Keywords: CDMA, Location, RFID Transponder.

Abstract: This paper describes a location system based on RFID technology. This system can be used to map the image of an endoscope. Compared with other endoscope location systems, this is a wireless system. The transponder mounted along the shaft of an endoscope does not increase the size of the endoscope or affect the flexibility of endoscope. We build the mathematical model of this location system and verify this model by experiments. Although experiments are for a transponder moving along one dimension, we can develop a three-dimensional system based on our results.

1 INTRODUCTION

The magnetic endoscope imager (MEI) system was developed independently by (Bladen, 1993) and (Williams, 1993). The physical principles of the three-dimensional location system are straightforward. The image of the endoscope is mapped in three dimensions using a set of coils that are installed in the instrument along the shaft. Coils measure the rate of change of the magnetic field flux according to Faraday’s law of induction: the change in magnetic flux through the coil produces a voltage in the coil circuit. Since the voltage depends uniquely on the location (x-y-z) and orientation (θ-φ) of coil, it is possible to get the location and orientation information of each coil. This information is collected several times each second, and is used to map the image of the endoscope.

The dimensions of the coil are critical to sensitivity (and to some extent to location accuracy). The larger is the coil, the greater is the sensitivity. However to fit the physical size of an endoscope, a typical size of coil is 1 cm long along the axis of instrument. Low frequency fields render the body transparent and low field strengths ensure safety (Bladen, 1993).

One advantage of a magnetic imager system is the elimination of the need to pass a separate imaging catheter during procedures, thereby freeing the accessory channel for suctioning, lavage, and therapeutic manoeuvres (Friedland, 2002). The other advantage is that the loops in the path of endoscope become clear in the MEI system (Bladen, 1993) compared with a conventional radiological imager system.

Research work by (Bladen, 1993), (Wehrmann, 2002), (Shah, 2002) and (Geng, 2004) proves that MEI is a reliable and accurate method. The systems studied in their work are cabled systems. Voltage signal are communicated by leads connected to coils. The more coils that are used, the more leads are mounted along the shaft of instrument. The leads increase the diameter and affect the flexibility of the instrument. In this paper, we describe a wireless location system which can be a replacement of the previous cabled system. In our system, we replace the coils with RFID (Radio Frequency Identification) transponders which load the read circuit in a changing pattern over time and identify each transponder by ID information stored in it.

2 RFID & CDMA BACKGROUND

We identify each transponder by ID information stored in it. Here, we use spread-spectrum codes to program transponders. The following parts describe the characteristics of RFID and CDMA technology and explain the reasons why we use spread-spectrum codes to program RFID transponders in our system.

2.1 RFID Technology

RFID technology has existed for many decades. An RFID system consists of two components: RFID transponder and reader (Finkenzeller, 2003).
Normally the reader is connected to a computer and used to read and write data. The RFID transponder is programmed with a unique code and located on object to be identified. The RFID transponder can be active or passive (Weinstein, 2005). The built-in battery of the active transponder increases its size and limits its applications. In this paper, when we say RFID transponder, it is a passive transponder.

Passive transponders operate at different frequencies. As mentioned before, we chose a low frequency transponder in our system. The size of the passive transponder can be as small as several millimetres, such as TK5552 (Atmel, 2003) which is only 12 mm long with plastic housing. The length of TK5552 is almost the coil’s length used in (Bladen, 1993). The power in the electromagnetic field received from the reader is the only power used for the data transmission between reader and transponder (Finkenzeller, 2003).

The operating principle of the RFID transponder is transformer-type coupling between transponder and reader. When the transponder is in a magnetic field, the alternating current in the reader coil induces a current in the transponder’s antenna coil and this current is used to power the transponder electronics. ID information stored in the transponder is sent back to the reader by loading the transponder’s coil in a changing pattern over time, which affects the induced e.m.f. in the reader (Want, 2004). This process is called load modulation.

2.2 CDMA Technology

As described in Part 1, we find the image of the endoscope by locating transponders installed along the shaft of the endoscope. Our system is a wireless system, and all transponders share the same frequency resource, so the received signal is a combination signal from all transponders. To separate each transponder’s signal at the receiver, we program each transponder with a different spread-spectrum code. When the received signal goes through a corresponding correlator, each signal is separated individually. But this does not mean we can use an unlimited number of transponders in our system. MAI (Multiple Access Interference) is a factor which limits our system performance.

The other reason that we use spread-spectrum code is to increase SNR. We know the modulation method for RFID system is load modulation. Since the RFID transponder we used is very small (diameter of TK5552 is less than 3 mm) (Atmel, 2003), and it works at a long distance (up to 35 cm), the coupling of transponder coil and reader coil is very weak. This results in the voltage variation on the reader coil being very small. The location and orientation of each transponder are calculated by this voltage. To get accurate location information, we need to increase SNR. The definition of processing gain of a spread spectrum system tells us if the spread-spectrum code is N bits long, the processing gain is N (Proakis, 2001) and SNR in dB increases by

$$SNR_{improved} = 20 \log_{10} N$$ (1)

Now we can see, by combining RFID and CDMA technology, it is possible to replace wired coils in previous systems with RFID transponders.

3 LOCATION THEORY

To estimate each transponder’s location (x-y-z), three pairs of circular coils are used in this system. Each axis has a pair of parallel co-axial reader coils like Figure (3). We use two parallel reader coils instead of one reader coil on each axis for two reasons. One reason is to get powerful enough fields to activate the RFID transponder, and the other is to cancel the effects due to modulation index. Part 4.1.2 gives the explanations in details.

We start from a simple case: the transponder moves along the x direction. If we can locate a transponder which moves along the x-axis or moves along a trail parallel to the x-axis, it is possible for us to extend it to three dimensions by replication.

The detected voltage signal at the reader coil is an induced voltage which depends on the mutual inductance of the reader coil and transponder coil. To locate a transponder is to find out the mathematical relation of this induced voltage and distance between reader coils and transponder coil. This part aims to find out this relation.

3.1 Mutual Inductance of Coils

The Biot-Savart law tells us that the magnetic field flux density B at any point P of a circular coil can be calculated by (Stratton, 1941):

$$B_{axis} = \frac{\mu_0 I}{2\pi} \int \frac{r \times [K(k) \frac{(a^2 - r^2 - x^2)}{(a - r)^2 + x^2} b \times E(k)]}{[K(k) \frac{(a^2 - r^2 - x^2)}{(a - r)^2 + x^2} + K(k) \frac{(a^2 + r^2 + x^2)}{(a - r)^2 + x^2} b \times E(k)]}$$ (2)

$$B_{radius} = \frac{\mu_0 I}{2\pi} \int \frac{\times b \times [K(k) \frac{(a^2 - r^2 - x^2)}{(a - r)^2 + x^2} + K(k) \frac{(a^2 + r^2 + x^2)}{(a - r)^2 + x^2} b \times E(k)]}{[K(k) \frac{(a^2 - r^2 - x^2)}{(a - r)^2 + x^2} + K(k) \frac{(a^2 + r^2 + x^2)}{(a - r)^2 + x^2} b \times E(k)]}$$ (3)
\[ \mu_0 \] is the permeability constant of free space;  
I is the current in the coil;  
\[ \eta = \frac{1}{\sqrt{(a + r)^2 + x^2}} \]  
a is the radius of the reader coil;  
x is the distance to the reader coil;  
\[ r = \sqrt{y^2 + z^2} \]  
is the distance to axis of reader coil;  
\[ k = \frac{4ar}{\sqrt{(a + r)^2 + x^2}} \]  
K(k) and E(k) are complete elliptical integrals of the first and second kinds.  

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When a small single loop coil is in the field of this circular coil, we assume that small coil is situated in a uniform flux density B. The mutual inductance of two coils is  
\[ M = \frac{B_{\text{radius}} S \cos \alpha + B_{\text{axis}} S \sin \alpha}{I} \] \hspace{1cm} (4)  
S is the plane area bounded by the small coil;  
\[ \alpha \] is the angle between S and the x-axis.  
Equation (4) gives the relationship between mutual inductance and distance of two coils.

### 3.2 Induced Voltage

Assuming a transponder moving in the x direction, we only need the pair of parallel coaxial reader coils on the x-axis to determine the transponder’s x-location. Our RFID location system’s topology for a pair of reader coils is shown in Figure 2. It is a 3-coil coupling system. This means that the received signal at any reader coil is a combination of signal from transponder coil and signal from the other reader coil.

In the system, AC power source is used to drive two reader coils and generate magnetic fields to activate RFID transponder. It does not contribute to the received signal.

A real system’s arrangement looks like Figure 3.

\[ M_{10} \] is the mutual inductance of reader coil-1 and transponder;  
\[ M_{20} \] is the mutual inductance of reader coil-2 and transponder;  
\[ M_{12} \] is the mutual inductance between reader coils;  
x_1 \ is the distance of transponder to reader coil-1;  
x_2 \ is the distance of transponder to reader coil-2;  
2D is the distance between two reader coils.

Assume the complex current in the transponder is \( I_0 \), and \( I = I_0 e^{j\omega t} \). The induced complex voltages on reader coil-1 \( V_{10} \) and on reader coil-2 \( V_{20} \) are  
\[ V_{10} = j\omega M_{10} I_0 \] \hspace{1cm} (5)  
and  
\[ V_{20} = j\omega M_{20} I_0 \] \hspace{1cm} (6)  

Complex current \( I_{20} \) in reader coil-2 due to \( V_{20} \) is,  
\[ I_{20} = \frac{V_{20}}{Z_2} = \frac{j\omega M_{20} I_0}{Z_2} \] \hspace{1cm} (7)
where $Z_2$ is the total impedance of terminals 2a and 2b in Figure 4.

The induced voltage on reader coil-1 $V_{12}$ due to $I_{20}$ is

$$V_{12} = -\omega^2 M_{12} M_{20} \frac{I_0}{Z_2}$$

(8)

So the received complex voltage on reader coil-1 is

$$V_1 = j\omega M_{10} I_0 - \omega^2 M_{12} M_{20} \frac{I_0}{Z_2}$$

(9)

In the same way, we can find the received complex voltage on reader coil-2 is

$$V_2 = j\omega M_{20} I_0 - \omega^2 M_{12} M_{10} \frac{I_0}{Z_1}$$

(10)

where $Z_1$ is the total impedance of terminals 1a and 1b in Figure 4.

Two reader channels are symmetric, so we have

$$Z = Z_1 = Z_2$$

(11)

When the distance of two reader coils is fixed, the mutual inductance of two reader coils $M_{12}$ is a constant. We define

$$\lambda = \frac{\omega M_{12}}{Z}$$

(12)

Equation (9) and (10) are

$$V_1 = \omega I_0 (jM_{10} - \lambda M_{20})$$

(13)

and

$$V_2 = \omega I_0 (jM_{20} - \lambda M_{10})$$

(14)

When we only take the coupling data signal into account, the system in Figure 2 can be converted into Figure 4. There are two signal sources in system $V_1$ and $V_2$. Both of them contribute to the received signal at reader coils.

When the resonant frequency of two read channels is equal to the frequency of AC power source, the impedance of the read channel is resistive and $\lambda$ in Equation (12) is real. The received complex signal at reader coil-1 $V_{r1}$ is

$$V_{r1} = \omega^2 L_0 \left[ \frac{M_{20}}{\rho_2} - \frac{M_{10}}{\rho_1} \right] + j\lambda \left[ \frac{M_{10}}{\rho_2} - \frac{M_{20}}{\rho_1} \right]$$

(15)

where $\rho_1 = \frac{R^2 + 2R \rho}{R + \rho}$ and $\rho_2 = \frac{R^2 + 2R \rho}{\rho}$. 

![Figure 4: Simplified system topology.](image)

The amplitude of received signal on reader coil-1 $V_{r1}$ is

$$|V_{r1}| = \omega^2 L_0 \sqrt{\left( \frac{M_{20}}{\rho_2} - \frac{M_{10}}{\rho_1} \right)^2 + \lambda^2 \left( \frac{M_{10}}{\rho_2} - \frac{M_{20}}{\rho_1} \right)^2}$$

(16)

The amplitude of received signal at reader coil-2 $V_{r2}$ is similar to reader coil-1

$$|V_{r2}| = \omega^2 L_0 \sqrt{\left( \frac{M_{10}}{\rho_2} - \frac{M_{20}}{\rho_1} \right)^2 + \lambda^2 \left( \frac{M_{20}}{\rho_2} - \frac{M_{10}}{\rho_1} \right)^2}$$

(17)

The ratio of two received signal is

$$\frac{|V_{r1}|}{|V_{r2}|} = \sqrt{(\rho - M_{\text{ratio}})^2 + \lambda^2 (\rho M_{\text{ratio}} - 1)^2}$$

(18)

where $\rho = \frac{\rho_1}{\rho_2}$ and $M_{\text{ratio}} = \frac{M_{10}}{M_{20}}$.

### 3.3 Induced Voltage and Distance

Equation (2), (3) and (4) tell us when the transponder only moves in the x direction, M is a function of x and $\alpha$. Equation (4) is:

$$\frac{B_{\text{axis}} S \cos \alpha + B_{\text{radius}} S \sin \alpha}{I} = F(x, \alpha)$$

(19)
\[ M_{ratio} = \frac{F(x_1, \alpha)}{F(x_2, \alpha)} \]  

Equation (20)

\[ x_1 = D + x \text{ and } x_2 = D - x \]

\[ M_{ratio} = \frac{F(D + x, \alpha)}{F(D - x, \alpha)} = M(x, \alpha) \]  

Equation (21)

We define

\[ V_{ratio} = \frac{V_{r_1} - V_{r_2}}{V_{r_1} + V_{r_2}} \]

\[ = \frac{\sqrt{\rho - M(x, \alpha) + \alpha^2 [\rho M(x, \alpha) - 1]^2}}{\sqrt{\rho M(x, \alpha) - 1 + \alpha^2 [\rho - M(x, \alpha)]^2}} - 1 \]  

Equation (22)

The left of Equation (22) is \( V_{ratio} \), which is determined by the received signal at two reader coils. The right of Equation (22) is a function of transponder’s location \( x \). Equation (22) gives out the relation of received signal at two reader coils and the location of the transponder. We will explain why we use voltage ratio \( \frac{V_{r_1} - V_{r_2}}{V_{r_1} + V_{r_2}} \) to describe our system in Part 4.1.2.

4 EXPERIMENTS

In Part 3 we gave the mathematical description of the location system. The theory is validated by experimental measurements in this section and it will be shown that the system performs as predicted, even though some difficulties arise in implementation.

4.1 Problems in Implementation

4.1.1 Physical Size Effects and Calibration

In Part 2, we assume the RFID transponder is an ideal small point and the reader coils are filamentary. This means that the physical sizes of transponder and reader coils are not taken into account. But for a real system, this is not true. The physical centres of the read coils and the transponder are not the magnetic centres of the read coils and the transponder any more. To make sure the theory in Part 2 works, we should choose a transponder as small as possible. And we need to determine the effective magnetic centres of the transponder and two reader coils.

The magnetic centre can be determined by experiments. First, we estimate any point of a transponder as magnetic centre and put the transponder at any point of the axis in two opposite directions like Figure 5. If this point is the real magnetic centre of transponder, the voltage ratios for these two directions should be the same. Otherwise we need to adjust the estimated centre position until the two ratios are equal.

![Figure 5: Determination of the magnetic centre of reader coils and transponder coil.](image)

After we find the magnetic centre of the transponder, we can use the transponder to determine the original point of our location system. Moving the transponder along the axis of the reader coils, the point which satisfies the voltage ratio in Equation (22) equal to zero is the centre of the reader coils and is the original point of the location system as well.

4.1.2 Modulation Index

The transponder is powered by the magnetic fields, so the strength of the magnetic fields also affects the received signal. Figure 6 is our measured modulation index of the RFID transponder TK5552 used in our experiments. And Figure 6 agrees with TK5552’s data sheet.

![Figure 6: Modulation index versus field strength.](image)
It is clear that the modulation index is not a constant. This means that the received signal is not only dependent on the distance but also affected by the modulated index. This is why we use voltage ratio (between two parallel reader coils) instead of voltage itself to describe our location system in Part 3.

The non-constant modulation index means there is no determinate relation between voltage of received signal and transponder’s location. If we use a voltage ratio based system, modulation index effects are cancelled. In fact, when we use voltage ratio, the numbers of turns in the reader coils and transponder coil are cancelled as well.

In Part 3, we use voltage ratio \( \frac{V_{r1} - V_{r2}}{V_{r1} + V_{r2}} \) instead of \( \frac{V_{r1}}{V_{r2}} \) to describe our system. Figure 7 and Figure 8 are voltage ratio curves of \( \frac{V_{r1}}{V_{r2}} \) and \( \frac{V_{r1} - V_{r2}}{V_{r1} + V_{r2}} \) when the transponder moves along the x-axis. The signal received on one reader coil becomes small when the transponder is close to the other one. Due to the limited precision of the ADC used to capture data, there is an error in the received signal. From Figure 7, we can see that an error in a small received signal will result in a large error in \( \frac{V_{r1}}{V_{r2}} \). To reduce this quantisation error, we use \( \frac{V_{r1} - V_{r2}}{V_{r1} + V_{r2}} \) to derive position as shown in Figure 8.

### 4.1.3 Constant \( \lambda \) and \( \rho \)

In Equation (22), we define two constants \( \lambda \) and \( \rho \), once the system is built, the values of these are also determined. However it is difficult to determine their exact values by the definition in Part 3 for a real system. Fortunately Equation (22) tells us that \( \lambda \) and \( \rho \) can be estimated by voltage ratio and distance.

\[
[\lambda, \rho] = \mathcal{X}(V_{ratio}, x, \alpha) \tag{23}
\]

In Part 4.1 we have determined the original centre of the location system, so it is possible to measure a transponder’s location and angle. By putting a transponder at any point on the axis, we can get a set of voltage ratio and location data. Using two sets of voltage ratios and locations \( (V_{ratio_1}, x_1) \) and \( (V_{ratio_2}, x_2) \), we can estimate the values of \( \lambda \) and \( \rho \). To get a much more accurate calculation of \( \lambda \) and \( \rho \), we can use several sets of \( (V_{ratio}, x) \) at different points to estimate their values.

![Figure 7: Voltage ratio curve of Vr1/Vr2.](image1)

![Figure 8: Voltage ratio curve of (Vr1-Vr2)/(Vr1+Vr2).](image2)

### 4.2 Experiment Results

After solving the problems faced in a real system, we have constructed a system to verify our theory in Part 3. The parameters of the system are as follows:
- The number of reader coil turns \( N=20 \);
- The radius of reader coil \( a=24.5 \) cm;
- The separation of two reader coils\( = 35.6 \) cm;
- RFID transponder is TK5552;
- The constant \( \lambda = 0.046 \) and \( \rho = 0.13 \).

Our experiments measure a transponder moving along the x direction. To extend a one dimension system to three dimensions, we need to test the transponder moving not only along the x-axis, but also along trails parallel to the x-axis. We also need to take angle into account. Since three axes are orthogonal each other, we only measure the worst case: angle is 45 degrees. When the angle is beyond 45 degrees with one axis, it means the angle with another axis is smaller than 45 degrees.

Figures 9 to 11 are experimental results. Figure 9 shows voltage ratio curves for a transponder moving along the axis. Figure 10 and 11 are voltage ratio...
curves for a transponder moving along a trail parallel to the x-axis (y=-15cm, \(\alpha=0\)). Each figure has two curves. One is the voltage ratio from experiments, and the other one is calculated by Equation (22).

The results show that our mathematical equation in Part 3 can correctly describe our system. Figure 10 tells us that Equation (22) still works when the transponder is off axis. Figure 11 shows that the transponder still works when the angle is 45 degrees. Experimental results show that it is possible to develop a three-dimensional system based on our experiments, and this is the subject of ongoing research.

5 CONCLUSIONS

In this paper, we describe a location system based on RFID technology. Although our experiments are for a one-dimensional system, the results and analysis show it is possible to develop a three-dimensional system. Also spread-spectrum coding, which is used to program transponders, lets us track multiple transponders. Using this location system with multiple RFID transponders, we expect to be able to map the images of endoscopes without attaching extraneous wires.

REFERENCES


