Abstract: In syntactic pattern recognition an object is described by symbolic data. The problem of recognition is to determine whether the describing mathematical structure, for instance a graph, belongs to the language generated by a grammar describing the mentioned mathematical structures. So called ETPL(k) graph grammars are a known class of grammars used in pattern recognition. The approach in which ETPL(k) grammars are used was generalized by using probabilistic mechanisms in order to apply the method to recognize distorted patterns. In this paper the next step of the method generalization is proposed. The ETPL(k) grammars are improved by fuzzy sets theory. It turns out that the mentioned probabilistic approach can be regarded as a special case of the proposed one. Applications to robotics are considered as well.

1 INTRODUCTION

The fundamental idea in syntactic pattern recognition is using of symbolic data like strings, trees and graphs for representation of a class of recognized objects (Chen et al., 1991; Fu, 1982; Jakubowski, 1997; Jakubowski and Stapor, 1999). The general scheme of syntactic pattern recognition and a scene analysis is following (Fu, 1982). After pre-processing the recognized object is segmented in order to recognize the primitives the pattern consists of and relations between them. Decision whether the analysed pattern representation belongs to the class of objects describing by a given grammar is made basing on the parsing algorithm. This classical approach can be applied in robotics, for instance in vision systems and in manufacturing for description and analysis of the production process (Chen et al., 1991; Yeh et al. 1993). It seems also be effective for applying in multi-agent systems, particularly in embodied cognitive ones because such agents should be equipped with symbolic and explicit representation of the surrounding world in order to analyse the scene they act on (Ferber, 1999; Scheier and Pfeifer, 1999). For instance in (Kok et al., 2005) so called coordination graphs are used for solving a behaviour management problem in a multi-robot system. In this graph a node represents an agent and an edge indices that a corresponding agents have to coordinate their actions.

The use of graph grammars for syntactic pattern recognition is relatively rare because of difficulties in building a syntax analyser of such grammars. Therefore every result in building efficient parser for graph grammars is valuable. An example of such result is a parser for, so called, ETPL(k) (embedding transformation-preserving production-ordered k-left nodes unambiguous) grammars introduced in (Flasiński, 1993 and 1998). An efficient parsing algorithm for ETPL(k) (embedding transformation-preserving production-ordered k-left nodes unambiguous) grammars introduced in (Flasiński, 1993). An efficient parsing algorithm for ETPL(k) graph grammars, which the computational complexity is $O(n^2)$, has been constructed in (Flasiński, 1993). The so-called IE (indexed edge-unambiguous) graphs have been defined in (Flasiński, 1993) for a description of pattern (scenes) in syntactic pattern recognition. Nodes in an IE graph denote pattern primitives. Edges between two nodes in an IE graph represent...
spatial relations between pattern primitives. However, in practice, structural descriptions may contain pattern distortions. An idea of a probabilistic improvement of syntactic recognition of distorted patterns represented by graphs is described in (Flasiński and Skomorowski, 1998, Skomorowski 1998) and (Skomorowski, 1999). To take into account all variations of a distorted pattern under study, a probabilistic description of the pattern was introduced. A random IE graph approach (Flasiński and Skomorowski, 1998, Skomorowski, 1999, Skomorowski, 2000) is proposed for such a description and an efficient parsing algorithm for IE graphs is presented. Its computational complexity is $O(n^2)$ as well.

The purpose of this paper is to present an idea of approach to syntactic recognition of fuzzy patterns represented by fuzzy IE graphs, followed the example of random IE graphs used for distorted pattern description. It turns out that, in a way, the fuzzy approach is a generalization of the probabilistic one. Fuzziness allows us not only described distortions in analysed patterns but also give us possibility to describe in proper way patterns that can not be presented univocally. Furthermore there are a wide class of problems in which objects and/or spatial relations are described by fuzzy sets.

## 2 MOTIVATIONS

In this section a few example, in which the fuzzy-syntactic approach seems to be natural, are presented.

### 2.1 First Example

Assume that during a manufacturing process a robotic inspection system checks type of a hole in a making elements, for instance plates, and spatial relations between holes. Assume also that there are a few standard types of holes and circular and quadratic ones are among them – Fig.1a. Let, furthermore, the inspection system be based on a syntactic pattern recognition approach in which the holes are represented by nodes of graphs and spatial relations between holes by graph edges – see Fig.1b. A quadratic hole with rounded vertices can be regarded as a fuzzy object with partial membership to classes of both circular and quadratic holes – see Fig.1. In this example nodes description as fuzzy sets is a natural approach. In this case membership functions describing fuzzy sets can be define basing on axiomatic method (Bielecka, 2006).

![Figure 1: Holes in plate and their graph representation.](image)

### 2.2 Second Example

Considering the previous example assume that robotic inspection of technological process is based on statistical distribution of inaccuracy frequencies (Flasiński and Skomorowski, 1998, Noori and Radford, 1995). If a hole is made in a sufficient accuracy it is accept by the system. Not only the hole shape but also its location should be taken into consider. Since inaccuracies of holes location influence each other, the simple statistical analysis can be insufficient to make a decision. In such a case a fuzzy inference can be applied. Then, holes and their locations can be represented by fuzzy sets and membership functions can be calculated using the statistical distribution according to the methodology described in (Bielecka, 2006). Let, like in the first example, the inspection system is based on a syntactic pattern recognition in which the holes are represented by nodes of graphs and spatial relations between holes by graph edges. In this example both the graph nodes and its edges would be described as fuzzy sets. Automatic focusing vision system for inspection of size and shape and positions of small holes in the context of precision engineering, described in (Han and Han, 1999), is an example of a system performing such type of task.

### 2.3 Third Example

Consider an autonomous mobile agent. Assume that it has to navigated in an unchanging environment. A helicopter flying autonomously in a textured urban environment is an example of such agent (Muratet et al., 2005). As it has been already mentioned it should be equipped with symbolic representation of the surrounding world in order to analyse the scene they act on (Ferber, 1999; Scheier and Pfeifer, 1999). Let its vision system be a syntactic one based on graph representation of the spatial relationships between obstacles the agent should navigate among. Let according to, for instance, the optimization requirements, the system prefers one direction but
admits also another ones allowing to navigate without collision. In such a case the scene would be represented by a classical (i.e. not fuzzy) IE graph but directions the agent can choose would be represented by fuzzy sets – see Fig.2. The decision making system would be based on fuzzy inference.

Figure 2: Detection of possible directions of motion.

2.4 Fourth Example

Let us consider computer-aided analysis and recognition of pathological wrist bone lesions (Tadeusiewicz and Ogiela, 2005; Ogiela et al., 2006). This method consists on analysis of the structure of the said bones based on palm radiological images. During pre-processing operations in the examined X-ray images the bones contours were separated and a graph representing bones and spatial relation between them was spanned. In the beginning, spatial relationships given by the graph edges were represented by single directions (Tadeusiewicz and Ogiela, 2005) but later each basic spatial relationship was represented as angular interval (Ogiela et al., 2006). The second approach can be interpreted in such a way that every basic spatial relationship is described as a fuzzy set for which its membership function has positive values on the specified angular interval and is equal to zero outside this interval. It should be mentioned that in (Ogiela et al., 2006) such interpretation was not considered.

Recapitulating, four examples in which various aspects of possibility of improve syntactic approach by fuzzy sets has been discussed. The classical IE graphs can be generalized by including fuzzy sets to description their nodes (example 1), edges (example 4), both the nodes and edges (example 2) and fuzzy inference approach can be applied to classical graphs (i.e. non-fuzzy ones) – example 3.

3 FUZZY IE GRAPHS

Recall a definition of IE graph (Flasiński, 1993).

Definition 3.1
An IE graph is a quintuple $H=(V, E, \Sigma, \Gamma, \varphi)$ where: $V$ is a finite, non-empty set of nodes of a graph with assigned indexes in univocally way, $\Sigma$ is a finite, non-empty set of node labels, $\Gamma$ is a finite, non-empty set of edge labels, $E$ is a set of graph edges represented by triplet $(v, \lambda, w)$ where $v, w \in V$, $\lambda \in \Gamma$ and an index of $v$ is smaller than an index of $w$, $\varphi : V \rightarrow \Sigma$ is a nodes labeling function.

Let us assume that, due to pattern fuzziness, possible IE graphs associated with a given example pattern (scene) may look like IE graphs shown in Fig.3.

Figure 3: Possible IE graphs describing a given scene.

In the case of the IE graph shown in Fig.3a fuzziness concerns pattern primitives represented by the node 2 labeled by tree and the node 3 labeled by bus. In the case of the IE graph shown in Fig.3b fuzziness concerns a pattern primitive represented by the node 4 labeled by bus and a spatial relation between pattern primitives represented by the edge connecting the node 3 with the node 4. Assume that both labeled objects in nodes of a graph and spatial relations are represented by fuzzy sets of a first order (Zadeh L.A., 1965) with membership functions $\mu_i$ and $\nu_j$ respectively. Let, furthermore, the set of all objects $\Sigma$ be $m$-elemental and the set of all spatial relations be $k$-elemental. Let us define, informally, a fuzzy IE graph as an IE graph in which nodes labels are replaced by a vector $\mu = [\mu_1, ..., \mu_m]$ of values of membership functions $\mu_i$, $i \in \{1, ..., m\}$ and edges labels are replaced by vector $\nu = [\nu_1, ..., \nu_k]$ of values of membership functions $\nu_j$, $j \in \{1, ..., k\}$.

Let propose a formal definition of a fuzzy IE graph
Definition 3.2
A fuzzy IE graph is a quintuple $H=(V, E, \Sigma, \Gamma, \Phi)$ where:
- $V$ is a finite, non-empty set of nodes of a graph with assigned indices in univocal way,
- $\Sigma$ is a finite, non-empty set of node labels, containing, say, $n$ elements,
- $\Gamma$ is a finite, non-empty set of edge labels, containing, say, $k$ elements,
- $E = V \times \Theta \times V$ is a set of fuzzy graph edges represented by triplet $(v, \psi, w)$ where $v, w \in V$ and $i(v) < i(w)$ i.e. an index of $v$ is smaller than an index of $w$, $\Theta = [\Theta_1^v, ..., \Theta_k^v]$ is represented by $[(\lambda_1, \nu_1^v), ..., (\lambda_k, \nu_k^v)]$ where $\nu^v_i$ is a value of a membership function of succeeding edge labels for a $s$-th edge,
- $\Phi(\nu^v_i) = (\Pi_1^v, ..., \Pi_m^v)$ where $\Pi_i^v = (\sigma^v_i, \mu^v_i)$, $\sigma^v_i \in \Sigma$, $\mu^v_i$ is a value of a membership function of succeeding nodes labels for an $i$-th node.

The fuzzy measure of an outcome IE graph, obtained form a given fuzzy IE graph, is equal to the value of T-norm of the values components of the node and edge vectors. Recall axiomatic definition of T-norms which is given in, for instance, (Rutkowski, 2005) - definition 4.22, page 80.

Definition 3.3
T-norm is a function $T:[0,1] \times [0,1] \rightarrow [0,1]$ satisfying the following axioms:
1. $T(a,b) = T(b,a)$,
2. $T(T(a,b), c) = T(a, T(b,c))$,
3. if $a \leq b$ and $c \leq d$ then $T(a,b) \leq T(c,d),$
4. $T(a,0) = 0$ and $T(a,1) = a.$

Theorem
The functions $T_\sigma$ and $T_\alpha$ given by the formulae

$$T_\sigma(a,b) = \min\{a,b\} \text{ and } T_\alpha(a,b) = a \cdot b$$

are T-norms. The function $T_\sigma$ given by the formulae

$$T_\sigma(a,1) = a, \quad T_\sigma(a,b) = 0 \text{ for } a \neq 1 \text{ and } b \neq 1$$

(2)

is a T-norm as well. Furthermore, for every $a,b \in [0,1]$ if a function $T$ is a T-norm then

$$T_\sigma(a,b) \leq T(a,b) \leq T_\alpha(a,b)$$

(3)

Thanks to the property (ii) in Definition 3.3 T-norm being a function of $n$ variables can be introduced:

$$T(a_1,...,a_n) = T \left( \frac{n}{T_{i=1}^n a_i} \right)$$

(4)

Having a fuzzy IE graph $R$ the fuzzy measure of an outcome graph $r$ is calculated as

$$\lambda(r) = T \left( \frac{p}{\prod_{\nu \in f(\alpha)} T_{\nu}^{a_{\nu}} \prod_{\sigma \in g(\beta)} T_{\sigma}^{b_{\sigma}}} \right)$$

(5)

where $\alpha$ is a number of a regarded node, $\beta$ is a number of an edge, $f(\alpha)$ - is a chosen component number of a vector $\mu^{\alpha}$ whereas $g(\beta)$ is a number of component of a vector $\nu^{\beta}$. If a product is used as a T-norm then the presented parsing (see section 4) is identical as the random parsing described in (Skomorowski, 1998). In calculations presented in the next section the minimum $T_\sigma$-norm is used.

4 PARALLEL PARSING

Given an unknown pattern represented by a fuzzy IE graph $R$, the problem of recognition of a pattern under study is to determine if an outcome IE graph $r$, obtained from the fuzzy IE graph $R$, belongs to a graph language $L(G)$ generated by an ETPL$(k)$ graph grammar $G$. In the proposed parallel and cut-off strategy of fuzzy IE graph parsing for an efficient, that is with the computational complexity $O(n^2)$, analysis of fuzzy patterns (scenes) a number of simultaneously derived graphs is equal to a certain number $\text{limit}$. In this case, derived graphs spread through the search tree, but only the best, that is with maximum measure value, $\text{limit}$ graphs are expanded. Let us consider a graph shown in Fig.4a and a production shown in Fig.4b. Suppose that the embedding transformation for the production shown in Fig.4b is $C(r, \text{input}) = \{(d, b, r, \text{input})\}$ and $C(u, \text{output}) = \{(e, B, r, \text{input})\}$. During a derivation, a non-terminal $A$ in the node 2 of a graph shown in Fig.4a is removed and the graph of the production shown in Fig.4b is put in the place of the removed non-terminal $A$. The first item of the embedding transformation for the production: $C(r, \text{input}) = \{(d, b, r, \text{input})\}$ means that the edge $r$ of the graph shown in Fig.3a should connect the node $d$ of the production graph with the node $b$ of the graph shown in Fig.4a. The second item of the embedding transformation for the production: $C(u, \text{output}) = \{(e, B, r, \text{input})\}$ means that the edge $e$ of the graph shown in Fig.4a should be replaced by the edge $r$ connecting the node $e$ of the production graph with the node $B$ of the graph shown in Fig.4a. Thus, after the application of the production shown in Fig.4b to
the node indexed by 2 of the graph shown in Fig.4a we obtain a graph shown in Fig.4c.

Figure 4: An example derivation step in an ETPL(k) graph grammar.

Suppose that we analyze an unknown fuzzy pattern represented by a fuzzy IE graph shown in Fig.5. (for clarity, only non-zero membership functions vectors components are specified).

Figure 5: An example fuzzy IE graph representing an unknown distorted pattern.

Let us assume that a number of simultaneously derived graphs is equal to 2 (that is limit = 2). Furthermore let us assume that we are given an ETPL(k) graph grammar G with a starting graph Z shown in Fig.6 and a set of productions shown in Fig.7.

Figure 6: A starting graph Z of an example ETPL(k) graph grammar G.

In the first step of the derivation, after the application of the production (1), shown in Fig.7, to the node indexed by 2 of the starting graph Z, shown in Fig.6, we obtain a graph q1, shown in Fig.8a. Similarly, after the application of the production (2)
to the node indexed by 2 of the starting graph \(Z\) we obtain a graph \(q_2\) shown in Fig.8b. The graphs \(q_1\) and \(q_2\) (Fig.8) are admissible for further derivation, that is they can be outcome graphs obtained from the fuzzy IE graph shown in Fig.5. The application of the production (3) to the node indexed by 2 of the starting graph \(Z\) does not lead to a graph which can be an outcome graph obtained from the fuzzy IE graph shown in Fig.5. Thus, a graph obtained after the application of the production (3) to the node indexed by 2 of the starting graph \(Z\) is not admissible for further derivation. As in the analyzed example a number of simultaneously derived graphs is equal to 2 we expand the graphs \(q_1\) and \(q_2\) in the second step of derivation.

In the second step of derivation, after the application of the productions (6) and (7) (Fig.7) to the node indexed by 3 of the graph \(q_1\) (Fig.8a) we obtain graphs \(q_{1,6}\) and \(q_{1,7}\) shown in Fig.9. Similarly, after the application of the productions (6) and (7) to the node indexed by 3 of the graph \(q_2\) (Fig.8b) we obtain graphs \(q_{2,6}\) and \(q_{2,7}\) shown in Fig.10. The application of the production (5) to the node indexed by 3 of the graphs \(q_1\) and \(q_2\) (Fig.8) leads to graphs which can not be outcome graphs obtained from the fuzzy IE graph shown in Fig.5, as they miss the node indexed by 7 and labeled by \(F\) in the fuzzy IE graph shown in Fig.5. Thus, graphs obtained after the application of the production (5) to the nodes indexed by 3 of the graphs \(q_1\) and \(q_2\) (Fig.8) are not admissible for further derivation. The graphs \(q_{1,6}\), \(q_{1,7}\) (Fig.9) and \(q_{2,6}\), \(q_{2,7}\) (Fig.10) are admissible for further derivation, that is they can be outcome graphs obtained from the fuzzy IE graph shown in Fig.5.

Because in the analyzed example a number of simultaneously derived graphs is equal to 2 we should choose only two graphs from among the graphs \(q_{1,6}\), \(q_{1,7}\) (Fig.9) and \(q_{2,6}\), \(q_{2,7}\) (Fig.8) for further derivation. In order to do it, compute the following values: \(\lambda(q_{1,6}) = 0.7\) and \(\lambda(q_{1,7}) = 0.3\). The contributions of nodes indexed by 6 of the graphs \(q_{1,6}\) and \(q_{1,7}\) are not taken into account in this case as the node indexed by 6 and labeled by \(F\) in the graph \(q_{1,7}\) is not a terminal one. Consequently, the contribution of the edge connecting nodes indexed by 3 and 6 as well as the contribution of the edge connecting nodes indexed by 6 and 7 in the graphs \(q_{1,6}\) and \(q_{1,7}\) are not taken into account. Similarly, we compute the following values: \(\lambda(q_{2,6}) = 0.2\) and \(\lambda(q_{2,7}) = 0.2\). As \(\lambda(q_{1,6}) > \lambda(q_{1,7}) > \lambda(q_{2,6}) = \lambda(q_{2,7})\) we choose the graphs \(q_{1,6}\) and \(q_{1,7}\) for further derivation, that is we choose two graphs with maximum value (\textit{limit} = 2). Similarly, in two next steps of derivation the final outcome IE graph is obtained – see Fig.11. The derived graph \(q_{1,7,10,8}\) is also an outcome IE graph obtained from the parsed fuzzy IE graph shown in Fig.5.
5 CONCLUSIONS

In this paper we have proposed an idea of a new approach to recognition of fuzzy patterns represented by graphs. The problem has been considered in the context of pattern recognition and scene analysis with references to robotics (Han and Han, 1999; Kok et al., 2005; Muratet et al., 2004; Petterson, 2005) and applications in medicine (Tadeusiewicz and Ogiela, 2005, Ogiela et al., 2006). To take into account variations of a fuzzy pattern under study, a description of the analysed pattern based on fuzzy sets of the first order was introduced. The fuzzy IE graph has been proposed here for such a description. The idea of an efficient, that is with the computational complexity $O(n^2)$, parsing algorithm presented in (Flasiński, 1993) is extended, so that fuzzy patterns, represented by fuzzy IE graphs, can be recognized. In the algorithm a T-norm is used for calculation of value of membership measure of output graphs. Such solution makes that the algorithm is very flexible. In particular if arithmetic product is used as a T-norm, the algorithm is the same as the random one described in (Skomorowski, 1998).

REFERENCES


