

# EXTERIOR ORIENTATION USING LINE-BASED ROTATIONAL MOTION ANALYSIS

A. Navarro, E. Villarraga and J. Aranda

*Technical University of Catalonia, ESAII Department, Barcelona, Spain*

**Keywords:** Exterior orientation, motion analysis, hand-eye calibration, pose estimation.

**Abstract:** 3D scene information obtained from a sequence of images is very useful in a variety of action-perception applications. Most of them require perceptual orientation of specific objects to interact with their environment. In the case of moving objects, the relation between changes in image features derived by 3D transformations can be used to estimate its orientation with respect to a fixed camera. Our purpose is to describe some properties of movement analysis over projected features of rigid objects represented by lines, and introduce a line-based orientation estimation algorithm through rotational motion analysis. Experimental results showed some advantages of this new algorithm such as simplicity and real-time performance. This algorithm demonstrates that it is possible to estimate the orientation with only two different rotations, having knowledge of the transformations applied to the object.

## 1 INTRODUCTION

Perceptual orientation of specific objects in a 3D scene is necessary in a diversity of action-perception applications where 2D information is the only input. Automated navigation or manipulation are examples of this kind of applications and rely heavily in 2D data obtained from a video camera as sensory input to fulfill 3D tasks. The exterior orientation problem serves to map the 2D-3D relation estimating the transformation between coordinate frames of objects and the camera.

There are several methods proposed to estimate the orientation of a rigid object. The first step of their algorithms consists on the identification and location of some kind of features that represent an object in the image plane. Most of them rely on feature points and apply closed-form or numerical solutions depending on the number of objects and image feature correspondences. Previous works focused with a small number of correspondences applied iterative numerical techniques as (Lowe, 1987) and (Haralick et al., 1991). Other methods apply direct linear transform (DLT) for a larger number of points as (Hartley, 1998) or reduce the problem to close-form solutions, as (Fiore, 2001) applying orthogonal decomposition.

Lines, however, are the features of interest in this work. They are the features to be extracted from a

sequence of images and provide motion information through its correspondences. There are several approaches that use this kind of features to estimate motion parameters. Some early works solve a set of nonlinear equations, as the one by (Yen and Huang, 1983), or use iterated extended Kalman filters, as showed by (Faugeras et al., 1987), through three perspective views. Works by (Ansar and Daniilidis, 2003) combined sets of lines and points for a linear estimation, and (Weng et al., 1992) discussed the estimation of motion and structure parameters studying the inherent stability of lines and explained why two views are not sufficient.

An important property in using lines is their angular invariance between them. Then, our purpose was to study this property to provide a robust method to solve orientation estimation problems. It is possible to compute the orientation of an object through the analysis of angular variations in the image plane induced by its 3D rotations with respect to the camera. It can be seen as an exterior orientation problem where objects in the scene are moved to calculate their pose.

Some action-perception applications can be seen as a fixed camera visualizing objects to be manipulated. In our case, these objects were represented by lines. Three views after two different rotations generate three lines in the image plane, each of them define a 3D plane called the projection plane of the line. These planes pass through the

Navarro A., Villarraga E. and Aranda J. (2007).

EXTERIOR ORIENTATION USING LINE-BASED ROTATIONAL MOTION ANALYSIS.

In *Proceedings of the Second International Conference on Computer Vision Theory and Applications - IU/MTSV*, pages 290-293

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projection center and their respective lines. Their intersection is a 3D line that passes through the origin of the perspective camera frame and the centroid of the rotated object, as seen in Figure 1.

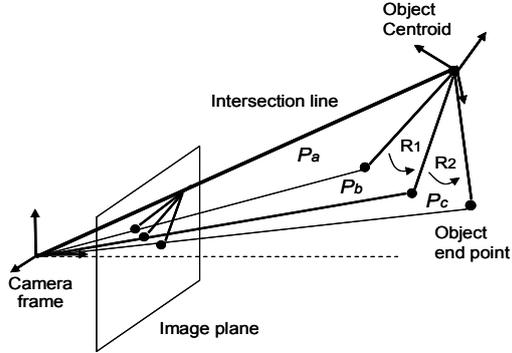


Figure 1: A 3D line through the origins is the intersection of the projection planes after two rotations of the object.

The motion analysis of angular variations between lines permitted us to estimate the orientation of a given object. Therefore, we propose a robust method to compute the orientation through rotations. These rotations must be known. They could be sensed or given and controlled, as is the case of robotic applications. Experimental results showed some advantages of this new algorithm such as simplicity and real-time performance.

## 2 ORIENTATION ESTIMATION ALGORITHM

Motion analysis of feature lines was the base of our orientation estimation algorithm. In this case known 3D rotations of a line and its subsequent projections in the image plane were related to compute its relative orientation with respect to a perspective camera. Vision problems as feature extraction and line correspondences are not discussed and we suppose the focal distance  $f$  as known. Our goal is, having this image and motion information, estimate the orientation of an object represented by feature lines with the minimum number of movements and identify patterns that permits to compute a unique solution without defined initial conditions.

### 2.1 Mathematical Analysis

A 3D plane is the result of the projection of a line in the image plane. It is called the projection plane and passes through the projection center of the camera and the 3D line. This 3D line, in this case, is the representation of an object. With three views after

two different rotations of the object, three lines are projected in the image plane. Thus three projection planes can be calculated. These planes are  $P_a$ ,  $P_b$  and  $P_c$ , and their intersection is a 3D line that passes through the projection center and the centroid of the rotated object, being the centroid the point of the object where it is rotated. Across this line a unit director vector  $v_d$  can be determined easily knowing  $f$  and the intersection point of the projected lines in the image plane. Our intention is to use this 2D information to formulate angle relations with the 3D motion data.

Working in the 3D space permits to take advantage of the motion data. In this case where the object is represented by a 3D line, the problem could be seen as a unit vector across its direction that is rotated two times. In each position of the three views this unit vector lies in one of the projection planes as seen in Figure 2. It first is located in  $P_a$ , then it rotates an angle  $\alpha_1$  to lie in  $P_b$  and ends in  $P_c$  after the second rotation by an angle  $\alpha_2$ . To estimate the relative orientation of the object we first must obtain the location of three unit vectors,  $v_a$ ,  $v_b$  and  $v_c$ , that coincide with the 3D motion data and lie on their respective planes. To do this we know that the scalar product of:

$$v_a \cdot v_b = \cos \alpha_1 \quad (1)$$

$$v_b \cdot v_c = \cos \alpha_2 \quad (2)$$

Calculating the angle  $\gamma$  between the planes formed by  $v_a v_b$  and  $v_b v_c$  from the motion information, we have

$$(v_a \times v_b) \cdot (v_b \times v_c) = \cos \gamma \quad (3)$$

And applying vector identities

$$v_a \cdot v_c = \cos \alpha_1 \cos \alpha_2 - \cos \gamma \quad (4)$$

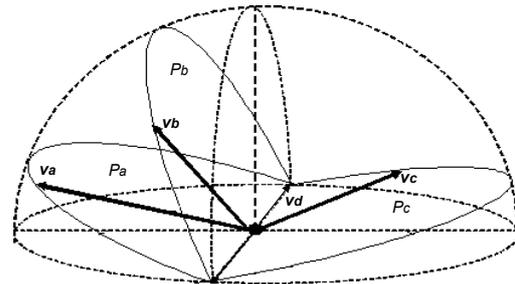


Figure 2: Unit vectors  $v_a$ ,  $v_b$  and  $v_c$  are constrained to lie on planes  $P_a$ ,  $P_b$  and  $P_c$  respectively. Their estimation can be seen as a semi sphere where their combination must satisfy the angle variations condition.

With the set of equations conformed by (1), (2) and (4) we have the three unit vectors to calculate.

However, there is not a unique solution, thus constraints must be applied.

## 2.2 Projection Planes Constraint

There are many possible locations where the three unit vectors can satisfy the equations in the 3D space. To obtain a unique solution unit vectors  $\mathbf{v}_a$ ,  $\mathbf{v}_b$  and  $\mathbf{v}_c$  must be constrained to lie in their respective planes. Unit vector  $\mathbf{v}_a$  could be seen as any unit vector in the plane  $P_a$  rotated through an axis and an angle. Using unit quaternions to express  $\mathbf{v}_a$  we have

$$\mathbf{v}_a = q_a \mathbf{v} q_a^* \quad (5)$$

where  $q_a$  is the unit quaternion applied to  $\mathbf{v}$ ,  $q_a^*$  is its conjugate and  $\mathbf{v}$  is any vector in the plane. For every rotation about an axis  $\mathbf{n}$ , of unit length, and angle  $\Omega$ , a corresponding unit quaternion  $q = (\cos \Omega/2, \sin \Omega/2 \mathbf{n})$  exists. Thus  $\mathbf{v}_a$  is expressed as a rotation of  $\mathbf{v}$ , about an axis and an angle by unit quaternions multiplications. In this case  $\mathbf{n}$  must be normal to the plane  $P_a$  if both unit vectors  $\mathbf{v}_a$  and  $\mathbf{v}$  are restricted to be in the plane.

Applying the plane constraints and expressing  $\mathbf{v}_a$ ,  $\mathbf{v}_b$  and  $\mathbf{v}_c$  as mapped vectors through unit quaternions, equations (1), (2) and (4) can be expressed as a set of three nonlinear equations with three unknowns

$$q_a \mathbf{v}_d q_a^* q_b \mathbf{v}_d q_b^* = \cos \alpha_1 \quad (6)$$

$$q_b \mathbf{v}_d q_b^* q_c \mathbf{v}_d q_c^* = \cos \alpha_2 \quad (7)$$

$$q_a \mathbf{v}_d q_a^* q_c \mathbf{v}_d q_c^* = \cos \alpha_1 \cos \alpha_2 - \cos \gamma \quad (8)$$

The vector to be rotated is  $\mathbf{v}_d$ , which is common to the three planes, and their respective normal vectors are the axes of rotation. Extending the equations (6), (7) and (8), multiplying vectors and quaternions, permits to see that there are only three unknowns which are the angles of rotation  $\Omega_a$ ,  $\Omega_b$  and  $\Omega_c$ .

Applying iterative numerical methods to solve the set of nonlinear equations, the location of  $\mathbf{v}_a$ ,  $\mathbf{v}_b$  and  $\mathbf{v}_c$  with respect to the camera frame in the 3D space are calculated. Now we have a simple 3D orientation problem that can be solved easily by a variety of methods as least square based techniques. However, in the case where motions could be controlled and selected movements applied, this last step to estimate the relative orientation would be eliminated. Rotation information would be obtained directly from the numerical solution. If we assume one of the coordinate axes of the object frame coincide with the moving unit vector and apply selected motions, as one component rotations, a unique solution is provided faster and easier.

## 3 EXPERIMENTAL RESULTS

Real world data was used to validate the algorithm. Experiments were carried out through a robotic test bed that was developed in order to get high repeatability. It consist on an articulated robotic arm with a calibrated tool frame equipped with a tool, easily described by a line, which is presented in different precisely known orientations to a camera. The camera field of view remains fixed during the image acquisition sequence. The tool center of rotation is programmed to be out of the field of view, as it would be presented in an action-perception application.

With this premises, a standard analog B/W camera equipped with a known focal length optics has been used. This generates a wide field of view that is sampled at 768x576 pixels resolution. After image edge detection, Hough Transform is used in order to obtain the tool contour and the straight line in the image plane associated to it. Tool contour is supposed to have the longest number of aligned pixel edges in the image.

Having this setup, feature lines were identified and located in a sequence of images. This images captured selected positions of the rotated tool. Once the equation of the lines projected in the image plane were acquired, unit vectors normal to the constraint planes and  $\mathbf{v}_d$  could be calculated. This unit vectors and the motion angles  $\alpha_1$  and  $\alpha_2$  served as the input to the proposed algorithm. The intersection of the lines was needed to calculate  $\mathbf{v}_d$ . This calculation is prone to errors due to be located out of the field of view. It means the intersection of a different number of lines is not usually the same point. Table 1 shows the standard deviation in pixels of the intersection points calculated through different motion angles. The intersections converge to a single point when the angles between lines are higher.

Table 1: Standard deviation of intersecting lines (in pixels) through different motion angles. Being the intersection point defined by  $X_{int}$  and  $Y_{int}$ .

Degrees	$X_{int}$	$Y_{int}$	$\sigma$
5	895,81	264,11	62,31
10	928,28	278,91	35,65
15	861,41	250,71	6,958
20	876,67	256,52	5,83

The 3D transformation resultant from the algorithm was tested projecting 3D lines, derived by new tool rotations, in the image plane and comparing them with the line detected by the vision system. Tests for motion angles between 5 and 20 degrees were

carried out. Figure 3 shows the error (in pixels) between lines through different angles.

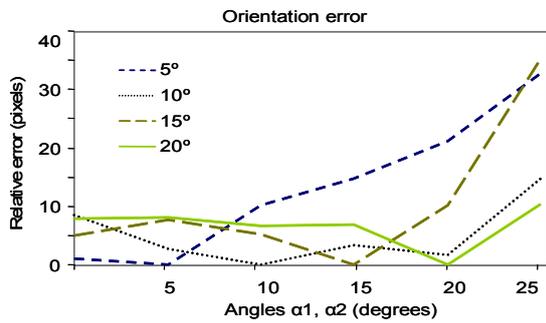


Figure 3: Relative error in pixels using the rotational motion analysis algorithm.

There can be seen how the error is minimum at the position where the transformation was calculated, it means at its second motion or third image. This error varies depending on the position of the tool, it increases with higher angles, when the position of the tool separates from the minimum error position. Figure 4 compares the algorithm performance for 5 and 20 degrees motion angles. There the error varies differently. In the case of 5 degrees the error increases greatly with each motion that separates the tool from the minimum error position. While for 20 degrees this error also increments, but remains stable.

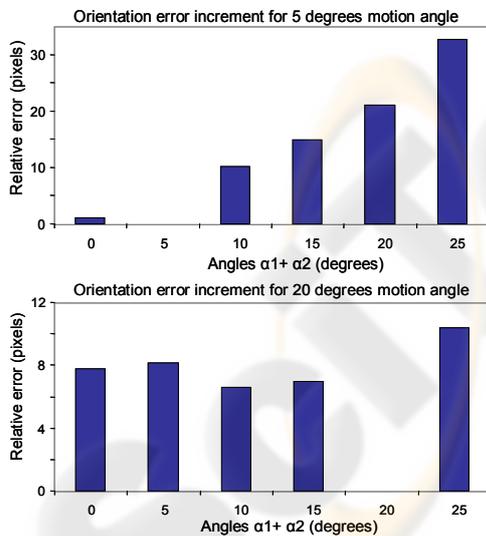


Figure 4: Algorithm performance comparison between 5 and 20 degrees, with a first rotation  $\alpha_1$  followed by a second  $\alpha_2$  of the same magnitude.

This results validate the line-based algorithm and its low computational cost demonstrate its real-time performance. The error increment with large position separations is mainly product of the

deviation at the intersection point. It can be seen that the calculation of  $vd$  has a great impact in the result and future work should be focused in this issue.

## 4 CONCLUSIONS

A method to estimate the relative orientation of an object with respect to a camera has been proposed. The object assumed was represented by feature lines. 2D correspondences of a line due to known 3D transformations of the object were the information used to calculate its orientation. We showed that with only two rotations the angular variation between lines provides sufficient information to estimate the relative orientation. This motion analysis led to address questions as the uniqueness of solution for the minimum number of movements and possible motion patterns to solve it directly. In the case of controlled motions, one component rotations through normal axes simplify calculations to provide a robust technique to estimate the relative orientation with no initial conditions defined.

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