

# Test Purpose of Duration Systems

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**Abstract.** The aim of conformance testing is to check whether an implementation conforms to its specification. We are interested to duration systems, we consider a specification of duration system that is described by a duration graph. Duration graphs are an extension of timed systems and are suitable for modeling the accumulated times spent by computations in the duration systems.

In this paper, we propose a framework to generate automatically test cases according to a test purpose for duration graphs. In the first, we define the synchronous product of the specification and the test purpose of an implementation under test. In the second, we demonstrate that timed words recognized by the synchronous product is also recognized by both specification and test purpose. This result allows us to generate tests according to test purpose from the synchronous product.

## 1 Introduction

Duration systems are an extension of real time systems for which in addition to constraints on delays separating certain events that must be satisfied, constraints on accumulated times spent by computation must also be satisfied.

Duration graphs are a formalism used to describe duration systems. They are an extension of real-time graphs supplied with a finite set of continuous real variables that can be stopped in some locations (rate=0) and resumed in other locations (rate=1). These variables are called *duration variables*.

Duration graphs model some temporal behaviors of real-time systems such as the accumulated times spent by computations at some particular locations. For instance, consider a real time scheduler with preemption which handles tasks that can be executed in parallel. If one task may be interrupted by other tasks of higher priority, then the constraint of the execution time of the considered task must be expressed using the accumulated times. Intuitively, we must use a continuous real variables that can be stopped when the task is interrupted and resumed when the task is active. Thus, these systems are modeled with automata supplied with duration variables that count accumulated times spent at some particular control locations.

Our work targets black box conformance testing for duration graphs. Conformance testing aims to check whether the behavior of some black box implementation conforms to that of its specification. By "black box" we mean that the tester has no knowledge about the implementation, thus can only rely on its observable inputs and outputs. Since, testing is difficult, expensive, time-consuming and labour-intensive process, moreover, it should be repeated each time an implementation is modified. A promising approach

to improve testing is to automatically generate test cases from formal models of specification. Using tools to generate test cases automatically may reduce the cost of test process. However, exhaustive test remains expensive and in some case is impossible. Springintveld et al in [16] proved that exhaustive testing of deterministic timed automata with dense time is theoretically possible, but highly infeasible. Some works define a criteria for selecting test cases to be generated automatically such as coverage criteria (transition or location coverage of the timed automata)[6,7,10]. Other works try to define purposes of test and generating test cases according to those purposes[13]. We hope that defining a purpose of test to select test cases converge with the way of tester reasoning. In practice, and in order to test an implementation the tester specifies informally some purposes and try to test implementation according to those purposes.

Our contribution is to propose a framework to generate automatically test cases according to a test purpose for duration graphs. In the first, we present the formalism used to model specification and test purpose called *Duration Variables Timed Graph with Inputs Outputs* (DVTG-IO for short), then we define a synchronous product of both specification and test purpose which is a duration variables timed graph that combines specification and test purpose, from this synchronous product we generate test cases according to the test purpose by applying The algorithm of Tretmans [17].

This paper is organized as follows : In section 1, we present the duration variables timed graphs with inputs outputs used to model specification. In section 2, we describe the test purpose. In section 3, we define the synchronous product of specification and test purpose. the test case is given in section 4.

## 2 Duration Variables Timed Graphs with Inputs Outputs (DVTG-IO)

We will introduce in this section formalisms used for describing both specification and test purpose of implementation under test, called Duration Variables Timed Graph with Inputs Outputs which are inspired from [15] and that are extensions of the well-known timed automata defined in [1].

A Duration Variables Timed Graph with Inputs outputs (DVTG-IO for short) is described by a finite set of locations and a transition relation between these locations. In addition, the system has a finite set of duration variables that are constant slope continuous variables, each of them changes continuously with a rate in  $\{0,1\}$  at each location of the system. Transitions between locations are conditioned by arithmetical constraints on the values of the duration variables. When a transition is taken, a subset of duration variables should be reset and an action should be executed, this action can be either input action, output action or unobservable action (known also as quiescent [17]).

### 2.1 DVTG-IO Formal Definition

We consider  $X$  a finite set of duration variables. A guard on  $X$  is a boolean combination of constraints of the form  $x \prec c$  where  $x \in X, c \in N, \prec \in \{<, \leq, >, \geq\}$ . Let

$\Gamma(X)$  be the set of guards on  $X$ . A Duration Variables Timed Graph with Inputs Outputs describing a specification is a tuple  $S = (Q^S, q_0^S, E^S, X^S, Act^S, \gamma^S, \alpha^S, \delta^S, \partial^S)$  where  $Q^S$  is a finite set of locations,  $q_0^S$  is the initial location,  $E^S \subseteq Q^S \times Q^S$  is a finite set of transitions between locations,  $Act^S = In \cup Out \cup \{\tau\}$  is a finite set of input actions (designed by  $a?$ ), output actions (designed by  $a!$ ) and unobservable action,  $\gamma^S : E^S \rightarrow \Gamma(X^S)$  associates to each transition a guard which should be satisfied by the duration variables whenever the transition is taken,  $\alpha^S : E^S \rightarrow 2^{X^S}$  gives for each transition the set of duration variables that should be reset when the transition is taken,  $\delta^S : E^S \rightarrow Act^S$  gives for each transition the action that should be done when the transition is taken,  $\partial^S : Q^S \times X^S \rightarrow \{0, 1\}$  associates with each location  $q$  and each duration variable  $x$  the rate at which  $x$  changes continuously while the computation is at  $q$ .

## 2.2 State Graph

The semantic of DVTG-IO is defined in terms of a state graph over states of the form  $s = (q, \nu)$  where  $q \in Q^S$  and  $\nu : X^S \rightarrow R$  is a valuation function that assigns a real value to each duration variables. Let  $St_S$  be the set of states of  $S$ . We notice that  $St_S$  is an infinite set due to the value of duration variables taken on  $R^+$ .

Given a valuation  $\nu$  and a guard  $g$ , we denote by  $\nu \models g$  the fact that valuation of  $g$  under the valuation  $\nu$  is true.

We define two families of relation between states :

- Discrete Transition  $(q, \nu) \xrightarrow{a} (q', \nu')$  where  $(q, q') \in E^S$ ,  $\delta^S(q, q') = a$ ,  $\nu^S \models \gamma(q, q')$  is true and  $\nu'(x) = \nu(x) \forall x \in X^S \setminus \alpha^S(q, q')$ ,  $\nu'(x) = 0 \forall x \in \alpha^S(q, q')$ , corresponds to moves between locations using transition in  $E^S$ .
- Timed transition  $(q, \nu) \xrightarrow{t} (q, \nu')$  such that  $t \in R$  and  $\nu'(x) = \nu(x) + \partial(q, x)t \forall x \in X^S$ , corresponds to transitions due to time progress at some location  $q$ .

## 2.3 Example

To illustrate duration variables timed graph with inputs outputs, we give, in figure 1, the specification of box phone inspired from [13] and described by DVTG-IO. The protocol is composed by ten locations, transitions between locations and three duration variables :  $x, y$  and  $z$ , and it has two phases : authenticity phase and communication phase, we suppose that authenticity phase does not exceed 5 units of time and communication phase does not exceed 15 units of time. Duration variables  $x$  and  $y$  are used respectively to make constraints on the time of execution of authenticity and communication phases,  $z$  is a timer used to make constraint on the order between actions.

In the initial location ( location 0) implementation wait that the user insert its card (the input action ?card-in) so it passes to location 1. In location 1, the implementation verifies the card validity and passes to location 2, if the card is accepted protocol generates the output !accept and passes to location 3 where the implementation waits that the user entries its code, if it is correct it passes to location 4 otherwise it remains at location 3. In location 5, system waits that user composes the number to phone and passes to locations 7 where it waits the bill and the connection. In locations 7,8 and 9 the user can hang up the connection.



purpose. We notice that we can define several test purposes for an implementation. We describe test purpose by a particular duration variables timed graph with inputs outputs having two particular locations : *Accept* and *Reject*. Location *Accept* defines the verdict Pass, such that all paths from the initial location to location *Accept* satisfy the purpose of test. However, all paths ending at location *Reject* don't satisfy the test purpose

A test purpose (TP for short) is a deterministic DVTG-IO ;

$TP = (Q^{TP}, q_0^{TP}, E^{TP}, X^{TP}, Act^{TP}, \gamma^{TP}, \alpha^{TP}, \delta^{TP}, \partial^{TP})$  where  $Q^{TP}$  is the set of locations containing *Accept* and *Reject* locations. We suppose that the set  $Act^{TP} = Act^S$  this allows to consider that actions of test purpose are also actions of the specification, and this allows  $TP$  to describe the test purpose with the same set of actions as the specification.

We impose that  $TP$  must be complete ( $\forall q \in Q^{TP}, \forall a \in Act^{TP}$ , we have  $q \xrightarrow{a}$ ), this hypothesis ensures that the synchronous product of  $S$  and  $TP$  has the same behaviors as  $S$ . With symbol "\*" we design complementary actions of one action  $a$  in transition of the form  $q \xrightarrow{a}$

### 3.1 Example

Figure 2 presents a test purpose of the example presented in figure 1. Informally, the aim of this test purpose is to test the return of card after more than one communication such that the total time of communication does not exceed 15 units of time.

The following test purpose is described by a DVTG-IO with five locations  $\{A, B, C, D, E, \}$  and transitions between locations, we extend this graph by one duration variables  $t$  used to count the accumulation of the durations spent in the communication phase.

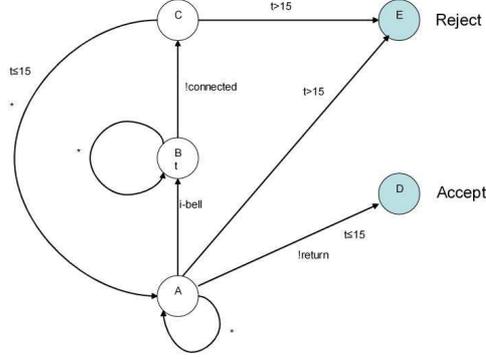
From location A, system can pass either to location E(Reject) if the time of communication exceed 15 units of time, we notice that this path does not satisfy the purpose of test or to location B, In location B, the system wait the input action !connected ( representing the fact that there is more than one communication). From location C, system can return to location A either to establish another communication or to return card during 15 units of time, in this case the system passes to location D (Accept) representing the fact that the purpose is satisfied.

## 4 Synchronous Product of DVTG-IO

In the previous paragraphs, we have defined graphs describing specification and test purpose of an implementation under test. In this section, we present the synchronous product of specification and test purpose.

Intuitively, synchronous product of two graphs describing respectively specification and test purpose is a duration variables timed graph with inputs outputs such that all timed words recognized by the synchronous product are recognized by both the specification and test purpose graphs.

Let  $S = (Q^S, q_0^S, E^S, X^S, Act, \gamma^S, \alpha^S, \delta^S, \partial^S)$  and  $TP = (Q^{TP}, q_0^{TP}, E^{TP}, X^{TP}, Act, \gamma^{TP}, \alpha^{TP}, \delta^{TP}, \partial^{TP})$  be two DVTG-IO's describing respectively specification and test purpose of implementation under test and having the same set of actions ( $Act$ ).



**Fig. 2.** Test purpose.

The synchronous product of  $S$  and  $TP$ ;  $M = S \otimes TP$  is a DVTG-IO defined by the tuple :

$$\begin{aligned}
 M &= (Q, q_0, E, X, Act, \gamma, \alpha, \delta, \partial) \text{ where} \\
 Q &\subseteq Q^S \times Q^{TP} \\
 q_0 &= (q_0^S, q_0^{TP}) \\
 E &\subseteq Q \times Q \text{ such that } e = ((q_1, q_2), (q'_1, q'_2)) \in E \text{ iff } e^S = (q_1, q'_1) \in E^S \text{ and} \\
 &e^{TP} = (q_2, q'_2) \in E^{TP} \\
 X &= X^S \cup X^{TP} \\
 \gamma &: E \longrightarrow \Gamma(X) \text{ such that } \gamma(e) = \gamma^S(e^S) \wedge \gamma^{TP}(e^{TP}) \\
 \alpha &: E \longrightarrow 2^X \text{ such that } \alpha(e) = \alpha^S(e^S) \cup \alpha^{TP}(e^{TP}) \\
 \delta &: E \longrightarrow Act \text{ such that } \delta(e) = \delta^S(e^S) = \delta^{TP}(e^{TP}) \\
 \partial &: Q \times X \longrightarrow \{0, 1\} \text{ such that } \partial((q_1, q_2), x) = \begin{cases} \partial^S(q_1, x) & \text{if } x \in X^S \\ \partial^{TP}(q_2, x) & \text{if } x \in X^{TP} \end{cases}
 \end{aligned}$$

#### 4.1 State Graph for Synchronous Product

A state of synchronous product of DVTG-IO is a pair  $s = ((q_1, q_2), \nu)$  where  $(q_1, q_2) \in Q$  ( $q_1 \in Q^S, q_2 \in Q^{TP}$ ) and  $\nu : X \longrightarrow R$  is a function that assigns a real value to each duration variables

$$\nu(x) = \begin{cases} \nu^S(x) & \text{if } x \in X^S \\ \nu^{TP}(x) & \text{if } x \in X^{TP} \end{cases}$$

Let  $St_M$  be the set of states

Two types of transition between states

- Discrete Transition  $((q_1, q_2), \nu) \xrightarrow{a} ((q'_1, q'_2), \nu')$  where  $((q_1, q_2), (q'_1, q'_2)) \in E$ ,  $\delta((q_1, q_2), (q'_1, q'_2)) = a$ ,  $\nu \models \gamma((q_1, q_2), (q'_1, q'_2))$

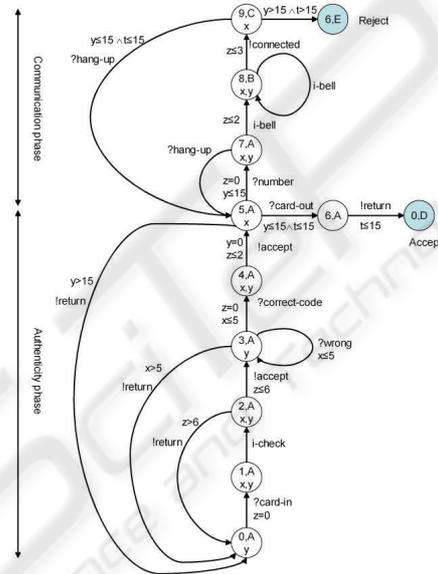
$$\nu'(x) = \begin{cases} \nu(x) & \forall x \in X \setminus \alpha((q_1, q_2), (q'_1, q'_2)) \\ 0 & \forall x \in \alpha((q_1, q_2), (q'_1, q'_2)) \end{cases}$$

- Timed transition  $((q_1, q_2), \nu) \xrightarrow{t} ((q_1, q_2), \nu')$  where  $t \in \mathbb{R}^+$ ,  $\nu'(x) = \nu(x) + \partial((q_1, q_2), x)t \forall x \in X$

Let  $(S_M, \rightsquigarrow)$  the state graph of  $M$

## 4.2 Example

The example of figure 3 describes the synchronous product of the previous specification and test purpose presented in figure 1 and 2



**Fig. 3.** The synchronous product specification test purpose.

Now, we present the theorem that demonstrates that all timed words recognized by  $M$  are also recognized by both  $S$  and  $TP$

### Theorem

Let  $M_1$  and  $M_2$  two DVTG-IO's,  $M = M_1 \otimes M_2$  the synchronous product  
We have

$$L(M) = L(M_1) \cap L(M_2)$$

**Proof**

Demonstrating that  $L(M) = L(M_1) \cap L(M_2)$  consists to demonstrate that  $\forall \omega \in L(M) \stackrel{?}{\iff} \omega \in L(M_1)$  and  $\omega \in L(M_2)$

Let  $\omega$  be a timed word of  $L(M)$ ,  $\omega = a_1\tau_1a_2\tau_2\dots a_n\tau_n$  where  $a_i \in Act$ ,  $\tau_i \in R^+$  for  $i = 1..n$

$\iff \exists \sigma = ((q_{01}, q_{02}), \nu_0, 0) \rightsquigarrow ((q_{11}, q_{12}), \nu_1, \tau_1) \rightsquigarrow \dots \rightsquigarrow ((q_{n1}, q_{n2}), \nu_n, \tau_n) \in CS(M)$

and  $\delta((q_{i-11}, q_{i-12})(q_{i1}, q_{i2})) = a_i \quad \forall i = 1..n$

$\iff \forall i = 1..n \quad ((q_{i-11}, q_{i-12}), \nu_{i-1}) \rightsquigarrow ((q_{i1}, q_{i2}), \nu_i)$  is a transition of  $(S_M, \rightsquigarrow)$

and  $\delta((q_{i-11}, q_{i-12})(q_{i1}, q_{i2})) = a_i \quad \forall i = 1..n$

$\iff \forall i = 1..n$

$(q_{i-11}, \nu_{i-11}) \rightsquigarrow (q_{i1}, \nu_{i1})$  is a transition of  $(S_{M_1}, \rightsquigarrow)$

$(q_{i-12}, \nu_{i-12}) \rightsquigarrow (q_{i2}, \nu_{i2})$  is a transition of  $(S_{M_2}, \rightsquigarrow)$

and  $\delta(q_{i-11}, q_{i1}) = \delta(q_{i-12}, q_{i2}) = a_i \quad \forall i = 1..n$

$\iff (q_{01}, \nu_{01}, 0) \rightsquigarrow (q_{11}, \nu_{11}, \tau_1) \rightsquigarrow \dots \rightsquigarrow (q_{n1}, \nu_{n1}, \tau_n) \in CS(M_1)$

$(q_{02}, \nu_{02}, 0) \rightsquigarrow (q_{12}, \nu_{12}, \tau_1) \rightsquigarrow \dots \rightsquigarrow (q_{n2}, \nu_{n2}, \tau_n) \in CS(M_2)$

and  $\delta(q_{i-11}, q_{i1}) = \delta(q_{i-12}, q_{i2}) = a_i \quad \forall i = 1..n$

$\iff a_1\tau_1a_2\tau_2\dots a_n\tau_n \in L(M_1)$  and  $a_1\tau_1a_2\tau_2\dots a_n\tau_n \in L(M_2)$  ■

This theorem is important because it allows us to generate test cases from specification and satisfying a test purpose. So, we generate test cases from the synchronous product of specification and test purpose.

## 5 Test Generation

In order to generate test cases that satisfies a test purpose, and thanks to the above theorem, we suggest to generate test cases from the synchronous product of specification and test purpose. We adapt the untimed test generation algorithm of Tretmans [17] to our context. The algorithm builds a test case in the form of tree duration variables timed graph with inputs outputs such that leaves of the tree represent the verdict of the test : *pass* or *fail*. If the test leads to leaf *pass* is considered conform to its specification, otherwise is considered not conform. In every location of the tree, the tester select the transition to be taken depending on the guard and the action assigned, it can either wait the emission by the implementation of an output or insert an input action and respecting the guard of transition. We notice that the graph representing test is not synchronous. In the sense, that an input action not imperatively succeeded by an output action.

## 6 Conclusion

We have presented in this paper our framework for generating test cases according to test purpose for duration variables timed graph with inputs outputs.. We have described the specification and the test purpose of an implementation under test by a duration variables timed graphs, and we have defined the synchronous product of the specification

and test purpose. Finally, we have demonstrated that all timed words recognized by the synchronous product are recognized by both the specification and test purpose. Thanks to this result, we can generate test cases according to a test purpose.

Regarding future work, we notice that in this paper, we don't treat the problem of the infinity of the state space, due to the infinite number of duration variables values, we can solve this problem by adapting the region graph approach or by approximation.

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