DEVELOPMENT OF AN ACCOUNTING SYSTEM
Applying the Incrementally Modular Abstraction Hierarchy to a Complex System

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Abstract: The new methodology for software development is introduced and applied to an accounting system. The new method is called the incrementally modular abstraction hierarchy (IMAH). IMAH has an abstraction hierarchy from abstract to concrete levels. Invariants defined on an abstract level are kept on a concrete level, which allows adding modules incrementally on each hierarchical level and avoiding combinatorial explosion of the serious problem in software engineering, while climbing down abstraction hierarchy in designing and modeling a complex system. This paper shows how IMAH is applied in developing an accounting system, which is fundamental in enterprise systems and a suitable example of complex software systems. At first, very simple example recording only journal vouches to a database system is used to describe methodologies of IMAH. Then, it is described how this simple system is incrementally developed to a conventional complex accounting system.

1 INTRODUCTION


The rational process model (RUP) is proposed to avoid this problem recently by repeating development process. (Jacobson, Booch and Rumbaugh, 1999). (Booch, Rumbaugh and Jacobson 1999). (Rumbaugh, Jacobson and Booch 1999). RUP divides a system into multiple subsystems. The most difficult subsystem among them is firstly developed in accordance with engineering disciplines starting from a business model and requirements and ending with test and development. When this subsystem is successfully completed, RUP moves to the following most difficult subsystem for development. As a result, RUP makes development feasibility clear at an early stage so that a project manager can control software development by giving correct judgments at each stage. RUP succeeds in raising the success rate of projects. RUP tries to decrease as much as possible opportunities of combinatorial explosion by iterative and incremental development. However, as it does not solve combinatorial explosion theoretically, RUP is still annoyed by the unsolved problem.

The incrementally modular abstraction hierarchy (IMAH), which is based on homotopy and topology, is introduced in this paper. (Sieradski 1992). (Spanier 1996). (Hatcher 2002). (Kunii 2005). (Kunii 2006). (Ohmori 2006). IMAH avoids combinatorial explosion by adding invariants linearly while climbing down abstraction hierarchy, keeping invariants defined on higher abstract levels and adding linearly new invariants on the current
abstract level. RUP enforces developers to design and model a system using unified modeling language (UML) diagrams. With the progress of development, many components are added to the diagram step by step. However, inevitability of these components, for example why these components are necessary or satisfy requirements, is not clear. When designing a class diagram, it is hard to explain theoretically why we need to put a new class in the diagram, or why this class has to have an association link with another class. This theoretical ambiguity leads to repeated modification of classes and class links, sometimes endlessly, as the development advances. This is a basic reason why RUP cannot avoid the combinatorial explosion problem theoretically.

In contrast, IMAH uses UML diagrams on a lower level in abstraction hierarchy. As IMAH provides theoretically fundamental properties for a class on higher abstract levels, the class has been already defined with necessary properties when it appears in a class diagram on a lower abstract level, which does not bring any modification of classes in the lower level.

In this paper, the accounting system development is described as an example for showing how a complicated enterprise information system is developed by IMAH. The accounting system is a fundamental system for enterprise resource planning (ERP) and gains importance with introduction of the Sarbanes–Oxley Act of 2002 (SOX). (Ohmori 2005). The SOX requires accountabilities of initiating, authorizing, processing, and reporting of financial data. A Web-base system, which allows financial data to be directly entered on site, is the most suitable system for SOX environments. In this paper, IMAH disciplines are firstly explained using a simple accounting system and designing of a full-scale Web-base accounting system is then described.

2 INCREMENTALLY MODULAR ABSTRACTION HIERARCHY

IMAH is based on the abstraction hierarchy in algebraic topology and consists of seven abstract levels.

1) The homotopy level is the most abstract level in the hierarchy. On the homotopy level, the developing system is described using a fiber bundle. The fiber bundle defines the most fundamental spaces constituting the developing system. The relation of these spaces is also provided on this level. As homotopy is continuous changes of continuous functions, dynamic changes can be represented using homotopy. Conceptual progress, which is thought as dynamic changes from the original concept to the current one, is described using homotopy extension properties (HEP) or homotopy lifting properties (HLP). A simple accounting system is described using a fiber bundle and a Web-base accounting system, which is conceptual progress originated from the simple accounting system, is described by HEP or HLP.

2) The set theoretical level is the following most abstract level. Elements consisting of the spaces defined on the above level are defined and incrementally added on this level. Logical operations are also available from this level.

3) The topology space level is the third highest abstract level. Continuity and closeness are defined on this level. Topological equivalence is one of the most important invariants, which are defined on this level. As the accounting system has discrete spaces, strong or weak topology is used to describe these spaces.

4) The adjunction space level is the middle abstract level. Dynamic changes are handled here. It occurs when two spaces are attached together or detached separately. The accounting system is the description of transactions, where entities such as products and cash are exchanged between two agents. When a transaction occurs, some entities are detached from one agent and attached to another agent. These dynamic changes are expressed using an attaching function.

5) The cellular structure level is the third lowest abstract level. The physical structure of the system is defined here. Up to now, the designing system is conceptual and difficult to capture its physical structure. On this level, an element constituting a space is represented by a cellular structure, which is imaginary similar to embryos and constructed by n-dimensional cells.

6) The presentation level is the following lowest abstract level. On this level, UML diagrams are used to represent the developing system. From this level, an accounting system is designed or modeled in the same way as RUP. However, as properties of classes have already been defined on the precedent abstract levels, UML diagrams are created almost automatically. This is the big difference with the traditional methodologies.

7) The view level is the lowest abstract level and the most concrete level. On this level, the system is represented by program codes. If program codes are installed in the system, the
behavior of the developing system can be observed.

3 DESIGNING AND MODELING OF AN SIMPLE ACCOUNTING SYSTEM

3.1 The Homotopy Level

A simple accounting system is the description of transactions in a company. A company buys parts and sells products. The difference of amount between buying and selling is profit or loss. When a transaction occurs, a journal voucher is issued. A journal voucher consists of a journal voucher number, header and details. A journal voucher number space is determined as the base space. The inverse map of the total space, the divides a total space into a base space and a fiber. As is of course the transaction space. A fiber bundle is the projection to the first factor

\[ \forall b \in B, F \mapsto p^{-1}(b) \]

is the projection to the first factor \( U \). Thus the bundle projection \( p: E \to B \) and the projection \( p_B: B \times F \to B \) are locally equivalent. The fiber \( F \) is homeomorphic to \( p^{-1}(b) \) for every \( b \in B \), namely

For each \( U \in Y \), a homeomorphism called a coordinate chart \( \varphi_U: U \times F \to p^{-1}(U) \) exists such that the composite

\[ \varphi \circ p^{-1}(U) \circ p \]

is a quadruple \( \xi = (E, B, F, p) \) consisting of a total space \( E \), a base space \( B \), a fiber \( F \), and a bundle projection that is a continuous surjection called \( F \)-bundle \( p: E \to B \) such that there exists an open covering \( Y = \{ U \} \) of \( B \) and, for each \( U \in Y \), \( B \to p^{-1}(U) \).

The total space for the simple accounting system is of course the transaction space. A fiber bundle divides a total space into a base space and a fiber. As the base space is projection of the total space, the journal voucher frame space \( J \), which is a frame to contain the description of a transaction, is determined as the base space. The inverse map of projection to an element of the base space represents a fiber. A journal voucher number space \( V \), a header space \( H \) and a detail space \( D \), which represent the description of a transaction, compose a fiber as shown in Figure 1.

When a transaction occurs, the elements are obtained from these spaces and combined each other, where a space and an element are similar concepts to a class and an instance of an object oriented language. Therefore, a journal voucher frame is obtained from the space \( J \). A journal voucher number, a header and a set of details are obtained from the spaces \( V \), \( H \) and \( D \). The journal voucher number, the header and the set of details are put in the journal voucher frame.

Figure 1. The fiber bundle for the simple accounting system.

In summary, the journal voucher frame space \( J \), the journal voucher number space \( V \), the header space \( H \) and the detail space \( D \) have been defined on this level.

3.2 The Set Theoretical Level

Elements of a space are defined on this level. The journal voucher frame space \( J \) consists of a set of journal voucher frames: \( J = \{ j_1, j_2, \ldots, j_n \} \), where \( j_i \) is a journal voucher frame with three variables such that \( j_i = (v_i, h_i, DS_i) \in J \), where \( v_i \), \( h_i \) and \( DS_i \) are a journal voucher number, a header and details. The journal voucher number space \( V \) consists of journal voucher numbers: \( V = \{ v_1, v_2, \ldots, v_t \} \), where \( v_i \) is a journal voucher number with one variable of \( s_i \in \mathbb{C} \).

The header space consists of headers: \( H = \{ h_1, h_2, \ldots, h_i \} \), where \( h_i \) is a header with three variables such that \( h_i = (t_i, a_i, r_i) \in H \), where \( t_i \), \( a_i \) and \( r_i \) are application date, applicant name and remarks. The detail space consists of a set of details: \( D = \{ d_1, d_2, \ldots, d_j \} \), where \( d_i \) is a detail with two variables such that \( d_i = (d_{ii}, d_{ai}) \), where \( d_{ii}, d_{ai} \) are an accounting item and amount. If amount is positive, the detail is debtor, otherwise it is creditor.

In summary, the elements of each space: \( J = \{ j_1, j_2, \ldots, j_n \} \); \( V = \{ v_1, v_2, \ldots, v_t \} \); \( H = \{ h_1, h_2, \ldots, h_i \} \) and \( D = \{ d_1, d_2, \ldots, d_j \} \) are defined on this level. The variables of each element are also defined on this level. The elements and variables are incrementally added on the basis of the invariants defined on the homotopy level.
3.3 The Topological Space Level

The strongest topology is introduced for the simple accounting system. The strongest topology for the journal voucher space \( J \) is introduced as follows. \( (J, T) = \{ \phi, j_1, j_2, j_3, \ldots, j_n, (j_1, j_2), (j_1, j_3), \ldots, (j_{n-1}, j_n), \ldots, (j_1, j_2, j_3, \ldots, j_n) \} \). Other spaces also have the strongest topology in the same way.

Here is an example how the topological space is used. When the general ledger is required, it is necessary to collect journal vouchers with a given accounting item. This requirement is achieved by gathering fibers as shown in Figure 2. For example, if all transactions traded with account receivable are projected to a subset \( J_S \subset (J, T) \), then the set of fibers of \( J_S \) gives the voucher numbers, headers and details of all the journal vouchers traded with account receivable.

![Figure 2. Fibers for account receivable.](image)

In summary, the topological space level defines topological spaces, for example \( (J, T) = \{ \phi, j_1, j_2, j_3, \ldots, j_n, (j_1, j_2), (j_1, j_3), \ldots, (j_{n-1}, j_n), \ldots, (j_1, j_2, j_3, \ldots, j_n) \} \), on the basis of invariants defined by the previous abstract levels.

3.4 The Adjunction Space Level

When a transaction occurs, the space \( J \) is attached to the spaces \( V, H \) and \( D \) such that the journal voucher number, header and details are put in the journal voucher frame. This behavior is clearly represented on the adjunction space level, where related spaces are adjoined by an attaching function.

Dynamic relations between the journal voucher frame space \( J \) and the detail space \( D \) are considered here as an example. When a transaction occurs, the spaces \( J \) and \( D \) are adjoined by identifying transaction details. That is, a journal voucher frame \( j_0 \) is attached to a journal voucher number \( v_0(s_0) \), a header \( h_0(t_0, a_0, r_0) \) and details \( d_0(d_0t_0, a_0t_0) \ldots \)

adjoining two spaces. Using an attaching function \( f \), the adjunction space \( D_f \)

\[ D_f = D + J \]

is obtained by identifying each of transaction details \( y \in D_0 \) with its image \( f(y) \), which is a journal voucher frame \( J \), so that \( j_i \sim f(y) \) \( \forall y \in D_0 \). The adjunction space \( D_f \) shows the shape of the space \( D \) where \( D_0 \subset D \) affected by this transaction is identical to the corresponding part of the space \( J \).

The attaching map \( f \) and the identification map \( g \) are:

\[ f: D_0 \rightarrow J | D_0 \subset D, \]

and

\[ g: D + J \rightarrow D_f = D + J / \sim = D + J / (j_i \sim f(y)) \Rightarrow y \in D_0 \]

The attaching map shows how \( D_0 \subset D \) is mapped into \( J \). The identification map shows how relations of two spaces \( D \) and \( J \) change after mapping \( f \). These relations are shown in Figure 3.

![Figure 3. The attaching function.](image)

The adjunction space level preserves the invariants defined at the homotopy level. The invariant showing that the journal voucher consists of three properties of a journal voucher number, a header and details is preserved by the attaching map \( f \). These spaces are attached by a disjoint union.

As an attaching map is continuous, the reverse function is defined. It means that the system can return at the point before applying the attaching map. Therefore, cancellation of any transaction can be accepted at any time, which gives flexibility to system development.

In summary, attaching maps from \( V, H \) and \( D \) to \( J \) are defined on the basis of invariants defined by the previous higher abstract levels.

3.5 The Cellular Structure Level

On this level, the physical structure is constructed using cells. At first, an element in each space is
For the skeleton header...

→ cellular structures using a follows. For any topological space these spaces are represented by \( +B_d \). A header \( h_i \) and a detail \( d_i \) have three and two variables. These spaces \( H \) and \( D \) are represented by \(+B_0\) and \(+B_2\). A journal voucher frame \( j_i \) is a container of these elements without variables, its space \( J \) becomes \(+B^3\).

A \( n \)-dimensional closed ball is represented by disjoint unions of an \((n-1)\)-dimensional open ball and a \((n-1)\)-dimensional surface and \((n-1)\)-dimensional closed balls such that

\[
B^k = \text{int}B^n + (S^{n-1} \setminus B^{n-1}) + B^{n-1}.
\]

A smaller dimensional closed ball \( B_j \) than the original ball is obtained by repeating this process. This ball is denoted by the following expression.

\[
\phi^{-j} B^k = B^j.
\]

A journal voucher frame \( j_i \) is attached to a journal voucher number \( v_i \), a header \( h_i \) and details \( DS_i \). These attaching is described by the following equations.

\[
f_1: \partial B_0^1 \rightarrow B_0^0, \quad \text{or} \quad B_0^0 + \partial B_0^1 \rightarrow B_0^0.
\]

\[
f_2: \partial B_0^2 \rightarrow B_0^1, \quad \text{or} \quad B_0^1 + \partial B_0^2 \rightarrow B_0^1.
\]

\[
f_3: \partial B_0^3 \rightarrow B_0^2, \quad \text{or} \quad B_0^2 + \partial B_0^3 \rightarrow B_0^2.
\]

Then, \( n \)-dimensional balls are transformed into cellular structures using a filtration.

Definition: A filtration space is a sequence of cells to represent a topological space. It is defined as follows. For any topological space \( X \), we can get a finite or infinite sequence of skeletons \( X^i \), where \( p \) is an integer, such that

\[
X = \cap_{0 \leq p \leq n} X^p
\]

\[
X^0 \subseteq X^1 \subseteq \ldots \subseteq X
\]

A skeleton \( X^i \) consists of cells, whose dimensions do not exceed \( n \). A cell is a topological space, equivalent topologically to an \( n \)-dimensional open ball \( \text{int}B^n \), where \( n \) is an arbitrary integer. A sequence of skeletons is called filtration. If it is finite, it becomes a CW-space.

Let’s consider details \( DS_i \). \( DS_i \) is represented by \(+B_2^0\) using open balls. A detail has an accounting item and amount as its variables. It also needs an index to be uniquely identified. These entities are used as a skeleton \( X_{\text{detail}} \) of \( DS_i \).

\[
X_{\text{detail}} = \{ e_{\text{did}}, e_{\text{id}}^0, e_{\text{item}}^0, \ldots, e_{\text{amout}}^0 \}
\]

For the skeleton \( X_{\text{detail}} \), every two entities among index, accounting item and amount are attached together via 1-dimensional cells as follows.

\[
e_{\text{item}}^0 \rightarrow f_3: e_{\text{item}}^0 \rightarrow e_{\text{amount}}^0 \rightarrow f_2: e_{\text{amount}}^0 \rightarrow e_{\text{amount}}^0 \rightarrow f_1: e_{\text{amount}}^0 \rightarrow e_{\text{amount}}^0.
\]

The keleton \( X_{\text{detail}} \) is also obtained as follows.

\[
X_{\text{detail}} = \{ e_{\text{detail}}^0, e_{\text{detail}}^1, \ldots, e_{\text{detail}}^k \}
\]

In the same way, a journal voucher number \( v_i \) and a header \( h_i \) are obtained. These are attached to a journal voucher frame \( j_i \) as shown in figure 4.

Figure 4. The cellular structures for the simple accounting system.

In summary, the cellular structures of elements for the spaces \( J, V, H \) and \( D \) are defined while preserving invariants defined on the previous higher levels.

3.5 The Presentation Level

The journal voucher frame \( j_i \in J \), the journal voucher number \( v_i \in J \), the header \( h_i \in J \), the details \( DS_i \subset D \) constituting a transaction have been represented as cellular structures \( X_{\text{frame}}, X_{\text{number}}, X_{\text{header}} \) and \( X_{\text{detail}} \) on the cellular structure level. These cellular structures are transformed into UML diagrams on the presentation level as shown in Figure 5. Each cellular structure is represented as a class such as \( X_{\text{frame}} \) is represented as class \( \text{VoucherFrame} \), with stereotype type \( \text{entity} \), which becomes an entity been of Enterprise Java Beans (EJB) on the next view level. The elements of a 0-dimensional skeleton are transformed into instant variables. The attaching function between two cells is transformed into an association links. Multiplicity of an association link reflects the number of cells connected by the attaching function. In a cellular structure, an index is used in a 0-dimensional
skeleton. It does not appear in its class. However, it becomes the primary key when a class is transformed into the database table using object-relational mapping on the next view level.

In summary, a space defined on the homotopy level is transformed into a class with stereotype entity on this level. An element defined on the set theoretical level is transformed into a cell on the cellular structure level and into an instance on this level. A variable defined on the set theoretical level is transformed into a 0-dimensional cell on the cellular structure level and into an instance variable on this level. An attaching function defined on the adjunction space level is transformed into an association link connecting two classes.

### 3.6 The View Level

The simple accounting system includes only classes with stereotype entity. These classes are automatically transformed into entity beans and database tables by AndroMDA. AndroMDA is a generator framework that adheres to the model driven architecture (MDA) paradigm. UML diagrams are transformed into deployable components for J2EE, Spring or .NET platform.

Any business logic is not included in the simple accounting system. Only, creating, reading, updating and deleting are necessary for the database. Java server pages (JSP) realizing these functions are also automatically generated by AndroMDA.

In summary, as everything which is necessary as an application program in EJB environment is automatically generated by AndroMDA, invariants defined on the previous levels are preserved here and components required for deploying the system is incrementally added on this level.

### 4 DESIGNING AND MODELING OF A FULL-SCALE WEB-BASE ACCOUNTING SYSTEM

#### 4.1 The Homotopy Level

A full-scale accounting system is provided by adding functions to the simple accounting system. A basic accounting system is obtained by adding general ledgers to the simple accounting system. An enterprise accounting system is equipped with financial statements to the basic accounting system. A full-scale Web-base accounting system with internal auditing functions is obtained by adding a business process model to the enterprise accounting system in Web-base environment. The conceptual progress from the simple accounting system to the full-scale Web-base accounting system is explained by HLP. HLP is defined mathematically as follows.

The function \( p: E \rightarrow B \) has the homotopy lifting property (HLP) for a space \( X \) if, for each continuous function \( k: X \rightarrow E \), each homotopy \( H: X \times I \rightarrow B \) of \( p \circ k(\hat{H}(\hat{x})_0 = \hat{k}(\hat{x})) \) has a lifting to a homotopy \( K: X \times I \rightarrow E \) of \( k (K(x)_0 = k) \) and \( K \) is constant on \( \{x\} \times I \) whenever \( H \) is constant on \( \{x\} \times I \).

If space \( X \) represents conceptual progress and \( E \) and \( B \) are a total space representing transactions and a base space representing journal vouchers as shown in Figure 6, the conceptual progress to the full-scale accounting system is considered as homotopical changes originated from the simple accounting system.

Therefore, the conceptual progress preserves invariants of the simple accounting system. For the full-scale Web-base accounting system, the journal voucher frame space \( J \), the journal voucher number space \( V \), the header space \( H \) and the detail space \( D \) are preserved. The invariants for conceptual progress part are incrementally added to the original part.

The basic accounting system, which enhances the simple accounting system by adding the function of the general ledger, is considered as the first conceptual progress. The general ledger is a permanent summary of all journals. The general ledger is sometimes divided into main accounting items, such as cash, account receivable and account payable ledgers. The general ledger is created from a set of journal vouchers. It is possible to create it whenever a journal voucher is processed or only...
when the reference of the general ledger is requested. The system is designed by the latter since it is expected the former takes time when the system becomes a full-scale system.

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To fulfill the above requirements, the basic accounting system is designed as follows. It adds the general ledger space and the processing list space to the spaces defined for the simple accounting system as shown in Figure 7. The general ledgers are generated on the general ledger space. The processing list space is divided into two subspaces: waiting and processed. Journal vouchers which have not been recorded yet in the general ledger are kept in the waiting space. When a journal voucher is recorded, it is moved to the processed space. The basic accounting system has two main procedures: 1) processing of a journal voucher, which carries out the same process as in the accounting system as well as saves it in the waiting space; 2) updating of the general ledger, which moves journal vouchers from the waiting space to the processed space and add amount recorded in the journal voucher to the current total amount of the corresponding accounting item as shown in Figure 8.

4.2 The Presentation Level

After designing the basic accounting system on the homotopy level, it goes to designing on the lower abstract levels in the same way as the simple accounting system. The results on the presentation level are shown in Figure 9. In the simple accounting system, all classes have stereotype entity. However, in the basic accounting system, processing of the general ledger requires business logic. A class with business logic has stereotype service. This class becomes a session bean of EJB. In the Figure 10, GeneralLegerHandler is equipped for this purpose. The method getGeneralLedger() of class GeneralLegerHandler updates the general ledger. The contents of this method, which is business logic of the basic accounting system, is not automatically generated by AndroMDA, a programmer has to supply its code. This program development is also carried out by abstraction hierarchy. A finite machine, where equivalent finite machines are homotopically equivalent, is defined on the homotopy level. As the explanation of this mechanism needs more space, it will be described in another paper.

After completing the development of the basic accounting system, the enterprise accounting system is developed in the same way starting from the homotopy level and ending with the view level. The full-scale accounting system is finally completed by repeating this process. This process is similar to RUP. However, RUP repeats it between the presentation and view levels, while IMAH repeats it between the homotopy and view levels with theoretical basis.
4 CONCLUSIONS

IMAH is applied to the development of accounting systems ranging from the simple accounting system to the full-scale Web-base accounting system. IMAH keeps invariants defined on higher abstract levels while climbing down the abstraction hierarchy. Using the simple accounting system, it is described how invariants are incrementally added to the developing system. The incremental invariant addition contributes to avoiding the combinatorial explosion problem.

The simple accounting system is enhanced to the basic accounting system and the full-scale Web-base accounting system. The conceptual progress also keeps invariants defined in the original system. IMAH is considered as the conceptual progress of RUP since both methodologies are performed by iterative process. However, IMAH is different from RUP since IMAH has theoretical background. Homotopy, fiber bundles, homotopy lifting properties, homotopy extension properties, topology, attaching functions and cellular structures give enough theoretical background to software engineering.

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Figure 9. The class diagram for the basic accounting system.