A DUAL MODE ADAPTIVE ROBUST CONTROLLER FOR DIFFERENTIAL DRIVE TWO ACTUATED WHEELED MOBILE ROBOT

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Abstract: This paper is addressed to dynamic control problem of nonholonomic differential wheeled mobile robot. It presents a dynamic controller to mobile robot, which requires only information of the robot configuration, that are collected by an absolute positioning system. The control strategy developed uses a linear representation of mobile robot dynamic model. This linear model is decoupled into two single-input single-output systems, one to linear displacement and one to angle of robot. For each resulting system is designed a dual-mode adaptive robust controller, which uses as inputs the reference signals calculated by a kinematic controller. Finally, simulation results are included to illustrate the performance of the closed loop system.

1 INTRODUCTION

The control of mobile robot is a well known problem with nonholonomic constraints. There are many works about it, and several of these works use kinematic model. Dynamic model, which is composed by kinematic model plus the dynamic of robot, is used in a few works.

An adaptive controller that compensates for camera and mechanical uncertain parameters and ensures global asymptotic position/orientation tracking was presented by Dixon (Dixon et al., 2001). Chang (Chang et al., 2004) proposes a novel way to design and analysis nonlinear controllers to deal with the tracking problem of a wheeled mobile robots with nonholonomic constraints.

The exact mobile robot model is complex and the model has a lot of parameters to be considered, so we will use a simplified model that consider some of them. But this isn't the only problem, because some parameters can suffer variations, as the mass and the frictional coefficients. For example, when robot is doing a task as to carry an object, the object mass isn't considered by the model. So, we are using an association between a dual mode adaptive robust controller (DMARC) and a kinematic controller to the robot linear model.

The DMARC controller was presented in (Cunha and Araujo, 2004) and it proposes a connection between a variable structure model reference adaptive controller (VS-MRAC, proposed by Hsu and Costa(Hsu and Costa, 1989)) and a conventional model reference adaptive controller (MRAC). The goal is to have a robust system with fast response and small oscillations (characteristics of VS-MRAC), and a smooth steady-state control signal (characteristics of the MRAC).

2 NONHOLONOMIC MOBILE ROBOT

The robot considered in this paper is a nonholonomic direct differential-drive two-actuated-wheeled mobile robot, which is showed in Fig. 1. The robot is of symmetric shape and the center of mass is at the geometric center C of the body. It has two driving wheels fixed to the axis that passes through C. Each wheel is controlled by one independent motor.

Based on Fig 1, we have that d is distance between wheels, $r_{d,e}$ are right and left wheels radius, $\omega_{d,e}$ are right and left wheels angle velocity, ω_r is the angle robot velocity, $v_{d,e}$ are right and left wheels linear ve-

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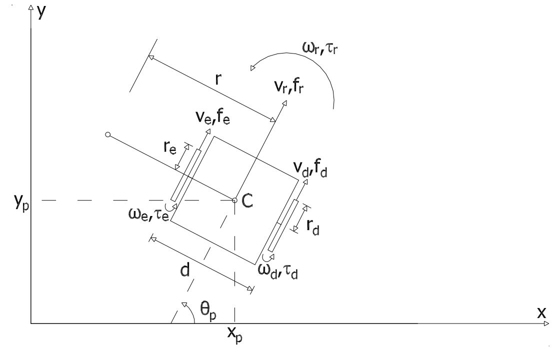

Figure 1: Differential-drive two-actuated-wheeled mobile robot.

locity, v_r is the linear robot velocity, $\tau_{d,e}$ are right and left wheels torque, τ_r is the robot torque, $f_{d,e}$ are right and left wheels force, f_r is the robot force, θ_p is the angle of robot, x_p is the x coordinate of C and y_p is the y coordinate of C.

2.1 Kinematic Model

The robot kinematic is described by equation (1),

$$\dot{q} = {}^{q}T_v v \tag{1}$$

$$q = \begin{bmatrix} x_p \\ y_p \\ \theta_p \end{bmatrix} \quad {}^{q}T_v = \begin{bmatrix} \cos(\theta_p) & 0 \\ \sin(\theta_p) & 0 \\ 0 & 1 \end{bmatrix} \quad v = \begin{bmatrix} v_r \\ \omega_r \end{bmatrix}$$

The relation between velocity vectors (ω_{at} and v) is given by the equation $\omega_{at} = {}^{\omega}T_v v$, with

$${}^{\omega}T_v = \begin{bmatrix} 1/r_d & d/2r_d \\ 1/r_e & -d/2r_e \end{bmatrix} \quad \omega_{at} = \begin{bmatrix} \omega_d \\ \omega_e \end{bmatrix}$$

This mobile robot has a nonholonomic constraint, because the driving wheels purely roll and don't slip. This constraint is described by

$$\frac{\partial y_p}{\partial x_p} = \frac{\sin(\theta_p)}{\cos(\theta_p} \rightarrow \dot{y}_p \cos(\theta_p) - \dot{x}_p \sin(\theta_p) = 0 \tag{2}$$

2.2 Dynamic Model

The robot dynamic is represented by the following equation

$$f = M_r \dot{v} + B_r v \tag{3}$$

which is composed by matrix M_r and B_r

$$f = \begin{bmatrix} f_r \\ \tau_r \end{bmatrix} \quad M_r = \begin{bmatrix} m & 0 \\ 0 & J \end{bmatrix} \quad B_r = \begin{bmatrix} \beta_{lin} & 0 \\ 0 & \beta_{ang} \end{bmatrix}$$

where β_{lin} is the frictional coefficient to linear movements, β_{ang} is the frictional coefficient to angle movements, m is the robot mass and J is the inertia moment.

It's very important to consider the following relation

$$f = {}^{\omega}T_v^T \tau \tag{4}$$

with τ is the vector $\begin{bmatrix} \tau_d & \tau_e \end{bmatrix}^T$.

The DC motors dynamic is obtained of electrical (5) and mechanical (6) equations,

$$i = R_m e - R_m K_m \omega_{at} \tag{5}$$

$$\tau = K_m i - J_m \dot{\omega}_{at} - B_m \omega_{at} \tag{6}$$

where the vectors are

$$i = \begin{bmatrix} i_d \\ i_e \end{bmatrix} \quad R_m = \begin{bmatrix} 1/R_d & 0 \\ 0 & 1/R_e \end{bmatrix} \quad e = \begin{bmatrix} e_d \\ e_e \end{bmatrix}$$

$$K_m = \begin{bmatrix} K_d & 0 \\ 0 & K_e \end{bmatrix} \quad J_m = \begin{bmatrix} J_d & 0 \\ 0 & J_e \end{bmatrix} \quad B_m = \begin{bmatrix} \beta_d & 0 \\ 0 & \beta_e \end{bmatrix}$$

and $e_{d,e}$ are armature motor voltages, $R_{d,e}$ are windings resistances, $K_{d,e}$ are constants of induced voltage, $J_{d,e}$ are inertia moments of rotors, $\beta_{d,e}$ are motors frictional coefficients and $i_{d,e}$ are armature motors currents.

Using equation (5) in (6) and the result in (4), and the force f , from (4), in equation (3), gives

$$K_v = M_v \dot{v} + B_v v \tag{7}$$

where

$$\begin{aligned} K_v &= {}^{\omega}T_v^T R_m K_m \\ M_v &= M_r + {}^{\omega}T_v^T \cdot J_m \cdot {}^{\omega}T_v \\ B_v &= B_r + {}^{\omega}T_v^T (R_m K_m^2 + B_m) {}^{\omega}T_v \end{aligned}$$

2.3 Linear Representation to Mobile Robot Dynamic Model

A state-space model, with output vector $Y = [S \quad \theta_p]^T$, is obtained from equation (7) and described by

$$\begin{cases} \dot{x} = Ax + Be \\ Y = Cx \end{cases} \tag{8}$$

where S is the linear displacement and

$$x = \begin{bmatrix} v_r \\ \omega_r \\ S \\ \theta_p \end{bmatrix} \quad A = \begin{bmatrix} -M_v^{-1} B_v & \vdots & 0_{2\times2} \\ I_{2\times2} & \vdots & 0_{2\times2} \end{bmatrix} \quad B = \begin{bmatrix} M_v^{-1} K_v \\ 0_{2\times2} \end{bmatrix}$$

3 INVERSE SYSTEM

The right inverse system is used as an output controller to force the original system output $Y(t)$ to track a given signal $\begin{bmatrix} U_S & U_\theta \end{bmatrix}$. The inverse system, in this paper, is applied to decouple the original MIMO

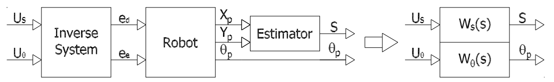

Figure 2: Decoupling based on an inverse system.

(Multiple-Input Multiple-Output) system (8) into two SISO (Single-input Single-Output) systems (Fig. 2).

To get a stable inverse system, it's necessary assume that the system discussed is minimum phase. The inversion algorithm of Hirschorn (Hirschorn, 1979) is applied to decouple the system (8). Deriving the output vector $Y = \begin{bmatrix} S & \theta_p \end{bmatrix}^T$ two times the resulting system is

$$\begin{cases} \dot{x} = Ax + Be \\ \ddot{Y} = C_2 x + D_2 e \end{cases} \tag{9}$$

where D_2 is an invertibility matrix, so the inverse system is

$$\begin{cases} \dot{\widehat{x}} = \widehat{A}\widehat{x} + \widehat{B}\widehat{u} \\ e = \widehat{Y} = \widehat{C}\widehat{x} + \widehat{D}\widehat{u} \end{cases}$$

$$\widehat{A} = A - BD_2^{-1}C_2 \qquad \widehat{C} = -D_2^{-1}C_2$$
$$\widehat{B} = BD_2^{-1}H_2 \qquad \widehat{D} = D_2^{-1}H_2$$
$$\widehat{u} = \begin{bmatrix} 0 & 0 & U_S & U_\theta \end{bmatrix}^T \qquad H_2 = \begin{bmatrix} 0_{2\times 2} & I_{2\times 2} \end{bmatrix}$$

The block diagram in Fig. 2 shows two independent systems, which have the same transfer function $W_S(s) = W_\theta(s) = 1/s^2$ obtained by

$$W_{S,\theta}(s) = [C(sI - A)^{-1}B] \cdot [\widehat{C}(sI - \widehat{A})^{-1}\widehat{B} + \widehat{D}]$$

where $W_{S,\theta}(S) = \begin{bmatrix} W_S(s) & W_\theta(s) \end{bmatrix}^T$.

It's important to remember that linear displacement (S) is not measurable, so an estimated signal is used. The linear model and the inverse system need the exact robot parameters, and we are using a simplified model with uncertain parameters. So a robust adaptive controller will be used to guarantee a good transient under unknown parameters and disturbances. Two controllers are designed, one to each transfer function.

4 CONTROLLER STRUCTURE

The controller structure is divided in five blocks (Fig. 3). First block, which is called kinematic controller, it will calculate reference values (S_{ref} and θ_{ref}) based on desired values (x_d, y_d, θ_d) and absolute positioning system (x_p, y_p, θ_p). Second block is composed by two DMRAC controllers, which are projected to do the robot to reach reference values. These controllers based on references values will supply two control signals (U_S and U_θ) to the inverse system. Third block is the inverse system, fourth block is the robot and the fifth block is the estimator.

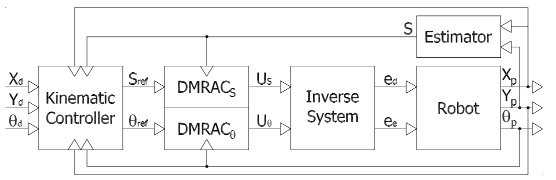

Figure 3: Controller strategy block diagram.

4.1 Estimator

The linear displacement estimator is given by

$$S = \sum_i \mathrm{sgn}(X) \cdot \sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2} \tag{10}$$

where X is

$$X = x_{i+1}\cos(\theta_i) + y_{i+1}\sin(\theta_i) - x_i\cos(\theta_i) - y_i\sin(\theta_i)$$

So, if $X > 0$, we have the linear displacement $(\widehat{\triangle l} > 0)$ and if $X < 0$, we have $\widehat{\triangle l} < 0$.

4.2 Kinematic Controller

The Fig 4 shows the new variables used in kinematic controller as Δl_{pos}, that is the distance between robot and any reference point (x_{ref}, y_{ref}), $\Delta\lambda_{pos}$ is the distance of robot to point R_{pos} that is nearest reference point in robot orientation axis. ϕ_{pos} is the angle of robot orientation axis, $(\Delta\phi_{pos} = \phi_{pos} - \theta_p)$. θ_d is desired orientation angle and γ is the difference between ϕ and θ_d angles $(\gamma = \phi - \theta_d)$.

To get that a robot leaves a point and reaches another point it's necessary $\Delta l_{pos} \to 0$ when $t \to \infty$. Based on decoupled linear model of robot a new auxiliar point R_{pos} (Fig 4) was proposed, to design a positioning controller, because only in this point $\Delta\lambda_{pos} = S_{ref} - S$. So, if $\Delta\lambda_{pos} \to 0$ and $\Delta\phi_{pos} \to 0$, the $\Delta l_{pos} \to 0$. The reference of DMARC$_S$ controller is calculated by

$$S_{ref} = \Delta l_{pos} \cdot \cos(\Delta\phi_{pos}) + S \tag{11}$$

and the reference signal to DMARC$_\theta$ controller is given by

$$\theta_{ref} = \phi_{pos} = \tan^{-1}\left(\frac{y_{ref} - y_p}{x_{ref} - x_p}\right) \tag{12}$$

where the equations (11) and (12) represent the positioning controller and are using only informations from absolute positioning system.

This work uses a mobile reference system to generate new points to the positioning controller for each step of the algorithm. It's based just in the robot kinematic model. The kinematic controller objective is to

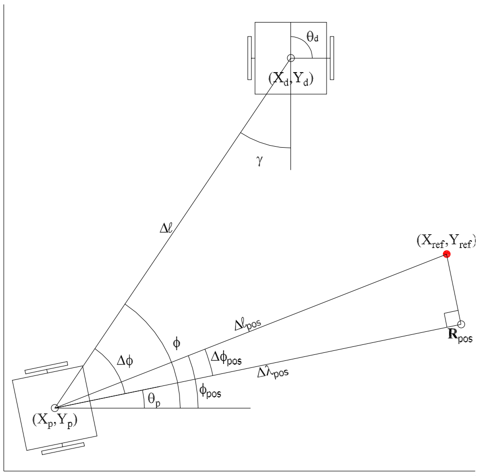

Figure 4: Mobile robot coordinates.

do $\theta_p = \theta_d$ and $\Delta l \to 0$, when $t \to \infty$. For this, we have to do $\theta_p \to \phi_{pos}$, $\phi_{pos} \to \phi$ and $\phi \to \theta_d$, at the same time that $\Delta l \to 0$ when $t \to \infty$.

Based on the functioning of the positioning controller, it was proposed that each new reference point has the same distance between robot and desired point. So

$$\Delta l = \Delta l_{pos} = \sqrt{(x_{ref} - x_p)^2 + (y_{ref} - y_p)^2} \qquad (13)$$

and that the angle of this point is

$$\tan^{-1}\left(\frac{y_{ref} - y_p}{x_{ref} - x_p}\right) = \phi + \gamma \qquad (14)$$

To the equations (13) and (13), we obtain the following equations to positioning controller

$$\begin{cases} x_{ref} &= x_p + \Delta l \cdot \cos(\phi + \gamma) \\ y_{ref} &= y_p + \Delta l \cdot \sin(\phi + \gamma) \end{cases}$$

4.3 Dual-Mode Adaptive Robust Controller

The DMARC uses the compact VS-MRAC structure, that was proposed by Araujo and Hsu (Araujo and Hsu, 1990), changing just the last control signal u_1 (Fig 5) to choose between a VS-MRAC and conventional MRAC controllers.

Consider a linear single-input/single-output time invariant plant with uncertain parameters and transfer function,

$$W(s) = k_p \frac{n_p(s)}{d_p(s)} = \frac{1}{s^2 + \alpha_1 s + \alpha_2}$$

with input u and output y. The reference model is

$$M(s) = k_m \frac{n_m(s)}{d_m(s)} = \frac{k_m}{s^2 + \alpha_{m1} s + \alpha_{m2}}$$

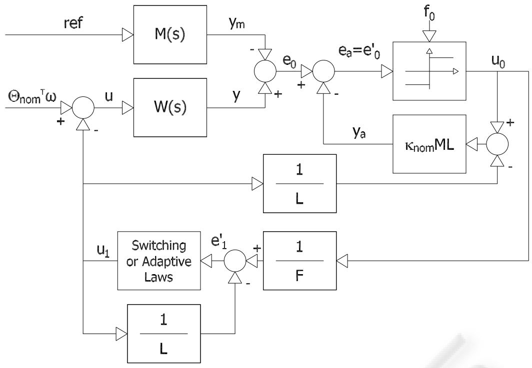

Figure 5: Block diagram of DMRAC controller with $n^* = 2$.

with input ref and output y_m.

The aim is to find u such that the output error

$$e_0 = y - y_m$$

tends to zero asymptotically for arbitrary initial conditions and arbitrary piece-wise continuous uniformly bounded reference signals $ref(t)$.

Following conventional assumptions are made:

1. the plant is observable, controllable, minimum phase ($n_p(s)$ is Hurwitz) and with unknown or uncertain bounded parameters. $d_p(s)$ and $n_p(s)$ are monics polynomials with degree $[d_p(s)] = 2$, degree $[n_p(s)] = 0$ and relative degree $n^* = 2$;
2. the reference model is stable and minimum phase ($n_m(s)$, $d_m(s)$ are Hurwitz). $d_m(s)$ and $n_m(s)$ are monics polynomials with relative degree known (n^*) ($[M(s)]$ has the same relative degree than $W(s)$ and signals $\mathrm{sgn}(k_p) = \mathrm{sgn}(k_m) > 0$ (positive, for simplicity).
3. only input/output measurements are used to find control law $u(t)$.

The following input and output filters are used

$$\begin{cases} \dot{Q}_1 = -\lambda Q_1 + gu \\ \dot{Q}_2 = -\lambda Q_2 + gy \end{cases}$$

where $Q_1, Q_2 \in \Re$ and λ is chosen such that $N_m(s)$ is a factor of $det(sI - L)$. The regressor vector is defined as $\omega^T = \begin{bmatrix} Q_1 & y & Q_2 & ref \end{bmatrix}^T$, and the control u is given by

$$u = \Theta^T \omega$$

where $\Theta^T = \begin{bmatrix} \Theta_{Q1} & \Theta_n & \Theta_{Q2} & \Theta_{2n} \end{bmatrix}^T$ is the adaptive parameter vector.

Considering the above assumptions exists a unique constant vector Θ^*, such that the transfer function of the closed-loop plant $W(S)$, with $u = \Theta^{*T}\omega$, is $M(s)$ (Matching condition). But θ^* is obtained of exact plant parameters. Usually this is not possible in practice, then the Θ is adapted until $e_0 \to 0$ when $t \to \infty$, and eventually (under some signal richness condition) $\Theta \to \Theta^*$.

The vector Θ^* is obtained as following

$$\Theta^* = \begin{bmatrix} (\alpha_1 - \alpha_{m1})/g \\ (\lambda(\alpha_1 - \alpha_{m1}) + (\alpha_2 - \alpha_{m2}) - \alpha_1 g \Theta^*_{v1})/g \\ (\lambda(\alpha_2 - \alpha_{m2}) - \alpha_2 g \Theta^*_{v1} - k_p \lambda \Theta^*_n)/(k_p \cdot g) \\ k_m/k_p \end{bmatrix}$$

The θ_{nom} is a nominal value of adaptive parameter vector (ideally, $\Theta_{nom} = \Theta^*$) and it is calculated as Q^* and based on the decoupled plants $(W_S(s), W_\theta(s))$ where $\alpha_1 = \alpha_2 = 0$.

Suppose that exists a polynomial $L(s) = (s+\delta)$ of degree $N = 1$ where $\delta > 0$ and $\delta \in \Re$ such that $M(s)L(s)$ is SPR (Strictly Positive Real). Consider the following auxiliary signal (a prediction of the output error e_0)

$$y_a = ML\Theta_{2n+1}[L^{-1}u - \Theta^T L^{-1}\omega]$$

where Θ_{2n+1} and Θ are estimates for $1/\Theta^*_{2n}$ and Θ^* (Matching parameters), respectively. The following augmented error is defined

$$e_a = (y - y_m) - y_a = e_0 - y_a$$

Narendra proposes the following modification to guarantee the global stability of the adaptive system

$$\begin{aligned} y_a &= ML[\Theta_{2n+1}(L^{-1}\Theta^T - \Theta^T L^{-1})\omega \\ &\quad + \alpha e_a (L^{-1}\omega)^T (L^{-1}\omega)], \quad \alpha > 0 \end{aligned}$$

To update Θ and Θ_{2n+1} are used the following adaptive integral laws to MRAC controller

$$\begin{aligned} \dot{\Theta} &= -e_a(L{-}1\omega) \\ \dot{\Theta}_{2n+1} &= e_a(L{-}1\Theta^T - \Theta^T L^{-1})\omega \end{aligned}$$

To VS-MRAC controller, we have to introduce the following filtered signals $\xi_0 = L^{-1}\omega$, $\xi_1 = \omega$, $\chi_0 = L^{-1}u$, $\chi_1 = u$.

The upper bounds are defined by

$$\begin{aligned} \overline{\Theta}_{11} &> |\Theta^*_{Q1} - \Theta_{nom,Q1}| & \overline{\Theta}_{12} &> |\Theta^*_n - \Theta_{nom,n}| \\ \overline{\Theta}_{13} &> |\Theta^*_{Q2} - \Theta_{nom,Q2}| & \overline{\Theta}_{14} &> |\Theta^*_{2n} - \Theta_{nom,2n}| \end{aligned}$$

$$\begin{aligned} \overline{\Theta}_1 &= \left[\overline{\Theta}_{11}\ \overline{\Theta}_{12}\ \overline{\Theta}_{13}\ \overline{\Theta}_{14}\right]^T \\ \overline{\Theta}_{0j} &> \rho \cdot \overline{\Theta}_{1j}, \quad j = 1,2,3,4 \end{aligned}$$

$$\overline{\kappa} > \left| \frac{\kappa^* - \kappa_{nom}}{\kappa_{nom}} \right|$$

where $\rho = \kappa^*/\kappa_{nom}$ (ideally $\rho = 1$) and κ_{nom} is a nominal value for $k^* = 1/\Theta^*_{2n}$ (it is assumed $k_{nom} \neq 0$). Further, it is defined

$$u_{nom} = \Theta^T_{nom}\omega \tag{15}$$

We have plants with $n^* = 2$ and in this case, a VS-MRAC structure, needs a chain of auxiliary errors to get the matching condition. The switching laws are chosen so that the auxiliary errors $e'_i (i = 0,1)$ become sliding modes after some finite time. The equivalent controls $((u_i)_{eq})$ are, in practice, obtained of (u_i) by means of a low pass filter (F) with high enough cut-off frequency.

To adjust the DMARC controller an expression for the μ parameter, based on the idea of the Takagi-Sugeno model, was used. This expression is given by (16), where ℓ is a parameter to be adjusted.

$$\mu = \mathrm{e}^{-(e'_1)^2/\ell} \tag{16}$$

Notice that when $e'_1 \to 0$, $\mu \to 1$ and the algorithm is the MRAC. When e'_1 becomes reasonably high, μ assumes a value sufficiently small, tending to the VSMRAC algorithm. The ℓ parameter has a great importance in the transition between MRAC and VS-MRAC. If ℓ is small the VS-MRAC action will be big.

The DMARC algorithm applied to plants with relative degree $n^* = 2$ is summarized in the Table 1.

Table 1: Algorithm of DMARC controller.

$$\begin{aligned} u &= -u_1 + u_{nom} \\ y_a &= \kappa_{nom} ML \cdot [u_0 - L^{-1}u_1] \\ e'_0 &= e_a = e_0 - y_a \\ (u_0)_{eq} &= F^{-1}(u_0) \\ e'_1 &= (u_0)_{eq} - L^{-1}(u_1) \\ f_0 &= \overline{\kappa}|\chi_0 - \Theta^T_{nom}\xi_0| + \overline{\Theta}^T_0 \cdot |\xi_0| \\ f_1 &= \overline{\Theta}^T_1 \cdot |\xi_1| \\ u_0 &= f_0 \cdot \mathrm{sgn}(e'_0) \\ u_1 &= -\Theta^T_1 \omega \\ \mu\dot{\Theta}_1 &= -\alpha\Theta_1 - \alpha\overline{\Theta}_1 \cdot \mathrm{sgn}(e'_1\omega), \quad \alpha > 0 \end{aligned}$$

5 RESULTS

The result showed in this paper is based on simulation of a micro robot soccer structure. The simulated result considers the main nonlinearities (see (Laura et al., 2006)) as dead zone between $\pm 150mV$ and saturation to values out of $\pm 10V$. It also has noise in input and output signals. The noise was calculated by a random variable with a normal distribution of probability. To inputs noises (e_d, e_e) we have values between $\pm 100mV$. To output noises (x_p, y_p and θ_p) we have values between $\pm 1cm$ to cartesian points and $\pm 8,5^o$ to angle.

The unknown parameters are in the dynamic model, and they are obtained of the exact mobile robot model with a normal distribution random variable of probability between 1% and 10%. The parameters used in the simulation are of a realistic robot, that is not symmetrical.

A controller DMRAC is applied to each plant ($W_S(s)$ and $W_\theta(s)$ with relative degree $\rightarrow n^* = 2$). Choosing a reference model to each plant, we have

$$M_S(s) = \frac{2.25}{s^2+3s+2.25} \qquad M_\theta(s) = \frac{100}{s^2+20s+100}$$

and as filters

$$\begin{cases} \dot{Q}_{S1} = -1.5Q_{S1} + 1.5U_S \\ \dot{Q}_{S2} = -1.5Q_{S2} + 1.5S \end{cases} \quad \begin{cases} \dot{Q}_{\theta 1} = -10Q_{\theta 1} + 10U_\theta \\ \dot{Q}_{\theta 2} = -10Q_{\theta 2} + 10\theta_p \end{cases}$$

We choose the polynomial $L_S(s) = s + 1.5$ to DMARC$_S$ with $F = 22s+1$, $\ell_S = 0.7551$, $\alpha_S = 0.05$ and

$$\begin{aligned} \Theta_{S,nom} &= [\ -2.0\ \ -6.75\ \ \ 4.5\ \ \ 2.25\ ]^T \\ \overline{\Theta}_{S,1} &= [\ 2.420\ \ 13.282\ \ 2.657\ \ 0.024\ ]^T \end{aligned}$$

We choose the polynomial $L_\theta(s) = s + 10$ to DMARC$_\theta$ with $F = 56s+1$, $\ell_\theta = 0.0051$, $\alpha_\theta = 0.05$ and

$$\begin{aligned} \Theta_{\theta,nom} &= [\ -2.0\ \ -300.0\ \ \ 200.0\ \ 9.7\ ]^T \\ \overline{\Theta}_{\theta,1} &= [\ 0.073\ \ 101.161\ \ 175.750\ \ 0.1\ ]^T \end{aligned}$$

The Fig. 6 shows a simulation result of the closed loop system. The robot going from the initial point $\begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$ to desired point $\begin{bmatrix} 1 & 1 & 0 \end{bmatrix}^T$. When the robot reaches a circle of $1cm$ radius with center in desired point, it's considered that the task was accomplished. The controller shows a good transient behavior that means in $3.49s$ the robot reaches the desired position without vibrations.

The simulation result is closed to real physical systems, so it includes the main nonlinearities, a plant with unknown parameters, bounded disturbances in the absolute positioning system and in motors driver system. It uses a sampling interval of $10ms$. These facts did not affect the closed loop system behavior.

In the Fig. 6 is possible identify the functioning of the DMARC controller. When the error is big a DMARC operates as VS-MRAC and when the error is small the DMRAC operates as MRAC.

6 CONCLUSIONS

In this paper a controller that decouples multivariable system into two monovariable systems was proposed. For each monovariable system an adaptive robust controller is applied to get desired responses.

The closed loop system with the proposed controller presented a good transient response and robustness to disturbances and errors in the absolute positioning system and in driver system.

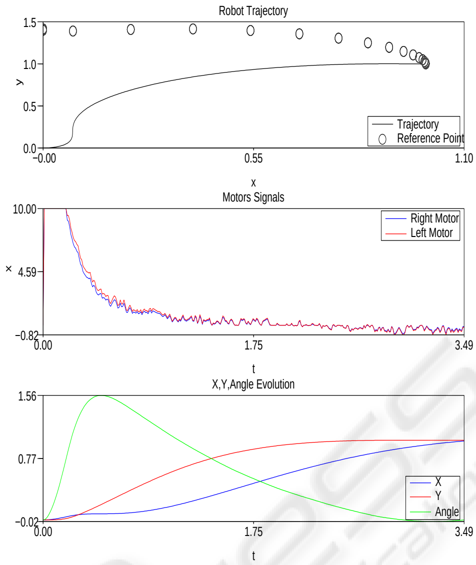

Figure 6: Robot signals (control signal and output plant).

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