A Hybrid Approach using Set Theory (HAST) for Magnetic Resonance (MR) Image Segmentation

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Abstract. This paper describes a new Hybrid Approach using Set Theory (HAST) for Magnetic Resonance (MR) Image segmentation based on two existing techniques, region-based and level set methods. In our approach, instead of using the typical pipeline methodology to integrate the two techniques, a hybrid set-based methodology will be proposed. To evaluate the effectiveness of HAST, MR images taken from a national hospital that reflects the quality of real world medical images are used. A comparison between the two individual techniques and HAST will also be made to demonstrate the effectiveness of the latter.

1 Introduction

Medical Resonance Imaging (MRI) [1,2] is a non-invasive technique used to capture internal body structures using magnetism and radio waves. MRI is used to obtain a two-dimensional view of the internal organs and the images that are produced are very crucial for early diagnosis of tumors and analysis of fluid flow in organs like kidney. Due to the importance of this imaging technique, it forms a motivation to work on new methods that can effectively perform image analysis tasks like image segmentation. Despite being a safer imaging technique compared to x-ray, images produced using MRI often suffer from problems of noise and geometrical complexity of inner body tissues. These two problems bring about two common errors during the segmentation process, under-segmentation (leaking) and over-segmentation. Under-segmentation [3] is defined as the inclusion of the unwanted region in the segmented image. Over-segmentation [3] is when the segmented image does not contain the complete desired region of interest. See Fig. 1.
2 MRI Segmentation Techniques

Much works had been carried out in the past to propose new segmentation techniques. Such works typically involve complicated mathematical formulas which form the basis of the techniques. Some of the most common techniques are the region-growing techniques [3,4], level set technique [3,4] and segmentation techniques based on watershed [4]. As we will be using the first two classes of techniques mentioned above in our approach, in this section we will provide a brief overview of the two techniques.

2.1 Region Growing

Region-based class of algorithm [3,4] typically makes use of information present in the input image to perform the segmentation task. One of the most commonly used information is the image intensity. In the implementation of a simple region-based technique, a seed index, \( x \) together with a lower intensity threshold \( t_1 \) and upper intensity threshold \( t_2 \) are initialized by the user. The algorithm first analyzes \( x \) to check if the intensity, \( I(x) \) falls between the intensity intervals. If so, \( x \) will be added to a set, \( S \) which forms the result of the algorithm and all the neighbors, \( N(x) \) of \( s \) will be added into a queue. The process of analyzing the intensity of each pixel is carried out repeatedly until the queue becomes empty. To prevent the same pixels from being repeatedly analyzed, the algorithm also keeps track of which pixels had already been analyzed. The equation below describes the relationship between the pixels in the result set, \( S \),

\[
\forall x, x \in S \rightarrow (I(x) \leq t_2) \land (I(x) \geq t_1) \land (\exists y, y \in N(x) \land y \in S)
\] (1)

Although region-based techniques tend to work ideally in images which have distinct intensity difference between adjacent tissues, due to its emphasis on image-based information, this class of algorithm is typically susceptible to noisy images, which is the common shortcoming in many imaging techniques.
2.2 Level Set

Level set technique [2,4], unlike region based which works on image intensities, works on the geometry of isosurfaces. It is a powerful approach which is able to handle geometrical issues like convolution, splitting and merging. This approach tracks the evolution of contours and surfaces in the image and uses a differential equation to control the evolution of contours. Mean intensities and gradient are typically used in the differential equation to perform segmentation. For this cause, level set techniques usually produce outputs that are based on the shape of the tissue of interest. However, this does not necessarily determine that level set algorithms typically work better. In a human body which contains many organs and tissues, the shape of the tissue of interest may sometimes be too complicated to be tracked by any level set algorithms. This makes the technique unsuitable to segment very complex tissues.

3 Hybrid Approaches

As we had given a brief overview of the two segmentation techniques used in this hybrid approach, it is not difficult to realize that each segmentation technique has its own strength and weakness, and they do not always work well on all types of images. This gives rise to the idea of hybrid approaches to integrate different segmentation techniques to amplify the benefits and minimize the weaknesses present in each of the techniques. Much works had been carried out so far, and some of them provide promising results. One such integration [11] involves the integration of Fuzzy-Connectedness, Voronoi Diagram-based [5] and the deformable model [6,7,8,9]. In this approach, the fuzzy-connectedness [10] method is first used to segment a region that consists of the tissue of interest from the input image. In this method, the segmentation task is achieved using the affinity between two elements in the image. The affinity between any two elements is defined by their degree of adjacency as well as the variation of their intensities. The segmentation result of this step is subsequently used to perform statistical analysis. In this analysis, statistical data is generated based on the average and variance of the strongest three channels in two color spaces, Red, Green and Blue (RGB) and Hue, Value and Chroma axis (HVC). This analysis is done to derive a homogeneity operator which will be used as a multi channel operator to perform classification based on the Voronoi Diagram-based algorithm [5]. The final result is then generated using the deformable model which will determine the smooth boundary of the segmented region. [3, 11, 12, 13, 14]

It can be seen that the pipeline methodology is used in the approach described above. This is not the only hybrid approach that adopts the pipeline methodology, many other approaches adopt the methodology in different ways. Although these hybrid methods provide segmentation results that are better than the individual techniques, one of the common problems is that all of them require user intervention in the form of intensive parameter initialization. In this paper, a hybrid approach that uses set operations to produce better segmentation results and reduce the hassle of parameter initialization will be discussed.
3.1 HAST

In HAST, instead of integrating the segmentation techniques using the common pipeline methodology, set theory will be used in the integrating process. Past works like SHA[15] had also adopted a similar approach. In SHA, the union and intersection operations are used for integration to reduce leaking and over-segmentation. In HAST, instead of using the same operations, we will focus on the union and difference operations to achieve the same aim. Furthermore, the two techniques, region-based and level set technique works on different basis. This makes the two techniques ideal for combination as the regions that are detected by both techniques are different. Notice that it will be less interesting to combine two techniques that have the same characteristic, as it would consequently mean that they have the same benefits and flaws. Therefore such combination will still encounter the same flaw. The region-based and level set combination will ideally be able to maximize the benefits of both techniques, and hopefully restrict each other’s flaws. The next paragraph and Fig.2 will give a detailed description of the steps involved in this approach.

Step 1: Generate using the region-based technique and the level set technique two segmentation images consisting of the tissue of interest. We call the two images generated $RG_1$ and $LS_1$, respectively.

Step 2: Combine $RG_1$ and $LS_1$ using the union operation to form the image, $I_1$.

Step 3: If $I_1$ is acceptable, GOTO step 5, Else GOTO step 4.

Step 4: Identify an adjacent tissue where the leaking occurs, and generate the images $RG_0$ and $LS_0$ using the two techniques. Combine $RG_0$ and $LS_0$ using the union operation and subtract the result from $I_1$. GOTO step 3.

Step 5: Apply the region growing technique using a seed point within the tissue of interest to segment the image, $I_1$. The intensity boundaries should be set to 1 and 300 respectively. The result of this step, $I_2$, will alienate any region that is disjoint with the tissue of interest.

Fig. 2. Overview of HAST.
3.1.1 Step 1 of HAST
In this step, the main task is to generate two segmentation images that consist of the tissue of interest using the two techniques. The two images, $\text{RG}_1$ and $\text{LS}_1$, generated should ideally consist of the entire tissue of interest. This is to say that over-segmentation is manifested as holes in the images. This may simply mean a bolder set of parameters may be used to produce such a segmentation image. At this point, we will first ignore the amount of leaking that may arise by such a bold set of parameters. We will handle the issue of leaking at a later stage.

3.1.2 Step 2 of HAST
The main purpose of this step is to reduce over-segmentation in the resulting image $I$. Such reduction is done by combining $\text{RG}_1$ and $\text{LS}_1$ using the union operation. As we had mentioned in the earlier sections that the two segmentation techniques used in HAST works on different basis, therefore the chance that $\text{RG}_1$ and $\text{LS}_1$ covers exactly the same pixels is extremely low. As we had mentioned earlier, this step serves to reduce over-segmentation, we will ignore leaking in the explanation of this step and leave the explanation to later.

Let $F$ be the set of pixels, denoted by $(x, y)$ contained in the region defined by the tissue of interest, and $U$, $O$ be the set of pixels contained in the under-segmented region and the over-segmented region respectively.

\[
O \subseteq F \quad (2)
\]

\[
U \cap F = \emptyset \quad (3)
\]

\[
O \cap F = O \quad (4)
\]

Let $R_i$ and $E_i$ be the set of pixels, denoted by $(x, y)$ contained in the region obtained $\text{RG}_i$ and, $\text{LS}_i$ respectively, and $O_i$ and $H_i$ be the set of pixels, denoted by $(x, y)$ contained in the over-segmented region of $\text{RG}_i$ and $\text{LS}_i$ respectively.

Since we know that all the pixels of the over-segmented regions must belong to the tissue of interest, we know $O_i$ and $H_i$ must be a subset of $F$.

\[
O_i \cap R_i = \emptyset \quad (5)
\]

\[
H_i \cap E_i = \emptyset \quad (6)
\]

3.1.2.1 $O_i$ and $H_i$ are disjoint (Case 1) Fig 2a
In this case, $O_i$ and $H_i$ are disjoint. This is the best case for over-segmentation reduction of the union operation. Since over-segmentation is defined as the set of pixels that are included in the set $F$ but not in the segmentation output, $\text{RG}_i$ or $\text{LS}_i$ therefore from this we know that the over-segmented region of $\text{RG}_i$, $O_i$ is included in the region of $\text{LS}_i$ and the over-segmented region of $\text{LS}_i$, $H_i$ must be included in the re-
The union operation in this case will cause the over-segmentation rate of the union result to be 0.

\[ O_i \subseteq E_i \quad (7) \]

\[ H_i \subseteq R_i \quad (8) \]

### 3.1.2.2 \( O_i \) and \( H_i \) intersect (Case 2) Fig 2b

In this case, like the explanation in case 1, the regions which \( O_i \) and \( H_i \) do not intersect will be included in the union region after the operation. Therefore in this case the over-segmentation of the union output will be the intersection between \( O_i \) and \( H_i \).

Since the intersection between \( O_i \) and \( H_i \) is a proper subset of both \( O_i \) and \( H_i \), therefore the over-segmentation of the union result in this case is better than both \( RG_i \) and \( LS_i \).

\[ (H_i - O_i) \subseteq R_i \quad (9) \]

\[ (O_i - H_i) \subseteq E_i \quad (10) \]

\[ (H_i \cap O_i) \not\subseteq (R_i \cup E_i) \quad (11) \]

### 3.1.2.3 \( O_i \) is a subset of \( H_i \) or vice versa. (Case 3) Fig 2c

This is the worst case for over-segmentation. The final over-segmentation after the union operation in this case is the region that the smaller set covers. In this case the over-segmentation rate of the larger set is reduced but remains the same for the smaller set.

\[ (H_i \cap O_i) = O_i \quad (12) \]

\[ O_i \not\subseteq (R_i \cup E_i) \quad (13) \]

![Fig. 3. Venn diagram describing the three cases of the union operation.](image)

### 3.1.3 Step 3 of HAST

Step 3 of HAST is a decision step which requires the user to decide whether the image is acceptable. If the output from step 2 is acceptable, the output is taken as the result and the process ends here. However such a situation may not be true all the time. Very frequently, we may find that leaking exists in the output from step 2. This
is because a union operation will generate an image with more leaking than the two source images. Note that if both source images, RG, and LS, do not contain any leaking; the union output will contain no leaking. In a case where one of the source images contain leaking, or in the worst case when both contain leaking, the union output will contain more leaking. In this step, when such a case occurs, users will carry out the steps in step 4 to reduce leaking.

3.1.4 Step 4 of HAST
This step is designed to reduce leaking in the union output when the user has identified leaking in the previous step. This step will begin with the user generating two segmentation images using the two techniques respectively. The segmentation will be generated based on the tissue adjacent to the tissue of interest where the leaking occurs. Upon generating the images, the two images are combined using the union operation before the combined image is subtracted from the union output in step 2. The leaking-reducing step lies in the subtraction operation. The two images generated in this step need not be a good segmentation of the adjacent tissue. In actual fact, the reduction in leaking will take place as long as the union of the two images of the adjacent tissue does not contain leaking. Unlike the first step where bold parameter values are used to generate the segmentation of the tissue of interest, in this step, more conservative values are chosen. As it is probably apparent that we have reduced over-segmentation in step 2, therefore we would want to minimize the increase in over-segmentation in this step of leaking-reduction. After this step, the control will be passed back to the user (step 3) to decide whether there is any more leaking to be reduced. Note that after the subtraction operation, the region where leaking reduction is directed to will consist of small portions of the adjacent tissue that are not yet removed. These portions will be removed only at the end when the bulk of all the leaking regions have been subtracted.

3.1.5 Step 5 of HAST
This is the last step of HAST. In the previous steps, the bulk of all the leaked regions have already been subtracted from the main union output, leaving behind small portions of the leaked regions on the union output, I. The purpose of this step is to touch up I by removing all these small portions that are disjoint from the tissue of interest. This is done using the region-based technique by specifying a seed point within the tissue of interest and setting the lower and upper intensity threshold to be 1 and 300 respectively. By doing this, all connected pixels to the seed point will be included in the final result, IBF, omitting all the unconnected pixels from step 4.

4 Experimental Results
In order to investigate the effectiveness of HAST, experiments were conducted on real MR images taken from a national hospital. Since HAST is part of the effort taken to reduce the hassle in manual segmentation, which is a process carried out in many hospitals, therefore tests and experiments should be conducted using images that can
reflect the quality of MR images in the real world. Before proceeding to view the experimental results, in order to measure the over-segmentation and leaking present in segmentation images, the calculation of the over-segmentation rate (OR), under-segmentation rate (UR) and the overall error rate (ER) are given as below. Three variables that are used in the formulas, namely the number of pixels missing from the segmentation image, denoted by $O_p$, the number of pixels outside the desired of the segmentation image, denoted by $U_p$, and the number of pixels inside the desired region of the template that we had generated for experimental purpose, denoted by $D_p$.

The formulas below describe the three calculations:

$$ OR = \frac{O_p}{U_p + D_p} \quad (14) $$

$$ UR = \frac{U_p}{U_p + D_p} \quad (15) $$

$$ ER = \frac{O_p + U_p}{D_p} \quad (16) $$

A total of ten experiments were carried out in the initial phase using ten different MR images taken from different patients. These MR images reflect the real-world quality and contain sufficient noises that make the images susceptible to the problems of segmentation. The following tables show the summary of the experimental results compared to the results obtained using each of the two techniques used in HAST.

Table 1. Relative Over-Segmentation rate and Relative Under-Segmentation Rate of 10 experiments.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$RG_1$</th>
<th>$LS_1$</th>
<th>$I$</th>
<th>$I_F$</th>
<th>$RG_1$</th>
<th>$LS_1$</th>
<th>$I$</th>
<th>$I_F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.64</td>
<td>0.11</td>
<td>0.08</td>
<td>0.09</td>
<td>0.04</td>
<td>0.33</td>
<td>0.26</td>
<td>0.21</td>
</tr>
<tr>
<td>2</td>
<td>0.69</td>
<td>0.25</td>
<td>0.09</td>
<td>0.09</td>
<td>0</td>
<td>0.42</td>
<td>0.29</td>
<td>0.29</td>
</tr>
<tr>
<td>3</td>
<td>0.36</td>
<td>0.77</td>
<td>0.21</td>
<td>0.21</td>
<td>0.08</td>
<td>0</td>
<td>0.08</td>
<td>0.08</td>
</tr>
<tr>
<td>4</td>
<td>0.25</td>
<td>0.56</td>
<td>0.19</td>
<td>0.24</td>
<td>0.51</td>
<td>0</td>
<td>0.19</td>
<td>0.01</td>
</tr>
<tr>
<td>5</td>
<td>0.21</td>
<td>0.56</td>
<td>0.12</td>
<td>0.16</td>
<td>0.60</td>
<td>0</td>
<td>0.35</td>
<td>0.18</td>
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<tr>
<td>6</td>
<td>0.44</td>
<td>0.61</td>
<td>0.37</td>
<td>0.37</td>
<td>0.10</td>
<td>0.05</td>
<td>0.11</td>
<td>0.10</td>
</tr>
<tr>
<td>7</td>
<td>0.57</td>
<td>0.47</td>
<td>0.34</td>
<td>0.45</td>
<td>0.14</td>
<td>0.20</td>
<td>0.24</td>
<td>0.03</td>
</tr>
<tr>
<td>8</td>
<td>0.71</td>
<td>0.49</td>
<td>0.54</td>
<td>0.55</td>
<td>0</td>
<td>0.09</td>
<td>0.01</td>
<td>0</td>
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<tr>
<td>9</td>
<td>0.38</td>
<td>0.25</td>
<td>0.23</td>
<td>0.24</td>
<td>0.29</td>
<td>0.13</td>
<td>0.15</td>
<td>0.13</td>
</tr>
<tr>
<td>10</td>
<td>0.37</td>
<td>0.51</td>
<td>0.46</td>
<td>0.53</td>
<td>0.39</td>
<td>0.11</td>
<td>0.13</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2. Relative Overall Error rate of the 10 experiments.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$RG_1$</th>
<th>$LS_1$</th>
<th>$I$</th>
<th>$I_F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.71</td>
<td>0.65</td>
<td>0.47</td>
<td>0.38</td>
</tr>
<tr>
<td>2</td>
<td>0.69</td>
<td>1.17</td>
<td>0.54</td>
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<tr>
<td>3</td>
<td>0.48</td>
<td>0.77</td>
<td>0.32</td>
<td>0.32</td>
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<tr>
<td>4</td>
<td>1.57</td>
<td>0.56</td>
<td>0.47</td>
<td>0.25</td>
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<tr>
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<td>0.56</td>
<td>0.72</td>
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<tr>
<td>6</td>
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<td>0.69</td>
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<td>0.54</td>
</tr>
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<td>0.85</td>
<td>0.77</td>
<td>0.50</td>
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<tr>
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<td>0.65</td>
<td>0.56</td>
<td>0.56</td>
</tr>
<tr>
<td>9</td>
<td>0.93</td>
<td>0.44</td>
<td>0.45</td>
<td>0.42</td>
</tr>
</tbody>
</table>
From the experimental results, we can see clearly that this method reduces the problem of both leaking and over-segmentation. Before we proceed further to elaborate on the results, note that in all experiments except for experiment 3, the subtraction step for HAST (Step 4) is performed. For over-segmentation, in all ten experiments, the over-segmentation of the final result is reduced when compared to the initial two images, $RG_i$ and $LS_i$. However, one necessary point to notice is that the over-segmentation in the union of $RG_i$ and $LS_i$, in most cases is lesser than the final result. As we had mentioned in the theoretical part above, the ideal result of step 4 of HAST is to generate a segmentation of the adjacent tissue where the leaking occurs which does not leaked into the tissue of interest. However, such an assumption is not reasonable. In all these experiments except for experiment 3 which stops before the subtraction step, the segmentation of the adjacent tissues leak into the desired region, thus causing the over-segmentation in the final result to increase. One encouraging observation of this behavior is that the leaking in the segmentation of the adjacent tissues are usually very minimal. This will result in the increase in over-segmentation unnoticeable when compared to the amount of leaking that is reduced. In terms of leaking as a whole, we can easily notice that the union of $RG_i$ and $LS_i$ will usually cause a steep increase in leaking. As we had mentioned in the theoretical explanation, the increase in leaking at this point is expected. In the later steps where the segmentation of the adjacent tissue which leaks is subtracted from the union image, in all these cases except for experiment 3, the leakages are reduced. This is once again due to the fact that the subtraction step is not performed in experiment 3. In order to summarize the experiments conducted, the overall error rates of all the 10 experiments are also provided. Finally the most crucial explanation in terms of how HAST serves to improve both over-segmentation and leaking is that even though in the initial steps, we reduce over-segmentation at the expense of leaking and in the later steps we reduce leaking at the expense of over-segmentation, the overall improvements that we get are still much more that the price paid. This is clearly reflected in the improvement in the overall error rate in all the experiments.

5 Conclusion and Future Work

In this paper we discuss a new hybrid segmentation approach; the Hybrid Approach using Set Theory (HAST), which is a procedure incorporating set operations into a pipeline. In HAST, union is performed on the segmented images to reduce over-segmentation at the expense of leaking. However, the leaking from the initial images and those that are caused by the union step are then reduced altogether in the subtraction stage. Experiments were carried out using HAST, out of the ten experiments carried out, all the ten experiments provide better results when compared to the segmentation generated using any of the two individual images.

Although this approach is still somewhat new and the process is still subjected to the hassle of parameter initialization, new works have already been done to reduce the problem of initialization. More experiments are likely to be carried out in the future.
Acknowledgements

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