Efficient Interpretation of Large Quantifications in a Process Algebra

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Abstract. The process algebra interpreter $\text{EB}^3\text{PAI}$ supports the $\text{EB}^3$ method, which was developed for the purpose of automating the development of information systems through code generation and efficient interpretation of abstract specifications. For general information system patterns, $\text{EB}^3\text{PAI}$ executes in linear time with respect to the number of terms and operators in the process expression and in logarithmic time with respect to the number of entities in the system. This paper described three optimization techniques of $\text{EB}^3\text{PAI}$.

1 Introduction

The $\text{EB}^3\text{PAI}$ project is part of the APIS research project [1], whose objective is to develop a case tool that generates executable information systems (IS) from formal specifications (abstract models). In traditional IS development, the bulk of the design, programming and testing is done manually by humans. These three activities consume up to 70% of the development effort. They are usually not hard to accomplish, but they are time-consuming and error-prone. The key to reducing development costs and increasing quality clearly resides in eliminating or mechanizing these three tasks. The $\text{EB}^3$ method was developed for the purpose of automating the development of IS through code generation and efficient specification interpretation. Designed by Frappier and St-Denis [2], the $\text{EB}^3$ language includes a process algebra to support an event-oriented specification style. Initial work led to a practical set of rules for an interpreter of $\text{EB}^3$ process expressions [3], called $\text{EB}^3\text{PAI}$. This paper describes the last step in efficiently implementing this interpreter.

2 The $\text{EB}^3$ Specification Method

The $\text{EB}^3$ method [2] has been specifically created to specify information systems. It relies on event-oriented specification notations like CSP, CCS and LOTOS. However, special features have been added to take the specificities of information systems into account. The input behavior of the system is defined by the process expression $\text{main}$. Each execution of an action (an input) generates a response (an output). The denotational semantics of $\text{EB}^3$ is given by a relation $R$ between the traces accepted by $\text{main}$ and the set of output events. A process expression is recursively defined over a set of
symbols $\Sigma$, called the action set, $\lambda$ (an internal action that denotes non-observable activity of the system) and $\boxplus$ (which denotes a successful termination) in combination with operators. Operators in $\text{EB}^3$ draw their inspiration from regular expressions, the sequential composition ($\cdot$), the choice ($|$) and the Kleene closure ($^*$), with the addition of some operators from CSP and LOTOS: synchronization, guard, process call, and synchronization quantification (also called indexing in CSP). The symbol $\parallel$ denotes the interleave operator, i.e., $[\emptyset]$.$\text{EB}^3\!\text{PAI}$ also have a guard operator and process call. The symbol $\mathcal{P}\mathcal{E}$ denotes the set of all process expressions.

It is important to note that quantification is a crucial operator in IS specification. This constitutes a major difference from other problem domains where process algebras are typically used (protocol specification for example). Another difference is that $\text{EB}^3$ is only concerned by trace equivalence. Since the main aim of $\text{EB}^3$ is to provide an executable specification, the specification style used to achieve this goal is also different. Frappier and St.-Denis in [2] have proposed a set of rules that give $\text{EB}^3$ its operational behavior. $\text{EB}^3\!\text{PAI}$ executes a specification by simply evaluating the inference rules. We do not generate code per se; $\text{EB}^3\!\text{PAI}$ can be considered as a virtual machine and each specification becomes a high-level program. The implementation is the combination of $\text{EB}^3\!\text{PAI}$ and the specification.

3 Dynamic Optimization in the $\text{EB}^3\!\text{PAI}$ Interpretation Process

In [3] we provide some static optimizations. However, this is not sufficient to yield efficient interpretation. To provide a useful tool, we need to optimize other aspects of the interpretation algorithm, primarily to reduce the time and space complexity for large quantifications.

3.1 Optimization of Process Expression Storage in Memory

During an execution, actions and process expressions can be copied several times. Some execution sequences can require a large amount of memory. The two main causes are quantification and choice resulting in the execution of some non-deterministic specifications. In order to minimize memory space, process expressions can be represented by a graph which allows them to be shared. For instance, if $E_1$ is a sub-expression of both $E_2$ and $E_3$, than $E_1$ can be instantiated once and referenced by both $E_2$ and $E_3$.

3.2 Optimizing Quantification Execution Time: Direct $\kappa$-optimization

The $\text{EB}^3$ language allows the use of quantification operators. A basic approach to executing quantification operators is too ineffective to be acceptable. To optimize these executions, we determine by static analysis of each quantified expression which value of the quantified set must be selected based on the parameters of the action to execute. We call these values $\kappa$-values, the positions of the values in the action parameters $\kappa$-positions, and this method direct $\kappa$-optimization.

This approach is sufficient to optimize a choice quantification, since the quantification disappears after the transition. In the case of a quantified interleave, the quantification remains in the result process expression, since it can spawn one interleave process.
for each value in the quantification set. The interleave of the instantiated process expressions is represented by a function \( K : T \to PE \) such that \( K(\Pi(\sigma)) \) is the only process expression that can execute \( \sigma \).

### 3.3 Extending Quantification Optimization: Indirect \( \kappa \)-optimization

We have found conditions under which a quantification can be optimized when the algorithm used for the direct \( \kappa \)-optimization fails to find a single \( \kappa \)-position for each action. We can summarize the general idea of indirect \( \kappa \)-optimization, as follows:

**During static analysis**:
1. Find the quantified choice operators to deduce the possible functional dependencies (below we refer to choice quantified variables occurring in the scope of other quantifications (interleave or choice) as the dependent variables and the enclosing quantified variables as the keys).
2. For all actions not optimized with the algorithm of the \( \kappa \)-optimization: identify the producer that binds the keys (\( bId \) in the example) to the dependent variable (\( mId \) in the example) under the condition that the choice and interleave quantifications to optimize are synchronized on these actions.

**At runtime**:
1. When a producer is executed, store the value of the functional dependency between the set of keys and the dependent variable.
2. Store the value of the new process expression for the operand of the quantified interleave in a mapping \( K \), as for direct \( \kappa \)-optimization.
3. When a consumer is executed, delete the stored value of the functional dependency between the keys and the dependent variable.

Complete \( \kappa \)-optimization is not effective for all specifications that can be written. Actually, it is not effective for all IS specifications. However, our aim is to optimize all specifications written with the patterns described in [2]. We have established that the following patterns satisfy the conditions for \( \kappa \)-optimization: the producer-modifier-consumer, the one-to-many association, the multiple (many-to-many) association, the \( n \)-ary association, the weak entity type, the recursive association, and the inheritance association.

### 4 Implementation and Performance

**Complexity analysis**  Let \( n \) denote the sum of all \( |X| \), where \( X \) is either an entity type or an association of an EB\(^3\)PAI specification. Let \( s \) denote the number of nodes in the tree representing a process expression, excluding the nodes of a \( \kappa \)-optimized quantification (they will be computed with \( n \)). Note that for most ISs, \( s \) is usually small (e.g., \( s \ll 10^3 \)), whereas \( n \) can be quite large (e.g., \( n \ll 10^{12} \)). Figure 1 summarizes the algorithmic and the space complexity of the K-optimization and compares them with no optimization in EB\(^3\)PAI and with manual implementation (i.e., outside EB\(^3\)PAI). Thus, for \( \kappa \)-optimizable specifications, EB\(^3\)PAI has an overhead of \( O(s) \) compared with manual implementation of an IS. With no \( \kappa \)-optimization, the difference is substantial and EB\(^3\)PAI becomes impractical as a tool, but it can still be useful for specification animation for validation purposes.
### Table 1: Algorithmic and space complexity of EB\textsuperscript{3}PAI.

<table>
<thead>
<tr>
<th>Optimization Type</th>
<th>Alg. complexity</th>
<th>Space complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>no optimization</td>
<td>$O(s,n)$</td>
<td>$O(n+s)$</td>
</tr>
<tr>
<td>direct $\kappa$-optimization</td>
<td>$O(\log(n) + s)$</td>
<td>$O(n+s)$</td>
</tr>
<tr>
<td>indirect $\kappa$-optimization</td>
<td>$O(\log(n) + s)$</td>
<td>$O(n+s)$</td>
</tr>
<tr>
<td>manual implementation</td>
<td>$O(\log(n))$</td>
<td>$O(n)$</td>
</tr>
</tbody>
</table>

**Fig. 1:** Algorithmic and space complexity of EB\textsuperscript{3}PAI.

**Performance for direct $\kappa$-optimization** Complete $\kappa$-optimization is not implemented, but direct $\kappa$-optimization is. The performance for indirect $\kappa$-optimization should be very close to that of direct $\kappa$-optimization, because it uses the same data structures plus an additional hash table to store the functional dependencies. Performance tests were conducted with a specification of a library management system on a Pentium III 800MHz with 384Mo of SDRAM, running GNU/Linux. Indirect $\kappa$-optimization has not been implemented yet; only direct $\kappa$-optimization. The transaction mean time of valid actions is 97ms. Invalid actions (for which the execution failed) are less expensive in time (approx. 40 ms per action) than valid actions. The same specification implemented in Java using an Oracle database provides an average transaction processing time of 10 ms, which is 10 times faster than EB\textsuperscript{3}PAI. Nevertheless, 100 ms is still acceptable for many IS systems where the transaction rate is low (e.g., a library management system).

### 5 Conclusion

In this paper, we have presented two optimization techniques to efficiently execute quantified process expressions in the EB\textsuperscript{3} process algebra. Their space and algorithmic complexities are comparable to those of a manual implementation for a large number of IS specifications which are determined by a set of specification patterns. Direct $\kappa$-optimization was implemented in the EB\textsuperscript{3}PAI interpreter. It performs 10 times slower than a manual implementation of the specification for the library system, but its average response time is acceptable for a large class of IS with low transaction rates, which demonstrates that abstract interpretation is a viable way of implementing IS.

### References