NEW INVARIANT DESCRIPTORS BASED ON THE MELLIN TRANSFORM

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Abstract: In this paper we introduce two new classes of radiometric and combined radiometric-geometric invariant descriptors. The first class includes two types of radiometric descriptors. The first type is based on the Mellin transform and the second one is based on central moments. Both descriptors are invariant to contrast changes and to convolution with any kernel having a symmetric form with respect to the diagonals. The second class contains two subclasses of combined descriptors. The first subclass includes central-moment based descriptors invariant simultaneously to translations, to uniform and anisotropic scaling, to stretching, to contrast changes and to convolution. The second subclass includes central-complex-moment based descriptors invariant simultaneously to similarity transformation and to contrast changes. We apply those invariants to the matching of geometrically transformed and/or blurred images.

1 INTRODUCTION

In pattern analysis, one important research axis is the analysis and characterization of objects and patterns corrupted by radiometric and/or geometric degradations. Real images can reveal geometric distortions as well as photometric degradations. Two important types of factors can be at the origin of those degradations. First, those that originate from the imaging system. They might be due to defects or limitations of the imaging device such as diffraction, bad image sensor, limited depth of field, limited dynamic range, etc. Second, the conditions under which the image is taken, i.e. ambient illumination, weather conditions, viewpoint position, etc. Several research fields are interested in the characterization of the ideal image from the acquired one by neither having recourse to restoration nor to geometric standardization as those two processes are often ill-posed problems (Katsaggelos, 1991). The obtained results may thus be not unique as they depend on the problem formulation. Moreover the algorithmic complexity may be high. To solve those problems, we are interested by invariant descriptors that are calculated from the image functions and allow the identification and characterization of the ideal scene image regardless of the photometric and/or geometric degradations. In the literature we can find four major descriptor types, namely, algebraic descriptors, visual descriptors, transform coefficient descriptors and statistical descriptors. Our research work is classified in the category of statistical descriptors.

In this paper, we introduce two new classes of invariant descriptors. The first class includes radiometric descriptors and the second one contains combined radiometric-geometric descriptors. The proposed invariants are inspired by a new relationship established between the Mellin transforms of all of the original image, the degraded image, and the convolution kernel. They have a high discriminant power in matching of geometrically transformed and/or radiometrically degraded images and their algorithmic complexities are low. In the rest of this paper we proceed as follows. In section 2, we give a brief synopsis of literature. In section 3, we introduce a new class of radiometric descriptors. Section 4 is devoted to the class of combined radiometric-geometric descriptors. Finally, experimental results are given in section 5.

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2 RELATED WORK

Two main degradation types are tackled in the field of analysis and interpretation of degraded images based on statistical invariant descriptors. In the case of geometric degradations, the author in (Hu, 1962) introduces the first class of statistical moment invariants based on the theory of algebraic invariants. The proposed descriptors are invariant to translation, rotation and scaling, and are used in the recognition of degraded planar objects. Authors in (Flusser and Suk, 1993) have developed the so called affine moment invariants, that is image descriptors that are invariant under general affine transformation. These descriptors are based on central moments and are used for the recognition of patterns and objects degraded by a general affine transformation. Several other researchers (Belkasim et al., 1991), (Reiss, 1991), (Teh and Chin, 1988) tackled the subject of moment based invariant descriptors for the recognition of geometrically degraded images.

For radiometric degradations, very little research work is interested in this topic. The research work of Flusser and his research group (Flusser et al., 1995), (Flusser and Suk, 1997), (Flusser and Suk, 1998) is the first significant contribution in this domain. They have developed new classes of radiometric descriptors invariant to blur degradations. The proposed descriptors are based on central moments. They have several application fields, such as the recognition of blurred images, the recognition and classification of 1-D degraded signals and the template matching on satellite image functions. Authors in (Stern et al., 2002) developed two new moment based methods for the recognition of motion blurred images.

In extension to previous work, Flusser and his research group (Flusser and Zitová, 1999), (Flusser et al., 2003), (Suk and Flusser, 2003) have developed new classes of combined invariant descriptors, that is descriptors that are simultaneously invariant to both geometric and radiometric degradations. The proposed descriptors are based on central and complex moments and are used in the recognition of affine transformed and blurred images, in template matching on blurred and rotated images, etc. In this line of thoughts, the authors in (Van Gool et al., 1996) propose a new class of combined affine radiometric invariants. These descriptors are used for the recognition of affine transformed and photometrically degraded gray level images. Authors in (Mindru et al., 1999) introduce the so called generalized color moments for the characterization of the multispectral nature of data in a limited area of the image. The proposed descriptors are used in the recognition of planar color patterns regardless of the viewpoint and illumination. A more detailed survey can be found in (Flusser, 2006) and (Flusser, 2007). In what follows we introduce our first class of radiometric features.

3 RADIOMETRIC INVARIANTS

In this section, we propose a new set of Mellin-transform based descriptors invariant simultaneously to uniform scaling, to contrast changes and to blur degradations that can be modelled by any convolution kernel h having a symmetric form with respect to the diagonals, i.e. $h(x,y) = h(y,x)$. Then, inspired by those invariant descriptors we introduce a new central-moment based descriptor which is simultaneously invariant to translations, to uniform scaling, to contrast changes and to convolution. Note that the symmetry constraint of the convolution kernel is not a severe limitation for the applicability of our invariant descriptors since the majority of convolution kernels used to model optical blur are symmetric with respect to the diagonals (e.g., Gaussian and pillbox filters (Chaudhuri and Rajagopalan, 1999)) as well as those used to approximate the Atmospheric Point Spread Function *APSF* which models the atmospheric veil on images (Metari and Deschênes, 2007a).

3.1 Radiometric Invariant based on the Mellin Transform

The Mellin integral transform of a function $f(x,y)$ is defined as (Zayed, 1996):

$$\mathcal{M}\,(f(x,y))(s,v) = \int_0^{+\infty}\int_0^{+\infty} x^{s-1}y^{v-1} f(x,y)dxdy, \tag{1}$$

with $s,v \in \mathbb{C}$. The idea behind the elaboration of our first radiometric invariant feature is based on two properties of the Mellin transform and Mellin convolution. The first one mentions that the Mellin convolution in $\mathbb{R}^+$ is equivalent to ordinary convolution in $\mathbb{R}$ (Korevaar, 2004): Let g, f and h be three functions defined and integrable on the reals, the ordinary convolution product of f with h is given by:

$$g(x) = (f * h)(x) = \int_{-\infty}^{+\infty} f(x-t)h(t)dt. \tag{2}$$

By carrying out the following change of variables

$x = \ln(x')$ and $t = \ln(t')$, we obtain:

$$\begin{aligned}
(g \circ \ln)(x') &= (f * h)(\ln(x')), \\
&= \int_0^{+\infty} f(\ln(x') - \ln(t'))h(\ln(t'))\frac{dt'}{t'}, \\
&= \int_0^{+\infty} (f \circ \ln)(\frac{x'}{t'})(h \circ \ln)(t')\frac{dt'}{t'}, \\
&= ((f \circ \ln) *_{Mel} (h \circ \ln))(x'), \qquad (3)
\end{aligned}$$

with $*_{Mel}$ denoting the Mellin convolution product. Thus, Mellin convolution in $\mathbb{R}^+$ is equivalent to ordinary convolution in $\mathbb{R}$.

The second one reveals that the Mellin transform of the Mellin convolution product of two functions is equal to the product of the Mellin transforms of these functions (Davies, 2002): Let f and h be two functions defined and integrable on the positive reals, the Mellin transform of the Mellin convolution product of f and h is given by:

$$\begin{aligned}
\mathcal{M}\,((f *_{Mel} h)(x))(s) &= \int_0^{+\infty} x^{s-1}\int_0^{+\infty} f(t)h(\frac{x}{t})\frac{dt}{t}dx, \\
&= \int_0^{+\infty} \frac{f(t)}{t}\int_0^{+\infty} x^{s-1}h(\frac{x}{t})dxdt, \\
&= \int_0^{+\infty} \frac{f(t)}{t}\int_0^{+\infty} (tx)^{s-1}h(x)tdxdt, \\
&= \int_0^{+\infty} t^{s-1}f(t)dt\int_0^{+\infty} x^{s-1}h(x)dx, \\
&= \mathcal{M}\,(f(x))(s)\mathcal{M}\,(h(x))(s). \qquad (4)
\end{aligned}$$

It results from the above mentioned properties the new relationship shown in equation (6). Let us consider the functions g, f and h related by the following relationship:

$$g(x,y) = f(x,y) * h(x,y), \qquad (5)$$

where $*$ is the ordinary convolution operator.

The Mellin transform of g can thus be obtained as follows:

$$\mathcal{M}\,(g) = \mathcal{M}\,(f * h) = \mathcal{M}\,(f)\mathcal{M}\,(h). \qquad (6)$$

Proof: From equation (3), we have:

$$(g \circ \ln)(x') = ((f \circ \ln) *_{Mel} (h \circ \ln))(x'). \qquad (7)$$

Applying the Mellin transform to equation (7), we obtain:

$$\begin{aligned}
\mathcal{M}\,((g \circ \ln)(x'))(s) &= \mathcal{M}\,(((f \circ \ln) *_{Mel} (h \circ \ln))(x'))(s), \\
\mathcal{M}\,(\underbrace{(g \circ \ln)(x')}_{g(x)})(s) &= \mathcal{M}\,(\underbrace{(f \circ \ln)(x')}_{f(x)})(s) \\
&\times\ \mathcal{M}\,(\underbrace{(h \circ \ln)(x')}_{h(x)})(s), \\
\mathcal{M}\,(g(x))(s) &= \mathcal{M}\,(f(x))(s)\mathcal{M}\,(h(x))(s). \qquad (8)
\end{aligned}$$

In addition to this, as we will show below, Mellin transform of order (s, v) of a symmetric filter with respect to the diagonals is equal to its Mellin transform of order (v, s). The theorem and the proof of the proposed invariant descriptor are thus given by:

Theorem 1: The radiometric invariant descriptor $k^f(s, v)$ of the image function $f(x, y)$ is defined as:

$$k^f(s,v) = \frac{\mathcal{M}\,(f(x,y))(s,v)}{\mathcal{M}\,(f(x,y))(v,s)}, \quad s,v \in \mathbb{N}. \qquad (9)$$

The proposed descriptor $k^f(s, v)$ is invariant to uniform scaling, to contrast changes and to convolution with any kernel h having a symmetric form with respect to the diagonals, i.e. $k^{\eta(f*h)(rx,ry))}(s,v) = k^{f(x,y)}(s,v)$, with η is a positive constant number and r denotes the uniform scaling factor.

3.1.1 Invariance to Convolution

In the case of a shift invariant imaging system the acquired image $g(x, y)$ is the result of the convolution product of a clear image $f(x, y)$ with a kernel $h(x, y)$ where models the Point Spread function *PSF* of the imaging system (cf. equation (5)). Applying the proposed invariant $k(s, v)$ to equation (5) and using the Mellin transform property (equation (6)), we obtain:

$$\begin{aligned}
k^g(s,v) &= \frac{\mathcal{M}\,(g(x,y))(s,v)}{\mathcal{M}\,(g(x,y))(v,s)}, \\
&= \frac{\mathcal{M}\,(f(x,y))(s,v)\mathcal{M}\,(h(x,y))(s,v)}{\mathcal{M}\,(f(x,y))(v,s)\mathcal{M}\,(h(x,y))(v,s)},
\end{aligned}$$

the ratio $\frac{\mathcal{M}\,(h(x,y))(s,v)}{\mathcal{M}\,(h(x,y))(v,s)} = 1$ according to the following proof:

$$\mathcal{M}\,(h(x,y))(s,v) = \int_0^{+\infty}\int_0^{+\infty} x^{s-1}y^{v-1}h(x,y)dxdy,$$

by making the following change of variables $x = y$ and $y = x$, we obtain:

$$= \int_0^{+\infty}\int_0^{+\infty} \underbrace{\left|\frac{\partial(x,y)}{\partial(y,x)}\right|}_{=1} y^{s-1}x^{v-1}h(y,x)dydx,$$

where $\left|\frac{\partial(x,y)}{\partial(y,x)}\right|$ denotes the jacobian of the transformation. If we suppose that h is a symmetric kernel with respect to the diagonals, i.e. $h(x,y) = h(y,x)$ then we have:

$$\begin{aligned}\mathcal{M}\,(h(x,y))(s,v) &= \int_0^{+\infty}\int_0^{+\infty} x^{v-1}y^{s-1}h(x,y)dxdy, \\ &= \mathcal{M}\,(h(x,y))(v,s). \qquad (10)\end{aligned}$$

Thus,

$$k^g(s,v) = \frac{\mathcal{M}\,(f(x,y))(s,v)}{\mathcal{M}\,(f(x,y))(v,s)} = k^f(s,v). \qquad (11)$$

3.1.2 Invariance to Uniform Scaling and to Contrast Changes

Following a global contrast change and a uniform scaling of the image function $f(x,y)$, the Mellin transform $\mathcal{M}\,(f'(u,w))$ of the resulting image function $f'(u,w)$ is related to Mellin transform $\mathcal{M}\,(f(x,y))$ of the original image function $f(x,y)$ by what follows:

$$\begin{aligned}\mathcal{M}\,(f'(u,w)) &= \int_0^{+\infty}\int_0^{+\infty} u^{s-1}w^{v-1}f'(u,w)dudw, \\ &= \int_0^{+\infty}\int_0^{+\infty} (rx)^{s-1}(ry)^{v-1}\eta f(x,y)rdxrdy, \\ &= \eta r^{s+v}\mathcal{M}\,(f(x,y)), \qquad (12)\end{aligned}$$

with η a positive constant number and r the uniform scaling factor. Applying $k(s,v)$ to a function $f'(u,w)$, we obtain:

$$\begin{aligned}k^{f'(u,w)}(s,v) &= \frac{\eta r^{s+v}\mathcal{M}\,(f(x,y))(s,v)}{\eta r^{v+s}\mathcal{M}\,(f(x,y))(v,s)}, \\ &= k^{f(x,y)}(s,v). \qquad (13)\end{aligned}$$

Thanks to the above proofs we can conclude that the proposed feature is simultaneously invariant to uniform scaling, to contrast changes and to convolution with any diagonal-symmetric kernel.

3.2 Invariant Descriptor based on Central Moments

Inspired by previous invariants, let us now propose a new set of radiometric invariants based on central moments. Specifically, if we replace each occurrence of the Mellin transform of order (s, v) by one central moment of order (s+v) in equation (9), we obtain a new set of image descriptors invariant simultaneously to vertical and horizontal translations, to uniform scaling, to contrast changes as well as to blur degradations that can be modelled by any kernel having a symmetric form with respect to the diagonals.

Theorem 2: The descriptor $P^f(s,v)$ is given by:

$$P^f(s,v) = \frac{\mu_{sv}^{f(x,y)}}{\mu_{vs}^{f(x,y)}}, \quad x,y \in \mathbb{R}^+, \qquad (14)$$

where $s,v \in \mathbb{N}$, $\mu_{sv}^{f(x,y)}$ is the $(s+v)th$ order central moment of a function $f(x,y)$ and is defined as:

$$\mu_{sv}^{f(x,y)} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(x-\overline{x})^s(y-\overline{y})^v f(x,y)dxdy, \qquad (15)$$

with $\overline{x} = m_{10}^f/m_{00}^f, \overline{y} = m_{01}^f/m_{00}^f$, representing the coordinates of the mass center of the function $f(x,y)$. The functional m_{sv}^f is the geometric moment of order $(s+v)$ of the image function $f(x,y)$ and is given by:

$$m_{sv}^f = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} x^s y^v f(x,y)dxdy,\ s,v \in \mathbb{N}. \qquad (16)$$

Since the descriptor $P(s,v)$ is based on central moments, it is trivial to prove that it is invariant to both horizontal and vertical translations. The proofs of the descriptor invariance with respect to uniform scaling, contrast changes and to convolution are identical to the ones of the descriptor $k(s,v)$. Note that without loss of generality we suppose that the gravity center of the image function coincides with the origin $(0,0)$.

4 COMBINED RADIOMETRIC GEOMETRIC INVARIANTS

In this section, we introduce a new class of combined invariant features. We start by introducing the combined invariant based on central moments.

4.1 Combined Descriptor based on Central Moments

In this subsection, we propose a new set of central-moment based descriptors invariant simultaneously to horizontal and vertical translations, to uniform and anisotropic scaling, to stretching, to contrast changes and to convolution with any diagonal-symmetric kernel.

Theorem 3: The combined descriptor $B(s,v)$ is given by:

$$B^f(s,v) = \frac{\mu_{sv}^{f(x,y)}}{\mu_{vs}^{f(x,y)}} \times \frac{\mu_{v+n,s+n}^{f(x,y)}}{\mu_{s+n,v+n}^{f(x,y)}}, \quad x,y \in \mathbb{R}^+, \qquad (17)$$

with $s,v \in \mathbb{N}$ and $n \in \mathbb{N}^*$. One can easily notice that $B^f(s,v)$ is a combination of the radiometric invariant descriptor $P^f(s,v)$ for different orders (s,v), i.e.

$B^f(s,v) = P^f(s,v) \times P^f(v+n, s+n)$. We chose to build the combined descriptor $B^f(s,v)$ based on the radiometric descriptor $P^f(s,v)$ because of its invariance to horizontal and vertical translations, to uniform scaling, to contrast changes as well as to convolution. In what follows, we show the invariance of $B^f(s,v)$ to anisotropic scaling and to stretching.

4.1.1 Anisotropic Scaling Invariance

Anisotropic scaling is given by (Jahne, 2005):

$$\begin{cases} u = rx, \\ w = ty, \end{cases}$$

with r and t the directional scale factors. Following an anisotropic scaling of the function $f(x,y)$, $\mu_{sv}^{f'(u,w)}$ is related to $\mu_{sv}^{f(x,y)}$ by what follows:

$$\begin{aligned} \mu_{sv}^{f'(u,w)} &= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} u^s w^v f'(u,w) du dw, \\ &= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} (rx)^s (ty)^v f(x,y) r dx t dy, \\ &= r^{s+1} t^{v+1} \mu_{sv}^{f(x,y)}. \end{aligned} \tag{18}$$

Applying the combined invariant $B(s,v)$ to a function $f'(u,w)$, we obtain:

$$\begin{aligned} B^{f'(u,w)}(s,v) &= \frac{r^{s+1} t^{v+1} \mu_{sv}^{f(x,y)}}{r^{v+1} t^{s+1} \mu_{vs}^{f(x,y)}} \frac{r^{v+n+1} t^{s+n+1} \mu_{v+n,s+n}^{f(x,y)}}{r^{s+n+1} t^{v+n+1} \mu_{s+n,v+n}^{f(x,y)}}, \\ &= B^{f(x,y)}(s,v). \end{aligned} \tag{19}$$

The above proof allows us to conclude that the combined feature is invariant to anisotropic scaling.

4.1.2 Stretching Invariance

To prove the invariance of $B^f(s,v)$ to stretching, we just have to replace the factor t by $\frac{1}{r}$ in the above proof (cf. equations (18) and (19)).

All of these proofs allow us to confirm the invariance of our combined descriptors $B(s,v)$ to horizontal and vertical translations, to uniform and anisotropic scaling, to stretching, to contrast changes and to convolution.

4.2 Combined Feature based on Central Complex Moments

In this section, we introduce a new class of combined features based on central complex moments. The proposed features are invariant simultaneously to similarity transformation and to contrast changes. Note that the similarity transformation includes horizontal and vertical translations, rotation and uniform scaling (Zisserman and Hartley, 2003). We start by giving some mathematical definitions.

Central Complex Moment: The central complex moment κ_{pq}^f of order (p+q) of the function $f(x,y)$ is defined as:

$$\begin{aligned} \kappa_{pq}^f &= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} ((x-x_c)+i(y-y_c))^p \\ &\times \quad ((x-x_c)-i(y-y_c))^q f(x,y) dx dy, \end{aligned} \tag{20}$$

with $p,q \in \mathbb{N}$, (x_c, y_c) the coordinates of the centroid of the image function $f(x,y)$ and i the imaginary unit. In polar coordinates, the central complex moment κ_{pq}^f is given by:

$$\kappa_{pq}^f = \int_0^{+\infty}\int_0^{2\pi} r^{p+q+1} e^{i(p-q)\theta} f(r,\theta) d\theta dr. \tag{21}$$

Theorem 4: The invariant descriptor $Z^f(p,q)$ of the image function $f(x,y)$ is given by:

$$Z^f(p,q) = \frac{\kappa_{pq}^{f(x,y)}}{\kappa_{qp}^{f(x,y)}} \times \frac{\kappa_{q+n,p+n}^{f(x,y)}}{\kappa_{p+n,q+n}^{f(x,y)}}, \quad n \in \mathbb{N}^*. \tag{22}$$

Proofs of the invariance of $Z(p,q)$ to the above mentioned geometric-radiometric degradations are given in what follows.

4.2.1 Translation Invariance

The invariance of the feature $Z(p,q)$ to horizontal and vertical translations is trivial to prove as the central complex moment is invariant to translations by definition.

4.2.2 Rotation - Contrast Change Invariance

Let g be a rotated (around the origin) and contrast changed version of the image function f, i.e. $g(r,\theta) = \eta f(r, \theta+\alpha)$ where α is the angle of rotation and η is a positive constant number. The central complex moments κ_{pq}^g of the image function g is related to the central complex moment κ_{pq}^f of the original image function f by the following equation:

$$\kappa_{pq}^g = \eta e^{-i(p-q)\alpha} \kappa_{pq}^f. \tag{23}$$

Applying $Z(p,q)$ to the image function $g(x,y)$, we obtain:

$$\begin{aligned} Z^g(p,q) &= \frac{\eta e^{-i(p-q)\alpha}\kappa^f_{pq}}{\eta e^{-i(q-p)\alpha}\kappa^f_{qp}} \times \frac{\eta e^{-i(q-p)\alpha}\kappa^f_{q+n,p+n}}{\eta e^{-i(p-q)\alpha}\kappa^f_{p+n,q+n}}, \\ &= \frac{\kappa^f_{pq}}{\kappa^f_{qp}} \times \frac{\kappa^f_{q+n,p+n}}{\kappa^f_{p+n,q+n}} = Z^f(p,q). \qquad (24) \end{aligned}$$

4.2.3 Uniform Scaling Invariance

Let g be a scaled version of the original image function f, i.e. $g(r,\theta) = f(\beta r,\theta)$, where β is the parameter of the uniform scaling. The central complex moment κ^g_{pq} is related to κ^f_{pq} by the following equation:

$$\kappa^g_{pq} = \beta^{-(p+q+2)}\kappa^f_{pq}. \qquad (25)$$

Applying the descriptor $Z(p,q)$ to the image function g, we obtain:

$$\begin{aligned} Z^g(p,q) &= \frac{\beta^{-(p+q+2)}\kappa^f_{pq}}{\beta^{-(q+p+2)}\kappa^f_{qp}} \times \frac{\beta^{-(q+p+2(n+1))}\kappa^f_{q+n,p+n}}{\beta^{-(p+q+2(n+1))}\kappa^f_{p+n,q+n}}, \\ &= \frac{\kappa^f_{pq}}{\kappa^f_{qp}} \times \frac{\kappa^f_{q+n,p+n}}{\kappa^f_{p+n,q+n}} = Z^f(p,q). \qquad (26) \end{aligned}$$

According to the above proofs, we conclude that the proposed feature $Z(p,q)$ is invariant simultaneously to similarity transformation (translations, rotation, uniform scaling) and to contrast changes.

5 EXPERIMENTAL RESULTS

In this section, we carry out a number of tests in order to express the discrimination power of our invariant descriptors. We start by applying the invariant based on the Mellin transform. Then, we show results of the radiometric descriptor based on central moments. Finally, we show the experimental results related to the combined invariants.

5.1 Mellin-Transform based Descriptor

To validate our invariant descriptor in practical situations we carried out tests on real images. Figure 1 shows real images (a,c-f) which were taken with a digital camera (Canon 1D professional) with different parameter settings. The radiometric invariant feature $k(s,v)$ is applied to real images in Figure 1. Experimental results are given in Table 1.

Results in Table 1 show that for any order (s,v) the numerical values obtained by applying the invariant

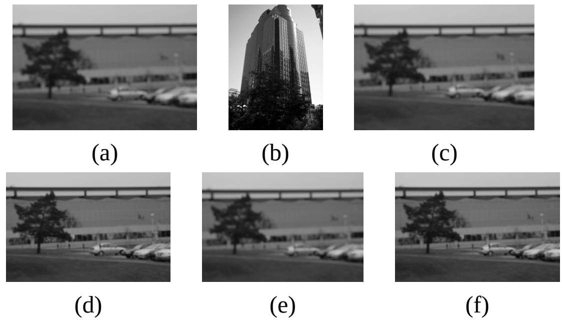


Figure 1: a, c-f: real images of the same scene with different blur degrees. b- foreign image.

descriptor $k(s,v)$ to images (a) and (c-f) are almost identical but are different from the numerical value obtained with image (b). Obtained results confirm the invariance of the descriptor $k(s,v)$ to both convolution and contrast changes.

Table 1: Results of the application of the radiometric invariant descriptor $k(s,v)$ to real images of Figure 1.

	$k(6,7)$	$k(4,12)$	$k(7,2)$	$k(10,6)$
Image a	1.55926	32.92205	0.09745	0.17464
Image b	**0.87445**	**0.277227**	**1.41440**	**1.92620**
Image c	1.55775	32.62386	0.09713	0.17579
Image d	1.55791	32.70831	0.09841	0.17529
Image e	1.55093	31.83843	0.10427	0.17771
Image f	1.54944	31.62592	0.10479	0.17840

With an aim of comparing our invariant feature to the state of the art and to the most related feature, we carried out tests on images using the radiometric feature $C(s,v)$ developed by Flusser and Suk in (Flusser and Suk, 1998). Results given in Table 2 provide an example of those tests using images in Figure 1. We may observe that for any order (s,v) the distance between the invariant descriptors of two degraded images of the same scene is smaller with the proposed feature $k(s,v)$ than the ones provided by $C(s,v)$. For instance, let us consider the order $(6,7)$. Following a data normalization, i.e., by dividing all descriptor values (related to the original image and to its degraded versions) by the descriptor value of the original image, the standard deviation of our results is equal to 0.0029, while the one obtained with $C(s,v)$ is equal to 0.0523. Computing times (relative to image (a) of Figure 1) of our invariant feature is equal to 0.047 s while the one with $C(s,v)$ is equal to 31.078 s. Note that computing time is evaluated in seconds. We hence notice that, for any order (s,v), computing time of our invariant feature is quite low. Finally notice that $k(s,v)$ can be computed for any values of (s,v), including even values.

Table 2: Results obtained using Flusser and Suk feature.

	$C^f(6,7)$ $[10^{-44}]$	$C(4,12)$	$C^f(7,2)$ $[10^{-31}]$	$C(10,6)$
Image a	-7.7130	0 (n/a)	5.7757	0 (n/a)
Image b	**4367.96**	**0 (n/a)**	**155.93**	**0 (n/a)**
Image c	-7.0798	0 (n/a)	5.7263	0 (n/a)
Image d	-6.9568	0 (n/a)	5.5082	0 (n/a)
Image e	-7.7330	0 (n/a)	5.7717	0 (n/a)
Image f	-7.1183	0 (n/a)	5.5061	0 (n/a)

5.2 Radiometric Descriptors based on Central Moments

In what follows, we apply the invariant descriptor $P(s,v)$ to images in Figure 2.

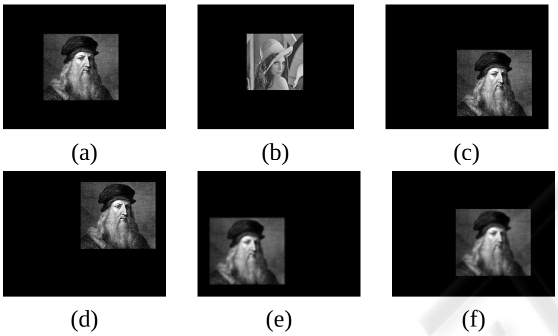


Figure 2: First row: a- original image, b- foreign image (lena), c- contrast change of translated image. Second row: d- translated images, e- translated and blurred image (filter size $= 7 \times 7$), f- contrast change of translated and blurred image.

From Table 3, we notice that the numerical values of the invariant descriptor $P(s,v)$ relative to images (a) and (c-f) are almost identical but are different from the one of image (b) as expected. Obtained results confirm the invariance of the descriptor $P(s,v)$ to contrast changes, to blur degradations as well as to horizontal and vertical translations.

Table 3: Results obtained by applying the invariant descriptor $P(s,v)$ to images of Figure 2.

	$P(4,3)$	$P(11,1)$	$P(9,2)$	$P(7,2)$
Image a	0.01999	-0.40269	2.76839	6.74357
Image b	**-3.30720**	**0.30403**	**-0.40323**	**-0.36359**
Image c	0.01706	-0.40277	2.77025	6.87348
Image d	0.01999	-0.40269	2.76839	6.74357
Image e	0.02040	-0.39920	2.81426	6.89165
Image f	0.01868	-0.40375	2.76167	6.80592

5.3 Combined Descriptor based on Central Moments

In this subsection, we apply the combined invariant $B(s,v)$ to images in Figure 3. Note that Image (f) includes at the same time, vertical and horizontal translations, stretching, uniform scaling, contrast change and blur degradation. Experimental results are given in Table 4.

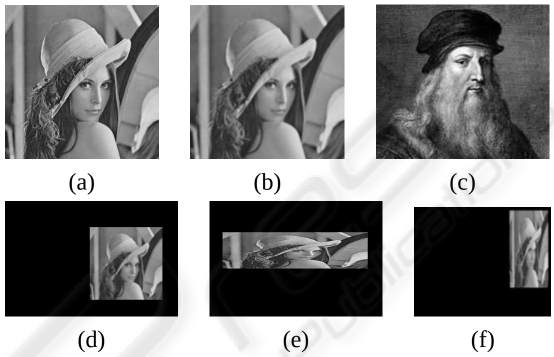


Figure 3: a- clear image, b- resized and blurred image, c- foreign image, d- translated and blurred image, e- stretched and translated image, f- contrast change of geometric degraded and blurred image (filter size $= 11 \times 11$).

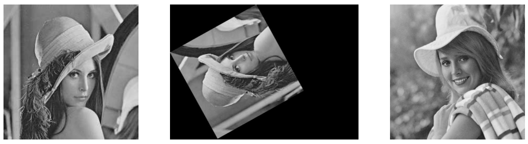
According to Table 4, we notice that for any order (s,v), the numerical values of the original image (Figure 3.a) and its degraded versions (Figure 3.b,d-f) are almost identical but are different from the one of the foreign image (Figure 3.c). The combined invariant descriptor $B(s,v)$ was tested for several orders and the obtained results express its invariance to the above mentioned radiometric and geometric degradations.

Table 4: Experimental results of the application of the combined invariant $B(s,v)$ to images of Figure 3.

	$B(2,3)$	$B(3,6)$	$B(8,5)$	$B(5,15)$
Image a	9.8918	0.0830	12.021	-0.3110
Image b	9.8666	0.0835	11.936	-0.3001
Image c	**-0.0007**	**109.26**	**0.0419**	**3.2301**
Image d	9.8277	0.0841	11.815	-0.2860
Image e	10.379	0.0782	12.725	-0.3207
Image f	9.7072	0.0816	12.626	-0.3204

5.4 Invariant Descriptor based on Central Complex Moments

In this subsection we evaluate the discrimination power of the descriptor based on central complex moments. For this purpose we apply $Z(s,v)$ to an image which includes at the same time vertical and horizontal translations, rotation, uniform scaling and contrast change (cf. Figure 4.b). Note that the invariant feature



(a) (b) (c)

Figure 4: a- original image, b- contrast change of similarity transformed image, c- foreign image.

values shown in Table 5 correspond to the multiplication of the real and imaginary parts of $Z(s,v)$. This representation was chosen since it has experimentally proven to provide a high discrimination power. It is obviously not unique, other representations can also be used. As can be seen from the table, for all of the orders, the values of $Z(s,v)$ related to images (a) and (b) are almost identical but are different from the one related to the foreign image (c), as expected.

Table 5: Results of the application of the combined invariant $Z(s,v)$ to images of Figure 4.

	$Z_{2,11}$	$Z_{3,7}$	$Z_{6,5}$	$Z_{8,5}$
Image a	0.04807	0.00905	0.00691	0.04281
Image b	0.05276	0.00944	0.00645	0.03912
Image c	**-0.02320**	**-0.00229**	**-0.00105**	**0.15769**

Obtained results express the invariance of $Z(s,v)$ to similarity transformation and to contrast changes.

Note that interested readers can find in (Metari and Deschênes, 2007b) a detailed comparisons made between the three above mentioned descriptors and the most widely used descriptors in the literature.

6 CONCLUSIONS

In this paper, we introduced two new classes of invariant features. The first class includes two radiometric features. The first one is based on the Mellin transform and the second one is based on central moments. Both radiometric descriptors are invariant to radiometric degradations that can be modelled by convolution as well as to global contrast changes and uniform scaling. The second class includes two combined invariant features. The combined descriptors based on central moments is invariant simultaneously to all of the following transformations : both horizontal and vertical translations, uniform and anisotropic scaling, stretching, contrast changes and degradations that can be modelled by convolution. The combined features based on central complex moments are invariant simultaneously to similarity transformation (translations, rotation, uniform scaling) and to contrast changes. These invariant features have been validated experimentally. All of the results confirm that they provide a high discrimination power at a low computing cost. They can be used for the recognition and classification of geometrically and/or radiometrically degraded images.

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REFERENCES

Belkasim, S. O., Shridhar, M., and Ahmadi, M. (1991). Pattern recognition with moment invariants - a comparative study and new results. *Pattern Recognition*, 24(12):1117–1138.

Chaudhuri, S. and Rajagopalan, A. N. (1999). *Depth from Defocus: A Real Aperture Imaging Approach*. Springer Verlag.

Davies, B. (2002). *Integral Transforms and their Applications - Third edition*. Springer.

Flusser, J. (2006). Moment invariants in image analysis. *In Proceedings of the International Conference on Computer Science. ICCS 2006*, pages 196–201.

Flusser, J. (2007). Moments and Moment Invariants in Image Analysis. *In Proceedings of the International Conference on Image Processing (to appear)*. San Antonio, Texas, USA. September 16-19.

Flusser, J., Boldya, J., and Zitová, B. (2003). Moment forms invariant to rotation and blur in arbitrary number of dimensions. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 25(2):234–246.

Flusser, J. and Suk, T. (1993). Pattern Recognition by Affine Moment Invariants. *Pattern Recognition*, 26(1):167–174.

Flusser, J. and Suk, T. (1997). Classification of degraded signals by the method of invariants. *Signal Processing*, 60(2):243–2492.

Flusser, J. and Suk, T. (1998). Degraded image analysis: An invariant approach. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 20(6):590–603.

Flusser, J., Suk, T., and Saic, S. (1995). Image feature invariant with respect to blur. *Pattern Recognition*, 28(11):1723–1732.

Flusser, J. and Zitová, B. (1999). Combined invariants to linear filtering and rotation. *International Journal of Pattern Recognition and Artificial Intelligence*, 13(8):1123–1136.

Hu, M. K. (1962). Visual pattern recognition by moment invariants. *Institute of Radio Engineers. Transactions. Information Theory*, 8(2):179–187.

Jahne, B. (2005). *Digital Image Processing: Concepts, Algorithms, and Scientific Applications*. Springer.

Katsaggelos, A. K. (1991). *Digital Image Restoration*. Springer-Verlag, New York.

Korevaar, J. (2004). *Tauberian Theory - A Century of Developments*. Springer - Verlag.

Metari, S. and Deschênes, F. (2007a). A New Convolution Kernel for Atmospheric Point Spread Function Applied to Computer Vision. *In proceedings of the Eleventh IEEE International Conference on Computer Vision*. Rio de Janeiro, Brazil, October 14-20.

Metari, S. and Deschênes, F. (2007b). New Classes of Radiometric and Combined Invariants inspired by Mellin Transform. *Technical Report no. 20, 30 pages, Université de Sherbrooke, Sherbrooke, Québec, Canada*.

Mindru, F., Moons, T., and Gool, L. V. (1999). Recognizing color patterns irrespective of viewpoint and illumination. *In proceedings of IEEE Computer Vision Pattern Recognition*, 1:368–373.

Reiss, T. H. (1991). The revised Fundamental Theorem of Moment Invariants. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 13(8):830–834.

Stern, A., Kruchakov, I., Yoavi, E., and Kopeika, S. (2002). Recognition of motion-blurred images by use of the method of moments. *Applied Optics*, 41(11):2164–2171.

Suk, T. and Flusser, J. (2003). Combined blur and affine moment invariants and their use in pattern recognition. *Pattern Recognition*, 36(12):2895–2907.

Teh, C. H. and Chin, R. T. (1988). On image analysis by the method of moments. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 10(4):496–513.

Van Gool, L., Moons, T., and Ungureanu, D. (1996). Affine/photometric invariants for planar intensity patterns. *In proceedings of the 4th European Conference on Computer Vision*, 1:642–651.

Zayed, A. I. (1996). *Handbook of Function and Generalized Function Transformations*. CRC Pr I Llc.

Zisserman, A. and Hartley, R. I. (2003). *Multiple View Geometry in Computer Vision*. Cambridge University Press.