NEW RESULTS ON DIAGNOSIS BY FUZZY PATTERN RECOGNITION

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Keywords: Pattern recognition, Fuzzy pattern matching, Nearest Neighbours techniques, Multi-criteria decision.

Abstract: We use the classification method Fuzzy Pattern Matching (FPM) to realize the industrial and medical diagnosis. FPM is marginal, i.e., its global decision is based on the selection of one of the intermediate decisions. Each intermediate decision is based on one attribute. Thus, FPM does not take into account the correlation between attributes. Additionally, FPM considers the shape of classes as convex one. Finally the classes are considered as equi-important by FPM. These drawbacks make FPM unusable for many real world applications. In this paper, we propose improving FPM to solve these drawbacks. Several synthetic and real data sets are used to show the performances of the Improved FPM (IFPM) with respect to classical one as well as to the well known classification method K Nearest Neighbours (KNN). KNN is known to be preferment in the case of data represented by correlated attributes or by classes with different a priori probabilities and non convex shape.

1 INTRODUCTION

In statistical Pattern Recognition (PR) (Dubuisson, 2001), historical patterns about system functioning modes are divided into groups of points, called classes, using unsupervised learning method (Duda, 2001) or human experience. These patterns, or points, with their class assignments, constitute the learning set. A supervised learning method uses the learning set to build a classifier that best separates the different classes in order to minimize the misclassification error. This separation, or classification, is realized by using a membership function, which determines the likelihood or the certainty that a point belongs to a class.

The membership function can be generated using Probability Density Function (PDF) estimation based methods or heuristics-based ones. In the first category, the membership function is equal to either the PDF or to the probability a posterior. The estimation of PDF can be parametric, as the bayesian classifier (Dubuisson, 1990), or non parametric, as the Parzen window (Dubuisson, 2001), voting k nearest neighbour rules (Denoeux, 1998), (Denoeux, 2001) and (Dubuisson, 2001), or by histograms (Sayed Mouchaweh, 2004), (Medasani, 1998). In heuristic-based methods (Medasani, 1998), the shape of the membership function and its parameters are predefined either by experts to fit the given data set, or by learning to construct directly the decision boundaries as the potential functions (Dubuisson, 1990) and neural networks (Ripley, 1996), or the clustering methods as Fuzzy C-Means (Medasani, 1998).

One of the applications of PR is the diagnosis of industrial systems for which no mathematical or analytical information is available to construct a model about the system functioning. Each functioning mode, normal or faulty, is represented by a class. The problem of diagnosis by PR becomes a problem of classification, i.e., the actual functioning mode can be determined by knowing the class of the actual pattern, or observation, of the system functioning state.

There are many fuzzy classification methods in the literature. The choice of one of them depends on the given application and the available data. We use the method Fuzzy Pattern Matching (FPM) (Devillez, 2004b), (Grabish, 1992) and (Sayed Mouchaweh, 2002a) because it is simple, adapted to incomplete database cases and has a small and constant classification time. FPM is a marginal classification method, i.e., its global decision is
based on the selection of one of the intermediate decisions. Each intermediate decision is calculated using a membership function based on a probability histogram for each class according to each attribute. Thus, FPM is not adapted to work with data represented in a space of correlated attributes. Additionally, it does not respect the shape of classes if this shape is non convex. Finally, all the classes are considered as equi-important, i.e., with the same a priori probabilities.

In this paper we propose a solution to develop FPM to take into account the correlation between attributes as well as the class importance and its shape if this shape is non convex. The paper is structured as follows. Firstly, the functioning of FPM is explained briefly. Then the limits of FPM are discussed using some synthetic examples. Next, several real examples are used to evaluate the performances of the Improved FPM (IFPM) with respect to the classical one as well as to the well known classification method K Nearest Neighbours (KNN). This evaluation is based on the misclassification rate. Finally a conclusion ends this paper.

2 FUZZY PATTERN MATCHING

FPM, described in (Devillez, 2004b), (Grabish, 1992) and (Sayed Mouchaweh, 2002a), is a supervised classification method based on the use of probability histograms. Let $C_1$, $C_2$, ..., $C_n$ denote the classes described by a attributes. These attributes provide different points of view about the membership of an incoming point in the different classes. The functioning of FPM involves two phases: the learning and the classification ones.

2.1 Learning Phase

In the learning phase, the data histograms are constructed for each class according to each attribute. The number of bins $h$ for a histogram is experimentally determined. This number has an important influence on the performances of FPM (Sayed Mouchaweh, 2002b). The histogram upper and lowest bounds can be determined either as the maximal and minimal learning data coordinates or by experts. In this paper, we adopted the first manner and we have added a tolerance $Tol$ to adjust this histogram in order to maximize FPM performances. The height of each bin is the number of learning points located in this bin. The probability distribution $p_j$ of the class $C_i$ according to the attribute $j$ is calculated by dividing the height of each bin $b_k$ by the total number $N_i$ of learning points belonging to the same class. Then these probabilities $\{p_j(y_{b_k}), k \in \{1,2,\ldots,h\}\}$ are assigned to bins centres $y_{b_k}$. The PDF $P_j$ is obtained by a linear linking between the bins heights centres. Indeed if we have a large number of data, the normalized histogram can be assumed to approximate the PDF. In order to take into account the uncertainty and the imprecision contained in the data, the probability distribution $p_j$ is converted into possibility one $\pi_j$.

This conversion is realized using the transformation of Dubois and Prade (Dubois, 1993) defined as:

$$\pi_j(y_{b_k}) = \frac{1}{h} \min\{p_j(y_{b_k}), p_j(y_{b_k})\} \quad (1)$$

We have chosen this transformation for the good results which it gives in PR applications (Sayed Mouchaweh, 2002b), (Sayed Mouchaweh, 2006). A linear linking between bins heights centres converts the distribution of possibilities $\{\pi_j(y_{b_k}), k \in \{1,2,\ldots,h\}\}$ into density one $\Pi_j$. This operation is repeated for all the attributes of each class.

2.2 Classification Phase

The membership function $\mu_j$ for each class $C_i$ and according to each attribute $j$ is considered to be numerically equivalent to the possibility distribution (Zadeh, 1978). Thus, the classification of a new point $x$, whose values of the different attributes are $x_1$, $x_2$, ..., $x_n$, is made in two steps:

- Determination of the possibility membership value $\pi_j$ of the point $x$ to each class $C_i$ according to the attribute $j$ by a projection on the corresponding possibility density $\Pi_j$;
- Merging all the possibility values $\pi_1, \ldots, \pi_n$, concerning the class $C_i$, into a single one by an aggregation operator $H$:

$$\pi_i = H(\pi_1, \ldots, \pi_n) \quad (2)$$

The result $\pi_i$ of this fusion corresponds to the global possibility value that the new point $x$ belongs to the class $C_i$. The operator $H$ can be a
multiplication, a minimum, an average, or a fuzzy integral (Grabish, 1992). Finally, the point \( x \) is assigned to the class for which it has the maximum membership value.

3 LIMITS OF FPM

3.1 Classes of Non Convex Shape

FPM is similar to the naive Bayesian classifier who supposes the attributes statistically independent. This classifier defines the probability a posteriori \( \Phi(C|\, x) \) that \( x \) belongs to the class \( C_i \) by:

\[
\Phi(C_i|\, x) = \prod_{j=1}^{d} p(x_j'|C_i)P(C_i)
\]

(3)

Where \( p(x_j'|C_i) \) is the marginal conditional density of the attribute \( j \) given the class \( C_i \) and \( P(C_i) \) denotes the a priori probability of the class \( C_i \). If the a priori probabilities of classes are the same, then the equation (3) becomes similar to the equation (2). Thus FPM, as all the other marginal methods, does not take into account neither the correlation between attributes nor the class importance or its shape if it is non convex. Indeed, FPM produces always rectangular membership level curves for all the possible class shapes. Figure 1.a and Figure 1.b present respectively the membership level curves obtained by FPM for a class defined either by two linear or non linear correlated attributes. We can notice that these levels do not respect the class shape.

Figure 1.a: Membership level curves obtained by FPM for a class defined by linear correlated attributes.

In (Devillez, 2004a), two improvements were proposed to integrate the information about class shape in the learning phase. These two improvements are based on the division of each class into several sub ones. However these improvements have some drawbacks as the critical determination of the value of some parameters, like the number of subclasses, the expensive computation time and the rejection areas inside classes, i.e., areas in which points are not assigned to any class.

3.2 Classes with Correlated Attributes

The XOR data are a classical example used in the literature to show the correlation between attributes. Indeed any classifier needs to use the information issued from all the attributes to take a correct decision. Thus FPM is not adapted for this type of data since its decision is based on the selection of one attribute. The Figure 2 shows XOR data in a representation space of two attributes as well as the membership level curves, obtained by FPM for the class 1. We can see that the classes have a convex shape and that FPM does not distinguish between the points of the class 1 and the ones of the class 2. Thus the improvements proposed by Devillez (Devillez, 2004a) cannot solve this problem since they were developed to be adapted to the class shape and not to the case of correlated attributes. Same remark can be noticed for the membership level curves of the class 2.

Figure 2: The membership level curves, obtained by FPM, for XOR data for the class 1.

In (Cadenas, 2004) a solution to make FPM operant in the case of data with correlated attributes is presented. This solution uses the Parzen Window method to construct the membership functions of each class according to one main feature and, if it is necessary, to one auxiliary feature. The
classification phase uses the fuzzy integral as an aggregation operator. However, this solution is consuming of time and has an exponential complexity according to the number of attributes. In addition, this solution does not work in the case of XOR described by more than two attributes.

4 IMPROVED FPM (IFPM)

We propose a solution to make FPM operant in the case of data with correlated attributes and classes of non convex shape. This solution looks for the relationship between the attributes of the representation space using the learning set. This relationship is represented by a correlation matrix between the bins of the histogram of the first attribute and all the other bins of the other attributes. This solution does not require any determination of any supplementary parameter. The functioning of IFPM is divided into two phases: learning and classification ones.

4.1 Learning Phase

The learning phase of IFPM is similar to the one of FPM but it integrates in addition the information about the zones of learning points inside the representation space. Each zone is results from the intersection of the bins of a learning point according to all attributes.

Let \( X \) denotes the learning set which contains \( N \) points \( x \) divided into \( c \) classes inside a representation space of \( a \) attributes. Each class \( C_i \) contains \( n_i \) points: \( i \in \{1,2,\ldots,c\} \). Each histogram for each attribute \( j \), \( j \in \{1,2,\ldots,a\} \), contains \( h \) bins \( b_{ij} \), \( k_j \in \{1,2,\ldots,h\} \). The correlation matrix \( B \) for the learning set \( X \) is defined as follows:

\[
B = [B^1, B^2, \ldots, B^a]
\]  

(4)

Where \( B^i \) is the correlation matrix for the class \( C_i \). This matrix can be calculated as follows:

\[
B^i = \{ \alpha_{bij} \in \{1, 0\} \}
\]

(5)

Where \( x \) belongs to the zone resulting by the intersection of the bins \( b_x = [b_{1x}, b_{2x}, \ldots, b_{ax}] \). \( \alpha_{bij} \) is the correlation factor between the bin \( b_{ij} \) of the first attribute and all the other bins of the other attributes according to the class \( C_i \). This correlation factor can be calculated using the following equation:

\[
\forall x \in X, C(x) = C_i : \\
\text{If } x \in b^i_1 \land b^i_2 \land b^i_3 \land \ldots \land b^i_h \Rightarrow \alpha_{ij} = 1 \\
\text{Else} \Rightarrow \alpha_{ij} = 0
\]

(6)

Where "\( \land \)" and "\( \land \)" denote respectively the AND and intersection operators.

The Figure 3 shows a simple example of the calculation of the matrix \( B \) for the case of a representation space containing one class defined by two attributes. We can notice that the bins \( b^1_1 \) and \( b^2_2 \) are correlated because a learning point is located in the zone resulting by the intersection of these two bins. Thus the correlation factor of this zone \( \alpha_{ij}^1 \) is equal to 1 in the matrix \( B \). While there is no learning point in the zone of the intersection of the bins \( b^1_1 \) and \( b^2_2 \). The correlation factor of this zone \( \alpha_{ij}^1 \) is equal to zero in \( B \).

![Correlation Matrix obtained by IFPM for the class \( C_i \) inside \( \mathbb{R}^2 \).](image)

4.2 Classification Phase

The classification of a new point \( x \) starts by determining its bins according to each attribute \( b_x = [b^1_x, b^2_x, \ldots, b^a_x] \). Then the possibility membership value \( \pi_i \) for each class \( C_i \) is calculated exactly as FPM if and only if \( \alpha_{ij}^i = 1 \). In order to take onto account the importance of a class, the possibility of each class is multiplied by its a priori probability. If \( \alpha_{ij}^i = 0 \), the point \( x \) will be rejected according to the class \( C_i \) i.e., \( \pi_i = 0 \). Finally the point \( x \) will be assigned to the class for which it has the highest possibility membership value. The Figure 4
represents the steps of the classification phase of IFPM

![Diagram of IFPM classification steps]

Figure 4: Algorithm of the classification phase of IFPM.

The Figure 5.a and Figure 5.b show the membership level curves issued from the application of IFPM on the two examples of the Figure 1.

![Figure 5.a: Membership level curves obtained by IFPM for a class defined by linear correlated attributes.](image)

![Figure 5.b: Membership level curves obtained by IFPM for a class defined by non linear correlated attributes.](image)

We can find that these curves respect the non spherical and non convex shapes of classes.

Equally, the Figure 6 shows these curves for the class 1 obtained by IFPM, for XOR data. We can find that IFPM discriminates well the points of the class 1 and the class 2. Same result can be obtained for the class 2.

![Figure 6: Membership level curves, obtained by IFPM, for XOR data for class 1.](image)

5 EXPERIMENTAL RESULTS

We will test the performances of IFPM with FPM and classical as well as Fuzzy KNN (FKNN) with crisp initialisation and several values of k between 1 and 15, using the misclassification rate as evaluation criterion. For that we use four data sets: XOR problem, Spiral, Pima Indians diabetes (Newman, 1998) and Ljubljana Breast Cancer (LBC) data sets (Newman, 1998). XOR problem data set is composed of 2 classes in a representation space of 5 attributes. We have chosen 5 attributes to test IFPM performances in the case of a representation space characterized by more than 2 correlated attributes. Spiral data are represented in terms of evenly spaced samples from a non linear two-dimensional transformation of the Cartesian coordinates. We have chosen this data set because the classes are non linearly separable. The Pima and LBC data sets are known to be strongly non Gaussian. The number of points in each class, the number of classes and the number of attributes are depicted in Table 1.

We have used the leave-one-out method to calculate the Misclassification Rate (MR) because it gives a pessimistic unbiased estimation of MR. We integrate also the Rejection Rate (RR) to indicate the number of points which are not assigned to any class. Indeed, it is better to reject a point than to misclassify it. The Table 2 shows the obtained results for the three methods. These results are obtained using the optimal values of h and Tol for FPM and IFPM, and K for KNN. We can conclude that IFPM provides better results, according to the evaluation criterion and for the four data sets, than FPM and both, KNN and FKNN.
Table 1: Data Sets used for the test.

<table>
<thead>
<tr>
<th>Data set</th>
<th>Classes</th>
<th>DIM</th>
<th>Per Class</th>
</tr>
</thead>
<tbody>
<tr>
<td>XOR</td>
<td>2</td>
<td>5</td>
<td>{400,400}</td>
</tr>
<tr>
<td>Spiral</td>
<td>2</td>
<td>2</td>
<td>{970,970}</td>
</tr>
<tr>
<td>Pima</td>
<td>2</td>
<td>8</td>
<td>{268,500}</td>
</tr>
<tr>
<td>LBC</td>
<td>2</td>
<td>9</td>
<td>{218,68}</td>
</tr>
</tbody>
</table>

Table 2: Comparison between IFPM, FPM, KNN and FKNN, using leave-one-out technique, according to Rejection Rate (RR) and to Misclassification Rate (MR).

<table>
<thead>
<tr>
<th>Method</th>
<th>FPM</th>
<th>IFPM</th>
<th>KNN</th>
<th>FKNN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Criterion</td>
<td>MR (%)</td>
<td>RR (%)</td>
<td>MR (%)</td>
<td>RR (%)</td>
</tr>
<tr>
<td>XOR</td>
<td>44.12</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Spiral</td>
<td>18.24</td>
<td>0</td>
<td>0</td>
<td>1.5</td>
</tr>
<tr>
<td>Pima</td>
<td>30.73</td>
<td>0.78</td>
<td>19.53</td>
<td>20.7</td>
</tr>
<tr>
<td>LBC</td>
<td>25.87</td>
<td>1.05</td>
<td>10.14</td>
<td>30.42</td>
</tr>
</tbody>
</table>

6 CONCLUSIONS

In this paper, we have proposed a solution to adapt the classification method Fuzzy Pattern Matching (FPM) to be operant in the case of classes with correlated attributes as well as the class importance and its shape if this shape is not convex. The integration of this solution in FPM is called Improved FPM (IFPM). The performances of IFPM are compared with the ones of FPM, K Nearest Neighbours (KNN) and Fuzzy KNN (FKNN) using the misclassification rate as evaluation criterion. This comparison is realized according to four data sets. We have also used XOR problem and Spiral data which are widely used to study the correlation between attributes. In addition, we have used Pima Indians diabetes and Ljubljana Breast Cancer data sets which are known to be strongly non Gaussian with different a priori probabilities. The misclassification rate obtained by IFPM is better than the one of FPM, KNN and FKNN. However, IFPM rejects more points than the previous methods. Anyway, it is better to reject a point than to misclassify it.

REFERENCES


