Multinomial Mixture Modelling for Bilingual Text Classification

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Abstract. Mixture modelling of class-conditional densities is a standard pattern classification technique. In text classification, the use of class-conditional multinomial mixtures can be seen as a generalisation of the Naive Bayes text classifier relaxing its (class-conditional feature) independence assumption. In this paper, we describe and compare several extensions of the class-conditional multinomial mixture-based text classifier for bilingual texts.

1 Introduction

Mixture modelling is a popular approach for density estimation in supervised and unsupervised pattern classification [1]. On the one hand, mixtures are flexible enough for finding an appropriate tradeoff between model complexity and the amount of training data available. Usually, model complexity is controlled by varying the number of mixture components while keeping the same (often simple) parametric form for all components. On the other hand, maximum likelihood estimation of mixture parameters can be reliably accomplished by the well-known Expectation-Maximisation (EM) algorithm.

Although most research on mixture models has concentrated on mixtures for continuous data, there are many pattern classification tasks for which discrete mixtures are better suited. This is the case of text classification (categorisation) [2]. In this case, the use of class-conditional discrete mixtures can be seen as a generalisation of the well-known Naive Bayes text classifier [3, 4]. In [5], the binary instantiation of the Naive Bayes classifier is generalised using class-conditional Bernoulli mixtures. Similarly, in [6, 7], its multinomial instantiation is generalised with multinomial mixtures. Both generalisations seek to relax the Naive Bayes (class-conditional feature) independence assumption made when using a single Bernoulli or multinomial distribution per class. This unrealistic assumption of the Naive Bayes classifier is one of the main reasons explaining its comparatively poor results in contrast to other techniques such as boosting-based classifier committees, support vector machines, example-based methods and regression methods [2]. In fact, the performance of the Naive Bayes classifier is significantly improved by using the generalisations mentioned above [5–7]. Moreover, there are other recent generalisations (and corrections) that also overcome the weaknesses of the Naive Bayes classifier and achieve very competitive results [8–12].

In this paper, we describe and compare several (minor) extensions of the (class-conditional) multinomial mixture-based text classifier for the case in which text data
is available in two languages. Our interest in this task of bilingual text classification comes from its potential use in statistical machine translation. In this application area, the problem of learning a complex, global statistical transducer from heterogeneous bilingual sentence pairs can be greatly simplified by first classifying sentence pairs into homogeneous classes and then learning simpler, class-specific transducers [13]. Clearly, this is only a marginal application of bilingual text classification. More generally, the proliferation of multilingual documentation in our Information Society will surely attract many research efforts in multilingual text classification. Obviously, most conventional, monolingual text classifiers can be also extended in order to fully exploit the intrinsic redundancy of multilingual texts.

The following section describes the different basic models we consider for multinomial mixture modelling of bilingual texts. In section 3, we briefly discuss how to plug these basic models in the Bayes decision rule for bilingual classification. Section 4 poses the maximum likelihood estimation of these models using the EM algorithm. Finally, section 5 will be devoted to experimental results and section 6 will discuss some conclusions and future work.

2 Multinomial Mixture Modelling

A finite mixture model is a probability (density) function of the form:

\[ p(x) = \sum_{i=1}^{I} \alpha_i p(x | i) \]  

(1)

where \( I \) is the number of mixture components and, for each component \( i \), \( \alpha_i \in [0, 1] \) is its prior or coefficient and \( p(x | i) \) is its component-conditional probability (density) function. It can be seen as a generative model that first selects the \( i \)th component with probability \( \alpha_i \) and then generates \( x \) in accordance with \( p(x | i) \).

The choice of a particular functional form for the components depends on the type of data at hand and the way it is represented. In the case of the bag-of-words text representation, the order in which words occur in a given sentence (or document) is ignored; the only information retained is a vector of word counts \( x = (x_1, \ldots, x_D) \), where \( x_d \) is the number of occurrences of word \( d \) in the sentence, and \( D \) is the size of the vocabulary \( (d = 1, \ldots, D) \). In this case, a convenient choice is to model each component \( i \) as a D-dimensional multinomial probability function governed by its own vector of parameters or prototype \( p_i = (p_{i1}, \ldots, p_{iD}) \in [0, 1]^D \),

\[ p(x | i) = \frac{x_+!}{\prod_{d=1}^{D} x_d!} \prod_{d=1}^{D} p_{id}^{x_d} \]  

(2)

where \( x_+ = \sum_d x_d \) is the sentence length. Equation (1) in this particular case is called multinomial mixture. Note that the first factor in (2) is a multinomial coefficient giving the number of different sentences of length \( x_+ \) that are equivalent in the sense of having identical vector of word counts \( x \). Also note that \( p_{id} \) is the \( i \)th component-conditional probability of word \( d \) to occur in a sentence and, therefore, the second factor in (2)
is the probability that each of these equivalent sentences has to occur. Thus, Eq. (2) (and Eq. (1)) defines a explicit probability function over all $D$-dimensional vectors of word counts with identical $x_+$, and an implicit probability function over all sentences of length $x_+$ in which equivalent sentences are equally probable.

In this work, we are interested in modelling the distribution of bilingual texts; i.e. pairs of sentences (or documents) that are mutual translations of each other. Bilingual texts will be formally described using a direct extension of the bag-of-words representation of monolingual text. That is, we have pairs of the form $(x, y)$ in which $x$ is the bag-of-words representation of a sentence in an input (source) language, and $y$ is its counterpart in an output (target) language. For instance, $x$ and $y$ may be bag-of-words in Dutch and English, respectively.

For modelling the probability of a pair $(x, y)$, we will consider five simple models:

1. **Monolingual input-language model:**
   \[ p(x, y) = p(x) \] (3)
   where $p(x)$ is given by (1) and (2).

2. **Monolingual output-language model:**
   \[ p(x, y) = p(y) \] (4)
   where $p(y)$ is a multinomial mixture model for the output bag-of-words,
   \[ p(y) = \sum_{i=1}^{I} \beta_i p(y | i) \quad \text{with} \quad p(y | i) = \frac{y_+!}{\prod_{e=1}^{E} y_e!} \prod_{e=1}^{E} q_{ie}^{y_e} \] (5)
   where $q_{ie}$ is the $i$th component-conditional probability of word $e$ to occur in an output sentence.

3. **Bilingual bag-of-words model:**
   \[ p(x, y) = p(z) \] (6)
   where $z$ is a bilingual bag-of-words obtained from the concatenation of the sentences originating $(x, y)$, and $p(z)$ is a monolingual, multinomial mixture model like the two previous models.

4. **Global (Naive Bayes) decomposition model:**
   \[ p(x, y) = p(x) p(y) \] (7)
   where $p(x)$ and $p(y)$ are given by the first two monolingual models above.

5. **Local (Naive Bayes) decomposition model:**
   \[ p(x, y) = \sum_{i=1}^{I} \gamma_i p(x, y | i) \quad \text{with} \quad p(x, y | i) = p(x | i) p(y | i) \] (8)
   where $p(x | i)$ is given by (2) and $p(y | i)$ by (5).
Note that the first two models ignore one of the languages involved and hence they do not take advantage of the intrinsic redundancy in the available data. The remaining manage bilingual data in slightly different ways.

3 Bilingual Text Classification

As with other types of mixtures, multinomial mixtures can be used as class-conditional models in supervised classification tasks. Let $C$ denote the number of supervised classes. Assume that, for each supervised class $c$, we know its prior $p_c$ and its class-conditional probability function, which is given by one of the five models discussed in the previous section. Then, the Bayes decision rule is to assign each pair $(x, y)$ to a class giving maximum a posteriori probability or, equivalently,

$$c(x, y) = \arg\max_c \log p_c + \log p(x, y | c)$$  \hspace{1cm} (9)$$

In the case of the monolingual input-language model, this rule becomes:

$$c(x, y) = \arg\max_c \log p_c + \log \sum_{i=1}^I \alpha_{ci} \prod_{d=1}^D p_{xid}$$  \hspace{1cm} (10)$$

Similar rules hold for the monolingual output-language model and the bilingual bag-of-words model. In the case of the global decomposition model, it is

$$c(x, y) = \arg\max_c \log p_c + \log \sum_{i=1}^I \alpha_{ci} \prod_{d=1}^D p_{xid} + \log \sum_{i=1}^I \gamma_{ci} \prod_{e=1}^E q_{yc}$$  \hspace{1cm} (11)$$

while, in the local decomposition model, we have

$$c(x, y) = \arg\max_c \log p_c + \log \sum_{i=1}^I \gamma_{ci} \prod_{d=1}^D p_{xid} \prod_{e=1}^E q_{yc}$$  \hspace{1cm} (12)$$

4 Maximum Likelihood Estimation

Let $(X, Y) = \{(x_1, y_1), \ldots, (x_N, y_N)\}$ be a set of samples available to learn one of the five mixture models discussed in section 2. This is a statistical parameter estimation problem since the mixture is a probability function of known functional form, and all that is unknown is a parameter vector including the priors and component prototypes. In what follows, we will focus on the local decomposition model; the rest of models can be estimated in a similar way.

The vector of unknown parameters for the local decomposition model is:

$$\Theta = (\gamma_1, \ldots, \gamma_I; p_1, \ldots, p_I; q_1, \ldots, q_I)$$  \hspace{1cm} (13)$$

We are excluding the number of components from the estimation problem, as it is a crucial parameter to control model complexity and receives special attention in Section 5.
Following the maximum likelihood principle, the best parameter values maximise the log-likelihood function

\[ L(\Theta | X, Y) = \sum_{n=1}^{N} \log \sum_{i=1}^{I} \gamma_i p(x_n | i) p(y_n | i) \]  

(14)

In order to find these optimal values, it is useful to think of each sample pair \((x_n, y_n)\) as an incomplete component-labelled sample, which can be completed by an indicator vector \(z_n = (z_{n1}, \ldots, z_{nI})\) with 1 in the position corresponding to the component generating \((x_n, y_n)\) and zeros elsewhere. In doing so, a complete version of the log-likelihood function (14) can be stated as

\[ L_C(\Theta | X, Y, Z) = \sum_{n=1}^{N} \sum_{i=1}^{I} z_{ni} [\log \gamma_i + \log p(x_n | i) + \log p(y_n | i)] \]  

(15)

where \(Z = \{z_1, \ldots, z_N\}\) is the so-called missing data.

The form of the log-likelihood function given in (15) is generally preferred because it makes available the well-known EM optimisation algorithm (for finite mixtures) [14].

This algorithm proceeds iteratively in two steps. The E(xpectation) step computes the expected value of the missing data given the incomplete data and the current parameters. The M(aximisation) step finds the parameter values which maximise (15), on the basis of the missing data estimated in the E step. In our case, the E step replaces each \(z_{ni}\) by the posterior probability of \((x_n, y_n)\) being actually generated by the \(i^{th}\) component,

\[ z_{ni} = \frac{\gamma_i p(x_n | i) p(y_n | i)}{\sum_{i'=1}^{I} \gamma_{i'} p(x_n | i') p(y_n | i')} \]  

(16)

for all \(n = 1, \ldots, N\) and \(i = 1, \ldots, I\), while the M step finds the maximum likelihood estimates for the priors,

\[ \gamma_i = \frac{1}{N} \sum_{n=1}^{N} z_{ni} \quad (i = 1, \ldots, I) \]  

(17)

and the component prototypes,

\[ p_i = \frac{1}{\sum_{n=1}^{N} z_{ni} \sum_{d=1}^{D} x_{nd}} \sum_{n=1}^{N} z_{ni} x_n \quad q_i = \frac{1}{\sum_{n=1}^{N} z_{ni} \sum_{e=1}^{E} y_{ne}} \sum_{n=1}^{N} z_{ni} y_n \]  

(18)

for all \(i = 1, \ldots, I\).

The above estimation problem and algorithm are only valid for a single multinomial mixture of the form (8). Nevertheless, it is straightforward to extend them in order to simultaneously work with several class-conditional mixtures in a supervised setting. In this setting, training samples come with their corresponding class labels, \(\{(x_n, y_n, c_n)\}_{n=1}^{N}\), and the vector of unknown parameters is:

\[ \Psi = (p_1, \ldots, p_C; \Theta_1, \ldots, \Theta_C) \]  

(19)
where, for each supervised class \( c \), its prior probability is given by \( p_c \) and its class-
conditional probability function is a mixture controlled by a vector of the form (13), \( \Theta_c \). The log-likelihood of \( \Psi \) w.r.t. the labelled data is

\[
L = \sum_{n=1}^{N} \log p_c \sum_{i=1}^{I} \gamma_{c_n} p(x_n|\theta, c_n) p(y_n|\theta, c_n)
\]

which can be optimised by a simple extension of the EM algorithm given above. More
precisely, the E step computes (16) using \( \Theta_{c_n} \), while the M step computes the con-
tventional estimates for the class priors and (class-dependent versions of) Eqs. (17) to (18)
for each class separately. This simple extension of the EM algorithm is equivalent to the
usual practice of applying its basic version to each supervised class in turn. However,
we prefer to adopt the extended EM, mainly to have a unified framework for classifier
training in accordance with the log-likelihood criterion (20).

5 Experimental Results

The five different models considered were assessed and compared on two bilingual
text classification datasets (tasks) known as the Traveller dataset and the BAF corpus.
The Traveller dataset comprises Spanish-English sentence pairs drawn from a restricted
semantic domain, while BAF is a parallel French-English corpus collected from a mis-
cellaneous "institutional" document pool. This section first describes these datasets and
then provides the experimental results obtained.

5.1 Datasets

The Traveller dataset comes from a limited-domain Spanish-English machine transla-
tion application for human-to-human communication situations in the front-desk of a
hotel [15]. It was semi-automatically built from a small “seed” dataset of sentence pairs
collected from traveller-oriented booklets by four persons. Note that each person had to
cater for a (non-disjoint) subset of subdomains, and thus each person can be considered
a different (multimodal) class of Spanish-English sentence pairs. Subdomain overlap-
ping among classes foresees that perfect classification is not possible, although in our
case, low classification error rates will indicate that our mixture model has been able to
capture the multimodal nature of the data. Unfortunately, the subdomain of each pair
was not recorded, and hence we cannot train a subdomain-supervised multinomial mix-
ture in each class to see how it compares to mixtures learnt without such supervision.

The Traveller dataset contains 8,000 sentence pairs, with 2,000 pairs per class. The
size of the vocabulary and the number of singletons reflect the relative simplicity of this
corpus. Some statistics are shown in Table 1.

The BAF corpus [16] is a compilation of bilingual "institutional" French-English texts
ranging from debates of the Canadian parliament (Hansard), court transcripts and
UN reports to scientific, technical and literary documents. This dataset is composed of
11 documents that are organised into 4 natural genres (Institutional, Scientific, Tech-
nical and Literary) trying to be representative of the types of text that are available in
multilingual versions. Institutional and Scientific classes comprises documents from the original pool of 11 documents, which were theme-related, but devoted to heterogeneous purposes or written by different authors. This fact provides the multimodal nature to the BAF corpus that can be adequately modelled by mixture models. The BAF corpus was aligned at the sentence level by human experts and it was initially thought to be used as a reference corpus to evaluate automatic alignment techniques in machine translation.

Prior to performing the experiments, the BAF corpus was simplified in order to reduce the size of the vocabulary and discard spurious sentence pairs. This preprocessing mainly consisted in three basic actions: downcasing, replacement of those words containing a sequence of numbers by a generic label, and isolation of punctuation marks. This basic procedure halved the size of the vocabulary and significantly simplified this corpus. Neither stopword lists, nor stemming techniques were applied since, as shown in [8], it is unclear whether this further preprocessing may be convenient. As it can be seen in Table 1, this corpus is more complex than the Traveller dataset.

<table>
<thead>
<tr>
<th></th>
<th>Traveller</th>
<th>BAF</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Sp</td>
<td>En</td>
</tr>
<tr>
<td>sentence pairs</td>
<td>8000</td>
<td>18509</td>
</tr>
<tr>
<td>average length</td>
<td>9</td>
<td>8</td>
</tr>
<tr>
<td>vocabulary size</td>
<td>679</td>
<td>503</td>
</tr>
<tr>
<td>singletons</td>
<td>95</td>
<td>106</td>
</tr>
<tr>
<td>running words</td>
<td>86K</td>
<td>80K</td>
</tr>
</tbody>
</table>

5.2 Experimental Results

Several experiments were carried out to analyse the behaviour of each individual classifier in terms of log-likelihood and classification error rate as a function of the number of mixture components per class (\( I \in \{1, 2, 5, 10, 20, 50, 100\} \)). This was done for a training and test sets resulting from a random dataset partition (1/2-1/2 split for Traveller and 4/5-1/5 for BAF).

Figure 1 shows the evolution of the error rate (left \( y \) axis) and log-likelihood (right \( y \) axis), on training and test sets, for an increasing number of mixture components (\( x \) axis). From top to bottom rows we have: the best monolingual classifier (English in both datasets), the bilingual bag-of-words classifier, and global and local classifiers. Each plotted point is an average over values obtained from 30 randomised trials.

From the results in Figure 1, we can see that the evolution of the log-likelihood on the training and test sets is as theoretically expected, for all classifiers in both, Traveller and BAF. The log-likelihood in training always increases, while the log-likelihood in test increases up to a moderate number of components (20 – 50 in Traveller and 5 – 10 in BAF). This number of components can be considered as an indication of the number of “natural” subclasses in the data. About this number of mixture components
is also commonly found the lowest classification test error rate, as it occurs in our case. As the number of components keeps increasing, the well-known overtraining effect appears, the log-likelihood in test falls and the accuracy degrades. For this reason we decided to limit the number of mixture components to 100, since additional trials with an increasing number of mixture components confirmed this performance degradation.

Figure 2 shows competing curves for test error-rate as a function of the number of mixture components for the English-based, bilingual bag-of-words-based, global and local classifiers; there are two plots, one for Traveller and the other for BAF. Error bars representing 95% confidence intervals are plotted for the English-based classifiers in both plots, and the global classifier in BAF.

From the results for Traveller in Figure 2, we can see that there is no significant statistical difference in terms of error rate between the best monolingual classifier and the bilingual classifiers. The reason behind these similar results can be better explained in the light of the statistics of the Traveller dataset shown in Table 1. The simplicity of the Traveller dataset, characterised by its small vocabulary size and its large number of running words, allows for a reliable estimation of model parameters in both languages. This is reflected in the high accuracy (≈ 95%) of the monolingual classifiers and the little contribution of a second language to boost the performance of bilingual classifiers. Nevertheless, bilingual classifiers seem to achieve systematically better results.

In contrast to the results obtained for Traveller, the results for BAF in Figure 2 indicate that bilingual classifiers perform significantly better than monolingual models. More precisely, if we compare the curves for the English-based classifier and the global classifier, we can observe that there is no overlapping between their error-rate confidence intervals. Clearly, the complexity and data scarcity problem of the BAF corpus lead to poorly estimated models, favouring bilingual classifiers that take advantage of both languages. However, the different bilingual classifiers have similar performance.

Additional experiments using smooth \( n \)-gram language models were performed with the well-known and publicly available SRILM toolkit [17]. A Witten-Bell [18] smoothed \( n \)-gram language model was trained for each supervised class separately and for both languages independently. These class-dependent language models were used to define monolingual and bilingual Naive Bayes classifiers. Results are given in Table 2.

From the results in Table 2, we can see that 1-gram language models are similar to our 1-component mixture models. In fact, both models are equivalent except for the parameter smoothing. The results obtained with \( n \)-gram classifiers with \( n > 1 \) are much better that the results for \( n = 1 \) and slightly better than the best results obtained with general \( I \)-component multinomial mixtures. More precisely, the best results achieved with \( n \)-grams are 1.1% in Traveller and 2.6% in BAF, while the best results obtained with multinomial mixtures are 1.4% in Traveller and 2.9% in BAF.

**Table 2.** Test-set error rates for monolingual and bilingual naive classifiers based on smooth \( n \)-gram language models in Traveller and BAF.

<table>
<thead>
<tr>
<th>Traveller</th>
<th>1-gram</th>
<th>2-gram</th>
<th>3-gram</th>
</tr>
</thead>
<tbody>
<tr>
<td>English classifier</td>
<td>4.1</td>
<td>1.9</td>
<td>1.3</td>
</tr>
<tr>
<td>Spanish classifier</td>
<td>2.8</td>
<td>1.2</td>
<td>1.2</td>
</tr>
<tr>
<td>Bilingual classifier</td>
<td>3.3</td>
<td>1.2</td>
<td>1.1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>BAF</th>
<th>1-gram</th>
<th>2-gram</th>
<th>3-gram</th>
</tr>
</thead>
<tbody>
<tr>
<td>English classifier</td>
<td>5.3</td>
<td>3.5</td>
<td>3.6</td>
</tr>
<tr>
<td>French classifier</td>
<td>6.7</td>
<td>4.4</td>
<td>4.4</td>
</tr>
<tr>
<td>Bilingual classifier</td>
<td>4.1</td>
<td>2.8</td>
<td>2.6</td>
</tr>
</tbody>
</table>
Fig. 1. Error rate and log-likelihood curves in training and test sets as a function of the number of mixture components, in Traveller (left column) and BAF (right column) for the four classifiers considered. Classifiers: the best monolingual, the bilingual bag-of-words (BBoW), the global and the local classifier.
6 Conclusions and Future Work

We have presented three different extensions of the multinomial mixture-based text classification model for bilingual text: the bilingual bag-of-words model and the global and local decomposition models. The performance of these extensions was compared to that of monolingual and smooth $n$-gram classifiers. Two outstanding conclusions can be stated from the results presented. First, mixture-based classifiers surpass single-component classifiers in all cases (monolingual, bilingual bag-of-words, global and local). In fact, we have taken advantage of the flexibility of the mixture modelisation over the ”single-component” approach to further improve the error rates achieved. This mixture modelling superiority is also reflected in the monolingual versions of our text classifiers and corroborated through smooth $n$-gram language model experiments with independent software. Second, bilingual classifiers outperform their monolingual and smooth 1-gram counterparts, and the excellence of bilingual classifiers is more clearly shown when the complexity of the dataset does not allow for monolingual well-estimated models, as in the BAF corpus. Therefore, the contribution of an extra source of information instantiated as a second language cannot be neglected.

As a future work, smooth $n$-gram language models for bilingual text classification provide an interesting starting point for future research based on more versatile language models, as mixtures of bilingual $n$-gram language models. A promising extension of this work would be the development of mixture of 2-gram language models.

All in all, the bilingual approaches described in this work are relatively simple models for the statistical distribution of bilingual texts. More sophisticated models, such as IBM statistical translation models [19], may be better in describing the statistical distribution of bilingual, correlated texts.
References