorted. Therefore, there is a great potential in the application of an adaptive signal post-processing that can undo some of the signal distortions, or to separate line-specific distortions from non-recoverable errors. Signal post-processing in optical data communication can offer new margins in system performance in addition to other enabling techniques. A variety of post-processing techniques have been already used to improve overall system performance, e.g. tunable dispersion compensation, electronic equalization and others (see e.g. [1-4] and references therein). Note that post-processing can be applied both in the optical and electrical domain (after conversion of the optical field into electrical current). Application of electronic signal processing for compensation of transmission impairments is an attractive technique that became quite popular thanks to recent advances in high-speed electronics.

In this work we apply standard machine learning techniques to adaptive signal post-processing in optical communication systems. To the best of our knowledge this is the first time that such techniques have been applied in this area. One key feature of this problem domain is that the trainable classifier must perform at an extreme speed,

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optical communication systems typically operate at bit rates of around 40 GHz. We demonstrate a feasibility of bit-error-rate improvement by adaptive post-processing of received electrical signal.

2 Background

At the receiver (typically after filtering) the optical signal is converted by a photodiode into the electrical current. Detection of the digital signal requires discrimination of the logical 1s and 0s using some threshold decision. This can be done in different ways (e.g. by considering currents at a certain optimized sample point within the bit time slots or by analyzing current integrated over some time interval) and is determined by a specific design of the receiver. Here without loss of generality we assume that discrimination is made using current integrated over the whole time slot. Note that the approach proposed in this paper and described in detail below is very generic and can easily be adapted to any particular receiver design. To improve system performance and minimize the bit-error-rate, we propose here to use a method to adjust the receiver by sending test patterns to transmission impairments specific for a given line. This is achieved by applying learning algorithms based on analysis of sampled currents within bit time slots and adaptive correction of the decisions taking into account accumulated information gained from analysis of the signal waveforms.

3 Description of the Data

The data represents the received signal taken in the electrical domain after conversion of the optical signal into an electrical current. The data consists of a large number of received bits with the waveforms represented by 32 real numbers corresponding to values of electrical current at each of 32 equally spaced sample points within a bit time slot. A sequence of 5 consecutive bits is shown in Figure 1. As already explained the pulse can be classified according to the current integrated over the width of a single bit. For each of time slots in our data we have the original bit that it represents. Therefore the data consists of 32-ary vectors each with a corresponding binary label.

In all we have a stream of 65536 bits to classify. As already explained categorising the vast majority of these bits is straightforward. In fact with an optimally set electrical current integrated over the whole time slot (*energy threshold*) we can correctly classify all but 1842 bits correctly. We can therefore correctly classify 97.19% of the data, an error rate of 2.81%. This is however an error rate significantly too high. The target error rate is less than one bit in a thousand, or 0.1%. Figure 2 (a) gives an example of a misclassification. The middle bit of the sequence is a 0 but is identified from its energy as a 1. This is due to the presence of two $1's$ on either side and to distortion of the transmitted signal. It would be difficult for any classifier to rectify this error.

However other cases can be readily identified by the human eye and therefore could be amenable to automatic identification. Figure 2 (b) shows an example where the bit pattern is obvious to the eye but where a misclassification actually occurs. The central bit is a 1 but is misclassified as a 0 from its energy alone.

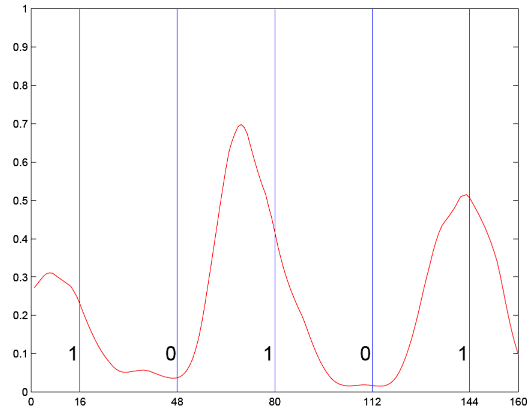
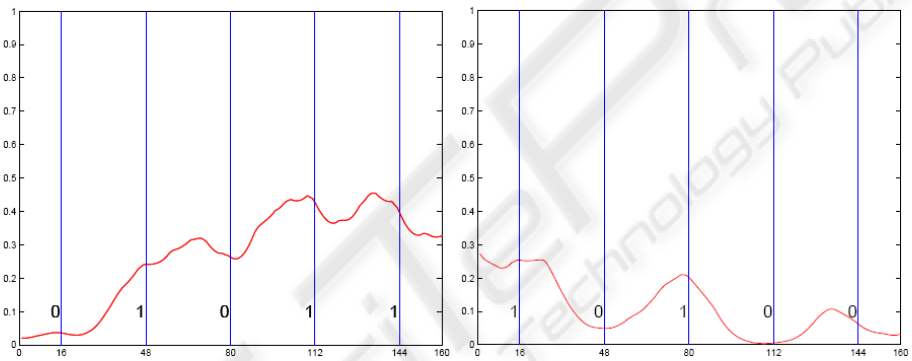


Fig. 1. An example of the electrical signal for a stream of 5 bits - 1 0 1 0 1.



(a) (b)

Fig. 2. (a) An example of a difficult error to identify. The middle bit is meant to be a 0, but jitter has rendered it very hard to see; (b) The central bit has been dragged down by the two 0s surrounding it and is classified as a 0 from its energy. However to the human eye the presence of a 1 is obvious.

3.1 Representation of the Data

Different datasets can be produced depending on how the original electrical current is represented. As well as representing a single bit as a 32-ary vector (called the *Waveform*-1 dataset), it can also be represented as a single energy value (the sum of the 32 values, *Energy*-1).

As described above of the 65536 bits, all but 1842 are correctly identified by an energy threshold. There are 32 distinct streams of 5 bits and 9 of these are represented in the misclassified class with a high frequency from 5.86% to 21.17% among the 1842 misclassified cases. These nine sequences are shown in Table 1.

Table 1. Nine sequences for which difficulties are most likely to occur.

0 0 1 0 0
0 0 1 0 1
0 1 0 1 0
0 1 0 1 1
1 0 0 1 1
1 0 1 0 0
1 0 1 0 1
1 1 0 1 0
1 1 0 1 1

As can be seen the majority of these involve a $1\ 0\ 1$ or $0\ 1\ 0$ sequence around the middle bit, and these are the patterns for which difficulties are most likely to occur. Therefore we may also want to take advantage of any information that may be present in adjacent bits. To this end we can form windowed inputs, in which the 3 vectors representing 3 contiguous bits are concatenated together with the label of the central bit being the target output (*Waveform*-3). It is also possible that using adjacent bit information by simply taking 3 energy values instead of the full waveform (*Energy-3*), or using information from a window of 3 bits, with 1 either side of the target bit (*Energy-Waveform-Energy*). Table 2 gives a summary of all the different datasets.

Table 2. The different datasets used in the first experiment.

Name	Arity	Description
Energy-1	1	The energy of the target bit
Energy-3	3	The energy of the target bit and one bit either side
Waveform-1	32	The waveform of the target bit
Waveform-3	96	The waveform of the target bit and the waveforms of the bits on either side
Energy-Waveform-Energy (E-W-E)	34	The waveform of the target bit and the energy of one bit either side of the target bit

4 Approaches Used

4.1 Easy and Hard Cases

One difficulty for the trainable classifier is that in this dataset the vast majority of examples are straightforward to classify. The hard cases are very sparsely represented, so that, in an unusual sense, the data is imbalanced. Figure 3 is a diagram of error rates of 0 and 1 as a function of the energy threshold. It shows that if the energy threshold is set to roughly 2.5, then those bits with energy less than this threshold are correctly classified into the 0 class; on the other hand, if the energy threshold is set to about 11, then those bits with energy greater than this are correctly classified into the 1 class. The optimal energy threshold to separate two classes is 5.01, in which case, 1842 of 65532 are incorrectly classified - a bit error rate of 2.81%. Using this threshold we divide the data into easy and hard cases, that is, those classified correctly by the method are easy ones, otherwise they are hard cases.

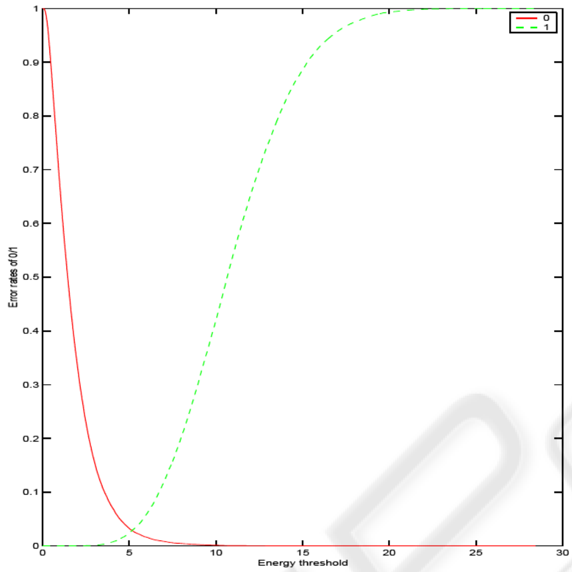


Fig. 3. A diagram of error rates of 0 and 1 as a function of an energy threshold.

4.2 Visualisation using PCA

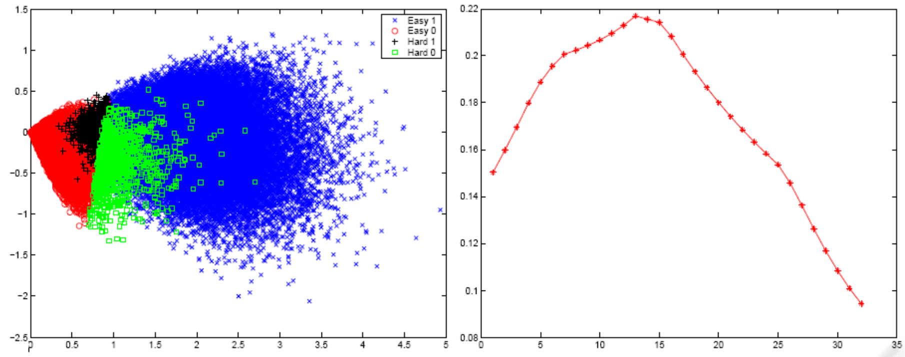
Before classifying bits into two classes, we first look at the underlying data distribution by means of Classical principal component analysis PCA [5], which linearly projects data into a two-dimensional space, where it can be visualised.

We visualise the easy cases using PCA, then project the hard cases into the same PCA projection space. The result is shown in Figure 4 (a). It shows that unsurprisingly the easy 0 and 1 classes are linearly separable. Interestingly, the hard 0 and 1 classes are for the most part also linearly separable. However, the hard 1s have almost complete overlap with the easy 0s, and the hard 0s have almost complete overlap with the easy 1s.

Figure 4 (b) is the eigenwave of the first component in the PCA analysis, which accounts for 86.5% of the total variance.

4.3 Single Layer Neural Network

As already described the classifiers need to be operationally very fast. Therefore the main classifier we use is a simple single layer neural network (SLN) [5]. Once trained (this is done off-line in advance) an SLN can be built in hardware and function with great speed. For comparison purposes a classifier that uses just an optimal energy threshold is implemented, where the threshold is the one giving the maximum accuracy rate (97.19%).



(a) (b)

Fig. 4. (a)Projection of the easy set using PCA, where the hard patterns are also projected into the easy ones' first two principal components space; (b) Eigenwave of the first principal component.

4.4 Identifying Difficult Cases using the Energy

As mentioned in Section 4.2, once the hard cases have been separated from the easy cases, it is possible to linearly separate the 1s and 0s for even the hard cases. Therefore, the question is how to distinguish between easy and hard cases. One way to tackle the problem is to use an energy threshold band. So we can set two thresholds, E_{min} and E_{max}, with $E_{min} \leq E_{max}$, such that if the energy of a bit is less than E_{min} then that bit is definitely a 0, and if is greater than E_{max} then it is a 1. For those bits whose energy lies between E_{min} and E_{max}, they can be considered as difficult cases. Experiment 2 describes how an energy threshold band can be used to select difficult cases which are subsequently used as training data to an SLN.

4.5 Gaussian Mixture Model (GMM)

Another approach we applied in this work is to use a Gaussian mixture model [5].

Distinguish between easy and hard cases can be performed by modelling the class-conditional probability $p(\mathbf{x}|c_i)$ for each easy and hard class first, then by calculating corresponding posterior probabilities using Bayes' theorem.

Since it is usually insufficient to model the conditional density by a single Gaussian distribution, we apply a Gaussian mixture model for each class-conditional probability density. In a Gaussian mixture model, the probability density function of each class is independently modelled as a linear combination of Gaussian basis functions. The number of basis functions, their position and variance and their mixing coefficients are all parameters of the model.

In a Gaussian mixture model, $p(\mathbf{x}|c_i)$ is of a linear combination of component densities $p(\mathbf{x}|j, c_i)$, and be written as follows:

$$p(\mathbf{x}|c_i) = \sum_{j}^{M} p(\mathbf{x}|j, c_i)P(j) \ , \tag{1}$$

where for each component j, we have a Gaussian distribution function

$$p(\mathbf{x}|j, c_i) = \frac{1}{\sqrt{(2\pi)^D |\boldsymbol{\Sigma}_j|}} \times \exp\left\{ -\frac{1}{2}(\mathbf{x} - \boldsymbol{\mu}_j)^T \boldsymbol{\Sigma}_j^{-1} (\mathbf{x} - \boldsymbol{\mu}_j) \right\} \,, \tag{2}$$

in which case $\boldsymbol{\mu}_j$ and $\boldsymbol{\Sigma}_j$ are mean and covariance matrix of each component j respectively. $P(j)$ in equation (1) satisfies

$$\sum_{j=1}^{M} P(j) = 1, \qquad 0 \leq P(j) \leq 1, \tag{3}$$

which guarantee that $p(x|c_i)$ is a valid density function.

The error function is defined as the negative log-likelihood for the dataset given by

$$E = -\ln \mathcal{L} = -\sum_{n=1}^{N} \ln p(\mathbf{x}^n|c_i) = -\sum_{n=1}^{N} \ln \left\{ \sum_{j}^{M} p(\mathbf{x}^n|j, c_i) P(j) \right\}. \tag{4}$$

The *expectation-maximisation* (EM) algorithm [6] is used to estimate parameters $P(j)$, $\boldsymbol{\mu}_j$, and $\boldsymbol{\Sigma}_j$ of a mixture model for an optimal fit to the training data.

In our experiment, we first estimated parameters of each class-condition density from the training dataset, where the data has been divided into an easy and hard subset using the optimal energy threshold. Then we reassign the class membership for each case using Bayes' theorem, that is

$$P(c_i|x) = \frac{P(c_i)p(x|c_i)}{\sum_{i=1}^{C} P(c_i)p(x|c_i)}. \tag{5}$$

Finally two SLNs were trained on these two reassigned classes. For a new test case, it is given either an *easy* or *difficult* class label using eq.(5), then it is forwarded to the corresponding trained SLN network based on its class membership to give the final discrimination.

5 Experiments

5.1 The First Experiment

We segment the data into 10-fold cross-training/validation sets and one independent test set. Each distinct segment has 5096 easy cases and 148 hard ones. Therefore, each training set includes 47196 cases and each validation set has 5244 cases in total; the independent test set has 12730 easy ones and 362 hard ones. The results reported here are therefore evaluations on the independent test set and averages over the 10 different validation sets. The main results are given in Table 3.

The classifiers do give an improvement over the optimal energy threshold method (*Energy*-1), with the SLN using the Waveform-3 dataset giving the best result. Interestingly the very simple classifier of the SLN/Energy-3 combination nearly decreased the error rate 42% on both validation and test sets when compared to the optimal threshold method. This classifier is simply a single unit with 3 weighted inputs.

To examine more closely how a threshold band performs on the waveform-3 dataset Experiment 2 was undertaken.

Table 3. The results of classifying the different validation and test sets for the different data representations. We also give the standard deviation for the validation sets.

Dataset	Validation Sets			The independent test set		
	mean errors		mean accuracy	errors		accuracy
	easy set	hard set		easy set	hard set	
Energy-1	0	148	97.180	0	362	97.23
Energy-3	22 ± 5	64 ± 7	98.366 ± 0.164	59	148	98.419
Waveform-1	22 ± 5	51 ± 5	98.608 ± 0.136	45	122	98.724
Waveform-3	22 ± 5	50 ± 8	98.633 ± 0.139	45	116	98.770
E-W-E	21 ± 6	50 ± 6	98.642 ± 0.140	49	123	98.686

5.2 The Second Experiment

Three different energy threshold bands are used to filter out the easy cases as discussed in Section 4.4. An SLN is then applied to each of the resultant difficult sets of size approximately 16700, 20600 and 24400, respectively. For the test set, above and below the threshold band, are classified appropriately, and the rest, the difficult set, are classified using the SLN. The results are given in Table 4.

Table 4. The results of classifying the different validation and test sets for each of the threshold bands. We also give the standard deviation for the validation sets.

Band	Validation Sets			The independent test set		
	mean errors		mean accuracy	errors		accuracy
	easy set	hard set		easy set	hard set	
$2.5 \leq x \leq 10.5$	23 ± 6	47 ± 5	98.659 ± 0.117	47	113	98.778
$2.0 \leq x \leq 11.0$	23 ± 5	48 ± 5	98.658 ± 0.122	42	114	98.808
$2.0 \leq x \leq 12.5$	22 ± 5	48 ± 5	98.661 ± 0.132	43	115	98.793

It can be seen that in general there is a classification improvement in the hard set on both validation and test sets when compared with results in Table 3. Using a band range from 2.0 to 11.0 give the best result on the independent test set over all our experimental results.

5.3 The Third Experiment

In this experiment, we divide the dataset into easy and hard subsets using Gaussian mixture models as discussed in section 4.5. For each class (easy/hard), we used a two-gaussian mixture with diagonal covariance matrices. The results are shown in Table 5.

It gives the best mean accuracy on the validation sets in all experiments, but not in the independent test set.

Table 5. The results of classifying the different validation and test sets. We also give the standard deviation for validation sets.

Model	Validation Sets			The independent test set		
	mean errors			errors		
	easy set	hard set	mean accuracy	easy set	hard set	accuracy
2 *gmms 'diag'*	21 ± 4	48 ± 5	98.675 ± 0.126	49	119	98.717

6 Discussion

The fast decoding of a stream of data represented as pulses of light is a commercially important and challenging problem. Computationally the challenge is in the speed of the classifier and the need for simple processing. We have therefore restricted our classifier to be, for the most part, an SLN and the data is either a sampled version of the light waveform or just the energy of the pulse. Experiment 1 showed that an SLN trained with the 96-ary representation of the waveform gave the best performance, reducing the bit error rate from 2.8% to 1.23%. This figure is still quite high and we hypothesised that the explanation was the fact that despite the data set being very large, 65532 items, the number of difficult examples (those misclassified by the threshold method) was very small and dominated by the number of straightforward examples. To see if we could correctly identify a significant number of these infrequent but difficult examples we undertook experiments 2 and 3 .

Although there was a small improvement obtained by using energy threshold band, further work is needed to determine if there is an optimal band size. Similar results were obtained by experiment 3.

This is early work and much of interest is still to be investigated, such as representational issues of the waveform, threshold band sizes, and other methods to identify difficult cases.

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