GROUP DECISION SYSTEMS FOR RANKING AND SELECTION
An Application to the Accreditation of Doping Control Laboratories

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Abstract: This paper presents a qualitative approach for representing and synthesising evaluations given by a team of experts involved in selection or ranking processes. The paper aims at contributing to decision-making analysis in the context of group decision making. A methodology is given for selecting and ranking several alternatives in an accreditation process. Patterns or alternatives are evaluated by each expert in an ordinal scale. Qualitative orders of magnitude spaces are the frame in which these ordinal scales are represented. A representation for the different patterns by means of k-dimensional qualitative orders of magnitude labels is proposed, each of these standing for the conjunction of k labels corresponding to the evaluations considered. A method is given for ranking patterns based on comparing distances against a reference k-dimensional label. The proposed method is applied in a real case in External Quality Assessment Schemes (EQAS) for Doping Control Laboratory contexts.

1 INTRODUCTION

In group decision-making processes, the ranking or selection of patterns depends on the judgments given by the evaluators in the group. The group decision method presented in this work is specially suitable when aiming at an evaluation via qualitative ordinal descriptions.

This work is a step further in terms of previous works in the area of multi-criteria decision making (Agell et al, 2006; Keeney, 1993) when information is represented by absolute orders of magnitude labels, such as "very bad", "bad", "acceptable", "good" and "very good". The proposed method is based on the representation of the initial evaluators’ judgments of each pattern via a k-dimensional label, which can be seen as a hyper-rectangle, and its synthesis by means of the distance to a reference k-dimensional label. This in turn is based on a qualitative generalisation of a type of goal programming method known as the reference point method for vectorial optimisation and decision-making support (Gonzza, 2001; Kallio, 1980).

In general, reference point methods for optimisation in $\mathbb{R}^n$ choose the points at a shorter distance from a previously fixed reference point as the optimal alternative (the “goal” to be reached) (Romero, 2001; Romero et al, 2001; Wier, 1980). In this work, ranking in the set of existing patterns is performed by selecting not a predetermined fixed reference k-dimensional label, but a “realistic” reference for the problem to be solved: with respect to the natural order, the proposed reference hyper-rectangle is the supreme of the set of available patterns, guar-
antecising consistency with the order between hyperrectangles. The distances between patterns and their \( n \) give the ranking of patterns directly. Moreover, any problem of selection or accreditation of patterns can be solved by using this ranking together with a suitable threshold.

The proposed methodology may be of interest to very different areas. Specifically, applications in candidate assessments (students in learning processes, recruiting processes) as well as project management (architectural, civil engineering and business projects) can be considered. It may also be of interest in decision-making processes in areas such as finance and marketing.

The proposed method is applied in the evaluation of data from an External Quality Assessment Scheme (EQAS) of accredited doping control laboratories. Laboratories involved in doping control have to be accredited by both the World Anti-Doping Agency (WADA) and the national accreditation body following the ISO17025 quality standard (World Anti-Doping Agency, 2004). This regulatory system is justified, on the one hand, to insure the highest quality in analytical standards in benefit of the athlete, and, on the other hand, to harmonise quality standards in laboratories regularly involved in legal disputes concerning positive test results (i.e. a substance banned by sport authorities). WADA accreditation is granted after the evaluation of analytical data provided by laboratories by an experts’ committee based on the information given to each expert, the judgments of each evaluator and a further synthesis of these judgments.

Section 2 presents some features related to the qualitative models of absolute orders of magnitude, and Section 3 provides a qualitative representation of patterns in a partially-ordered set. Section 4 introduces the new group decision-making process by defining a total order in the set of patterns in such a way that the set of labels corresponding to the available alternatives becomes a chain (ranking). The consistency property for the ranking method is established. In Section 5, the application to accreditation of doping control laboratories is given. Lastly, conclusions and open problems are presented.

2 ABSOLUTE ORDERS OF MAGNITUDE MODELS

The one-dimensional absolute orders of magnitude model (Trave, 2003) works with a finite number of qualitative labels corresponding to an ordinal scale of measurement. The number of labels chosen to describe a real problem is not fixed, but depends on the characteristics of each represented variable.

Let’s consider an ordered finite set of basic labels \( S = \{B_1, \ldots, B_n\} \), each one of them corresponding to a linguistic term, in such a way that \( B_1 < \ldots < B_n \), as for instance “very bad” < “bad” < “acceptable” < “good” < “very good”.

The complete universe of description for the Orders of Magnitude Space \( OM(n) \) is the set \( S \):

\[
S = S_a \cup \{B_i, B_j | \forall i < j \},
\]

where the label \( [B_i, B_j] \) with \( i < j \) is defined as the set \( \{B_i, B_{i+1}, \ldots, B_j\} \).

The order in the set of basic labels \( S_a \) induces a partial order \( \leq \) in \( S \) defined as:

\[
[B_i, B_j] \leq [B_r, B_s] \iff B_i \leq B_r \text{ and } B_j \leq B_s.
\]

with the convention \( [B_i, B_j] = \{B_i\} = B_i \). This relation is trivially an order relation in \( S_a \), but a partial order, since there are pairs of non-comparable labels.

3 PATTERN REPRESENTATION IN A PARTIALLY ORDERED SET

In the proposed group decision-making problem, each pattern is characterized by the judgments of \( k \) evaluators, and these evaluations are given by means of qualitative labels belonging to an orders of magnitude space. So, each pattern is represented by a \( k \)-dimensional label, that is to say, a \( k \)-tuple of labels.

Let \( S \) be the orders of magnitude space with the set of basic labels \( S_a \).

The set of \( k \)-dimensional labels or patterns’ representations \( \mathbb{E} \) is defined as:

\[
\mathbb{E} = S \times \ldots \times S = \{(E_1, \ldots, E_k) | E_i \in S \forall i = 1, \ldots, k\}.
\]

Each \( k \)-dimensional label \( \mathbf{E} = (E_1, \ldots, E_k) \) is a set of \( k \) qualitative labels (each one associated to an expert judgment) that define a pattern in such a way that, on every component, the relation \( E_r \leq E_t \) means that \( E_r \) represents better results than \( E_t \).

This order in \( S \) is extended to the Cartesian product \( \mathbb{E} \):

\[
E = (E_1, \ldots, E_k) \leq E' = (E'_1, \ldots, E'_k) \iff E_i \leq E'_i, \forall i = 1, \ldots, k.
\]

\( E < E' \), that is to say, \( E \leq E' \) and \( E \neq E' \), means that pattern \( E' \) is better than pattern \( E \).

This order relation in \( \mathbb{E} \) is partial, since there are pairs of non-comparable \( k \)-dimensional labels.
4 THE GROUP DECISION-MAKING PROCESS

The proposed method for ranking or selection among the existing patterns consists in:

- Fixing a distance \( d \) in \( E \).
- Building a reference label \( \hat{E} \): a proposition of consistency determines that it has to be the supreme, with respect to the order \( \preceq \), of the set of patterns which are to be ranked.
- Assigning to each \( k \)-dimensional label \( E \) the value \( d(E, \hat{E}) \); so, the patterns will be totally ordered as a chain.
- This chain giving the ranking of the patterns directly. In an accreditation process, this chain, along with a threshold, solves the problem.
- After this process, if a subset of the patterns is at the same distance to \( \hat{E} \), the same algorithm being applied to this set, just beginning at the second point.

4.1 A Distance in the Pattern Representation Set

A method for computing distances between \( k \)-dimensional labels is presented.

The first step involves codifying the labels in \( S \) by a location function (Agell et al, 2006). Through this function, each element \( E_h = [B_i, B_j] \) in \( S \) is codified by a pair \( (l_1(E_h), l_2(E_h)) \) of integers: \( l_1(E_h) \) is the opposite of the number of basic elements in \( S_o \) that are between \( B_i \) and \( B_j \), that is to say, \( l_1(E_h) = -(i - 1) \), and \( l_2(E_h) \) is the number of basic elements in \( S_o \) that are between \( B_i \) and \( B_j \), i.e., \( l_2(E_h) = n - j \). This pair of numbers permits each element in \( S \), where all different levels of precision are considered, to be "located".

This "location" can be extended to any pattern defined by \( k \) orders of magnitude labels; the extension to the set of \( k \)-dimensional labels \( E \) is:

\[
L(E_1, \ldots, E_k) = (l_1(E_1), l_2(E_1), \ldots l_1(E_k), l_2(E_k))
\]

which provides the relative position of a \( k \)-tuple of qualitative labels with respect to the basis of \( E \).

Then, a distance \( d \) between labels \( E, E' \) in \( E \) is defined via any metric \( R \) in \( \mathbb{R}^{2k} \) and their codifications:

\[
d(E, E') = d((E_1, \ldots, E_k), (E'_1, \ldots, E'_k)) = \sqrt{(L(E) - L(E'))^T R (L(E) - L(E'))}.
\]

This function inherits all properties of the distance in \( \mathbb{R}^{2k} \).

4.2 Ranking of the Patterns

Starting from a distance \( d \) in \( E \) and a reference \( k \)-dimensional label \( \hat{E} \), a total order \( \preceq \) will be defined in \( E \), in such a way that the set of labels \( E^1, \ldots, E^l \) corresponding to the available patterns becomes a chain \( E^1 \preceq \cdots \preceq E^l \), and so a ranking of the patterns is established.

4.2.1 A Total Order in \( E \)

Let \( \hat{E} \in E \) be a \( k \)-dimensional label and let us call it the reference label. Let \( d \) be the distance defined in \( E \) in Section 4.1. Then the following binary relation in \( E \):

\[
E \preceq E' \iff d(E', \hat{E}) \leq d(E, \hat{E})
\]

is a pre-order, i.e., it is reflexive and transitive.

This pre-order relation establishes an equivalence relation \( \equiv \) in \( E \) by means of:

\[
E \equiv E' \iff [E \preceq E', E' \preceq E] \iff d(E', \hat{E}) = d(E, \hat{E}).
\]

Then, in the quotient set \( E/\equiv \) the following relation between equivalence classes:

\[
\text{class}(E) \preceq \text{class}(E') \iff E \preceq E' \iff d(E', \hat{E}) \leq d(E, \hat{E})
\]

is an order relation. It is trivially a total order.

In this way, given a set of patterns \( E^1, \ldots, E^l \), these can be ordered as a chain with respect to their proximity to the reference label: \( \text{class}(E^1) \preceq \cdots \preceq \text{class}(E^l) \).

If each class \( E^j \), \( j = 1, \ldots, l \), contains only the label \( E^j \), the process is finished and we obtain the ranking \( E^1 \preceq \cdots \preceq E^l \). If there is some class \( E^j \), with more than one label, then the same process is applied to the set of the labels belonging to class \( E^j \), and continued until the final ranking \( E^{j_1} \preceq \cdots \preceq E^{j_q} \) is obtained.
4.2.2 Consistency of the Ranking Method

The method of ranking via a distance to a reference label previously selected is really necessary when the order relation \( \leq \) does not provide a total order in the set of available patterns.

When the set \( \{E^1, \ldots, E^l\} \) is totally ordered with respect to \( \leq \), that is to say, when a priori the patterns are already ranked \( E^1 \leq \cdots \leq E^l \), then the proposed method (via choosing of a suitable reference label) has to reproduce the same ranking. This means that the method has to be consistent.

Formally, the method will be consistent if the reference label \( \bar{E} \) is selected in such a way that:

\[
(\forall E^1, \ldots, E^l \in \mathcal{E}) (E^1 \leq \cdots \leq E^l \implies \bar{E} \leq \cdots \leq \bar{E}^l) \quad (9)
\]

This requirement is equivalent to the following:

\[
(\forall E, E' \in \mathcal{E}) (E \leq E' \implies \bar{E} \leq \bar{E}') \quad (10)
\]

In effect, (9) \( \implies \) (10) is obvious. Reciprocally, if (10) is satisfied, then when \( E^1 \leq \cdots \leq E^l \) it suffices to apply (10) to each pair \( E^i \leq E^{i+1} \).

Before establishing the way of choosing the reference label \( \bar{E} \), in order for property of consistency to be accomplished, let us compute the supreme of a set of hyper-rectangles with respect to the partial order \( \leq \) introduced in Section 3.

Given any \( E^1, \ldots, E^l \), let \( E^{\sup} \) be the supreme of the set \( \{E^1, \ldots, E^l\} \), i.e., the minimum label in \( \mathcal{E} \) which satisfies \( E^i \leq E^{\sup}, i = 1, \ldots, l \).

Its computation is as follows:

Let \( E^i = (E^i_1, \ldots, E^i_l) \), with \( E^i_h = [B^i_h - B^i_{j_h}, B^i_{j_h}] \) for all \( h = 1, \ldots, k \), and for all \( r = 1, \ldots, l \). Then

\[ E^{\sup} = \sup\{E^1, \ldots, E^l\} = (\bar{E}_1, \ldots, \bar{E}_k), \]

where

\[ \bar{E}_h = [\max\{B^1_h, \ldots, B^l_h\}, \max\{B^1_{j_h}, \ldots, B^l_{j_h}\}] \quad (11) \]

(see Figure 2).

Figure 2: The supreme of a set \( \{E^1, \ldots, E^l\} \).

Proposition 3 (of consistency) The ranking method is consistent in the above sense if and only if, for any set of patterns \( E, E', E'' \), the reference label \( \bar{E} \) is chosen as the supreme of the set \( \{E^1, \ldots, E^l\} \).

\[ \text{Proof.} \quad \text{If the label associated to any set of labels is its supreme, statement (10) is trivial, because if } E \leq E' \text{ then } \bar{E} = \sup\{E, E'\} = \bar{E} \text{ and } d(E', \bar{E}) = 0 \leq d(E, \bar{E}), \text{ that is to say, } E \leq E'. \]

To prove that it is necessary to choose the supreme to assure the consistency, it suffices to present as a counterexample of (10) the case in which \( \bar{E} \) is not the supreme. The easiest of these consists of the pair \( E, E' \), with \( E \leq E' \) and \( \bar{E} = E \). It is clear that \( E \nleq E' \).

5 AN APPLICATION TO ACCREDITATION OF DOPING CONTROL LABORATORIES

There is considerable concern over the abuse of drugs by athletes trying to improve their performance. Doping in sport is becoming increasingly sophisticated and sampling methods and testing procedures vary from one country to another. The absence of harmonisation in this area has led to an increasing number of doping accusations being contested (Warmuth, 2002). Establishing an external quality assurance scheme for all Doping Control Laboratories is necessary to increase the legal weight behind drugs tests and reduce the number of positive drug tests being challenged in the courts (Donike, 1992). Experimental data evaluated in the present report were generated in the framework of the EU funded project ALADIN 2002 (Analytical Laboratories for Antidoping Control: International Network for External Quality Assessment), coordinated by the Institut Municipal d’Investigació Mèdica (IMIM) in Barcelona, in close collaboration with the IOC laboratories in London (United Kingdom), Cologne (Germany) and Oslo (Norway), that aims to develop the external quality assurance scheme that would meet these needs. The group decision system developed in this work is applied to summarise the opinion of experts and to order the laboratories involved in the study according to their quality.

5.1 Description of Data

Nine independent experts evaluated analytical data provided by laboratories when analyzing samples containing a banned substance (nandrolone) in several rounds of the EQAS spread in four consecutive years, according to their expertise. A total of 105 reports generated by laboratories, were evaluated. Evaluations of each analytical report were rated as follows: Unacceptable; Insufficient; Sufficient; Good; and Very Good. No intermediate scores were possible. All experts were management staff from four
European IOC/WADA accredited anti-doping control laboratories (King’s College London, United Kingdom; Hormone Laboratory in Oslo, Norway; German Sports University in Cologne, Germany; and Institut Municipal d’Investigación Médica in Barcelona, Spain). All of the experts have excellent skills in the analysis and evaluation of doping control samples and they have more than ten years’ experience in this analytical field.

Each laboratory has initially been considered as a 9-tuple of basic qualitative labels belonging to an OM(5), where the basic elements are: B1= Unacceptable, B2= Insufficient, B3= Sufficient, B4= Good and B5= Very Good. The reference point -the “optimal laboratory”- is chosen as the maximum basic label assigned to all laboratories for each expert. Then the distance, i.e., level of quality (LQ) from each pattern to the optimal is used to rank the laboratories in the sample. As shown in Section 4, LQ established a total order in the set of laboratories, where the smaller LQ is, the better the quality of the laboratory. As an example, seven of the patterns are shown in Table 1, in which distances to the optimal laboratory (LQ) and a refined order (RLQ) are included in the last columns.

As can be seen in Table 1, some patterns are equidistant to the “optimal laboratory”. To avoid coincidence between distances as much as possible, and to be able to discriminate laboratories, the group decision process is reapplied in each group with the same level of quality, by defining a new ad-hoc reference point. In that sense, a refined order of the initial set of patterns (RLQ) is obtained. These can be seen in the last column of Table 1 for the example considered.

It is important to notice that, although in this case experts have been asked to evaluate each laboratory with basic labels, this group decision methodology can be considered with evaluations including different levels of precision and even missing values.

5.2 Experiment and Results

According to the ratings assigned to the 105 reports by the team of experts, these have been first ordered by LQ values. The distribution of the results obtained is shown in Figure 3.

As it can be seen in Figure 3, there are 14 pairs of indistinguishable laboratories with respect to their distance to the "optimal laboratory", 7 groups of three laboratories with the same LQ, 4 groups with four laboratories equally ranked and 3 groups of five of them. As an example, it can be considered the case given in Table 1 by the laboratories coded as #64, #65, #55,#17,#70. These five laboratories have the same LQ equal to 5,4, so they form one of the three groups of five laboratories previously described. In a second step, the 28 groups in which laboratories are not distinguished are internally re-ranked and scored with the RLQ value. Following the former example, Table 1 shows the new values allowing a better distinction in terms of their position to the "optimal laboratory". This second step produces a final order, making it possible to rank almost all the laboratories in the sample. At this point, if the group decision desired is a binary classification (in an accreditation process) experts will decide the LQ value to be considered as the threshold.

5.3 Conclusions and Future Research

This paper proposes a methodology that synthesises evaluations given by an experts’ committee. Evaluations are considered in an ordinal scale, for this reason a set of labels describing orders of magnitude is considered. A group decision method is given in order to rank the patterns based on comparing distances to a reference k-dimensional label. The methodology presented allows, on the one hand, the ordinal information given by experts on the specific application to be handled without previous normalisation, and, on the other, the methods of “goal programming” to be generalised without the need for previous knowledge of the ideal goal.

The results applied to a real case show the applicability of the methodology. In the experiment a set of laboratories are evaluated by a group of experts
Table 1: Level of quality and Refined level of quality.

<table>
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<th>code</th>
<th>Exp1</th>
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<th>Exp3</th>
<th>Exp4</th>
<th>Exp5</th>
<th>Exp6</th>
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that provide their individual inputs for some posterior manipulation. It is important to point out that the methodology presented allows imprecision in the evaluations given by the experts to be considered.

As a future work the design of an automatic system to perform the group decision process described will be implemented. In addition, within the AURA project framework, it is planned to build up an artificial intelligence application that includes a learning machine able to interpret and evaluate the laboratories’ analytical reports and a software tool to implement the presented methodology.

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