THE USE OF DYNAMICS IN GRAYLEVEL QUANTIZATION BY MORPHOLOGICAL HISTOGRAM PROCESSING

Franklin César Flores, Leonardo Bespalhuk Facci
Department of Informatics, State University of Maringá
Av. Colombo, 5790 - Bloco 19 - Zona 07, Zip Code 87020-900 - Maringá - PR - BRAZIL

Roberto de Alencar Lotufo
School of Electrical and Computer Engineering, State University of Campinas
Av. Albert Einstein, 400 - PO Box 6101, Zip Code: 13083-970 - Campinas - SP - BRAZIL

Keywords: Morphological processing, histogram classification, dynamics, watershed.

Abstract: In a previous paper, it was proposed a method applied to image simplification in terms of graylevel and flat zone reduction, by histogram classification via morphological processing. It this method, it is possible to reduce the number of graylevels of an image to n graylevels by selecting n regional maxima in the processed histogram and discarding the remaining ones, in other to classify the histogram via application of watershed operator. In the previous paper, it was proposed the choice of the n highest regional maxima. By far, it is not the best criterion to choose the regional maxima and other criteria had been were tested in order to obtain a better histogram classification. In this paper we propose the selection of the regional maxima via application of dynamics, a measurement of contrast usually applied to find markers to morphological segmentation.

1 INTRODUCTION

Color quantization is a very important research field in digital image processing and computer graphics, and among its contributions, it may be found several techniques applied to simplification of images by color reduction (Gonzalez and Woods, 1992; Heckbert, 1982; Soille, 1996). The importance of image simplification by color reduction is clear when dealing with problems of image display and image compression (Gomes and Velho, 1994).

The application of color reduction techniques may also provide the reduction of *flat zones* (connected regions of pixels with constant color) in the image, such a connected filter (Crespo et al., 1997; Salembier and Serra, 1995; Heijmans, 1999; Meyer, 1998). The reduction of flat zones does not introduct borders in the image, but, by supressing some borders, two of more flat zones may be joined in one.

Flat zone reduction have a great number of applications. They can be, for example, applied to image compression and image segmentation (Meyer and Beucher, 1990; Beucher and Meyer, 1992). They are also applied to reduce the statistics of the image, in order to simplify the number of attributes used in pattern recognition techniques (Hirata Jr. et al., 1999; Flores et al., 2000; F. C. Flores and Zuben, 2002).

One simple way to reduce graylevels in an image is the classical thresholding. Given a thresholding value, the graylevels are classified by setting the pixel value to white (maximum graylevel) if its graylevel is higher than the thresholding value, or to black (minimum graylevel), otherwise. The classical thresholding can be extended by applying a set of n thresholding values, in order to reduce the number of graylevels of an image to $n + 1$ graylevels: if a graylevel value belongs to an interval given by a pair of thresholding values, it should be replaced by the graylevel assigned to that interval.

In a previous paper (Flores and Lotufo, 2001), it was proposed a method which gives not only an image simplification in terms of graylevel reduction but also in terms of flat zone reduction. The proposed method is given by application of a set of morphological operators to the image histogram. The main motivation behind the project of this operator is that each object in the image has a significative graylevel distribution. So, to simplify an object in the image, that is enough to classify its corresponding distribution in the histogram.

In that paper it was also proposed a method to reduce an image to n graylevels. It consists in to choose the n highest regional maxima in the processed histogram and to filter the other peaks. The chosen maxima will provide the classification of the graylevels in the histogram by application of watershed operator.

César Flores F., Bespalhuk Facci L. and de Alencar Lotufo R. (2006).
THE USE OF DYNAMICS IN GRAYLEVEL QUANTIZATION BY MORPHOLOGICAL HISTOGRAM PROCESSING.
In *Proceedings of the First International Conference on Computer Vision Theory and Applications*, pages 121-128
DOI: 10.5220/0001374501210128

Note that, by far, it is not the best criterion to choose the regional maxima and other criteria were tested in order to obtain a better histogram classification.

In this paper we propose the application of *dynamics* (Grimaud, 1992) to select the regional maxima in order to achieve a better graylevel reduction. Dynamics consists in a valuation of extrema of the image by a measure of contrast that does not consider the size or shape of valleys and peaks. It is usually applied to find markers to morphological segmentationand achieve hierarquical segmentation (Meyer, 1996).

Section 2 presents some preliminar definitions, and section 3 presents the dynamics. Section 4 proposes the technique to connected filtering by graylevel classification and a variation when it is possible to choose the desired number of graylevels in the resulting image. Section 5 presents some experimental results and in the Section 6 we conclude this paper with a brief discussion.

2 PRELIMINARY DEFINITIONS

Let $E \subset \mathbb{Z}\times\mathbb{Z}$ be a rectangular finite subset of points. Let $K = [0, k]$ be a totally ordered set. Denote by $Fun[E, K]$ the set of all functions $f : E \rightarrow K$. An *image* is one of these functions (called graylevel functions). Particularly, if $K = [0, 1]$, f is a binary image. An *image operator* (*operator*, for simplicity) is a mapping $\psi : Fun[E, K] \rightarrow Fun[E, K]$.

Let $N(x)$ be the set containing the *neighbourhood* (Hirata Jr., 1997; Flores, 2000) of x, $x \in E$. We define a *path* (Hirata Jr., 1997) from x to y, $x, y \in E$ as a sequence $P(x, y) = (p_0, p_1, ..., p_n)$ from E, where $p_0 = x$, $p_n = y$ and $\forall i \in [0, n-1], p_i \in N(p_{i+1})$.

A *connected subset* of E is a subset $X \subset E$ such that, $\forall x, y \in X$, there is a path C entirely inside X.

Let $f \in Fun[E, K]$. A *flat zone* of f is a connected subset $X \subset E$, such that $f(x) = f(y), \forall x, y \in X$.

Definition 1 *The inf - reconstruction and sup - reconstruction operators are given, respectively, by,* $\forall f, g \in Fun[E, K]$,

$$\rho_{B,g}(f) = \delta^{\infty}_{B,g}(f)$$

$$\rho^{*}_{B,g}(f) = \varepsilon^{\infty}_{B,g}(f)$$

where $B \subset E$ *is the structuring element,* $n \in \mathbb{Z}_+$ *and* $\delta^n_{B,g}$ *and* $\varepsilon^n_{B,g}$ *are, respectively, the* n*-conditional dilation and the* n*-conditional erosion operators (Serra, 1982; Heijmans, 1994).* $\delta^{\infty}_{B,g}(f)$ *(*$\varepsilon^{\infty}_{B,g}(f)$*) means that the dilation (erosion) is applied till idempotency.*

Let $\tau_i : Fun[E, K] \rightarrow Fun[E, [0, 1]]$, $i \in K$, be a threshold function, where $\tau_i(f)(x) = 1$, if $f(x) \geq i$, and $\tau_i(f)(x) = 0$, otherwise.

Definition 2 *Let* $f \in Fun[E, K]$*. A regional maximum is a flat zone* Z *such that* $f(z) > f(n)$*,* $z \in Z$*,* $n \in N, N \in \mathcal{F}_Z$*, where* $\mathcal{F}_Z$ *is a set of all flat zones adjacent to* Z *(Flores, 2000). The regional maxima of* f *is found by application of a operator* $\mu^{\max}_{B_c} : Fun[E, K] \rightarrow Fun[E, [0, 1]]$*, given by*

$$\mu^{\max}_{B_c}(f) = \tau_1(\rho_{B_c,(f+1)}(f)) \vee \tau_k(f)$$

where $B_c \subset E$ *is the structuring element defining connectivity.*

A regional miminum is a flat zone Z *such that* $f(z) < f(n)$*,* $z \in Z$*,* $n \in N, N \in \mathcal{F}_Z$*, where* $\mathcal{F}_Z$ *is a set of all flat zones adjacent to* Z*.*

3 DYNAMICS

Dynamics (Grimaud, 1992; Meyer, 1996) is a transformation which valuates the extrema of an image according to a contrast measurement. One advantage of application of dynamics is that, while some methods such as morphological filters need a size parameter to evaluate constrast, the dynamics measurement does not take in account the size and the shape of image structures.

The evaluation of constrast of a regional minimum is a good way to provide markers to application of watershed operator in the morphological segmentation framework: an hierarquical segmentation may be achieved by selecting the regional minima which dynamics is higher than a thresholding value and assigning markers to them (Meyer, 1996).

Definition 3 *Let* $x, y \in E$*. The dynamics* Dyn_f *of a path* $P(x, y)$ *on an image* $f \in Fun[E, K]$ *is given by,*

$$Dyn_f(P(x, y)) = \{\bigvee |f(x_i) - f(x_j)| : x_i, x_j \in P(x, y)\}$$

i.e., the dynamics of $P(x, y)$ *is given by the difference in altitude between the points of highest and lowest altitude of P(x,y).*

Figure 1 shows the dynamics of a path between x and y.

Grimaud (Grimaud, 1992) also defines the dynamics between two points $x, y \in E$ on an image $f \in Fun[E, K]$ as

$$Dyn_f(x, y) = \{\bigwedge Dyn_f(P(x, y)) : P(x, y), \}$$

where $P(x, y)$ is a path between x and y. However, it will not be applied here, since the histogram is an 1-D signal and, therefore, there is only one path between any two points from the domain of histogram function. So, it will be considered here that $Dyn_f(x, y) = Dyn_f(P(x, y))$.

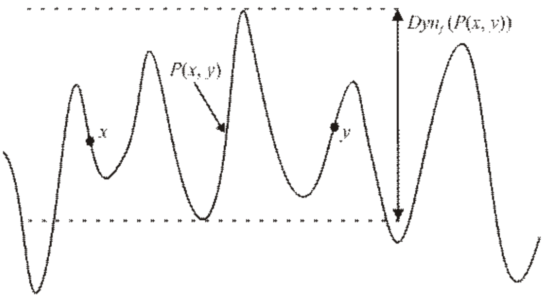


Figure 1: Dynamics of the Path Between x and y.

Definition 4 *Let $a(Z) \in K$ be the altitude of a regional minimum Z in f. The dynamics of Z is given by,*

$$Dyn(Z) = \{\bigwedge Dyn_f(x, y), x \in Z, y \in M : a(M) < a(Z)\}.$$

i.e., the dynamics of Z is given by the dynamics of the path with the lowest dynamics that links Z to a point y thats belongs to a catchment basin which regional minimum has an altitude lower than Z.

Figure 2 illustrates the dynamics of a regional minima Z.

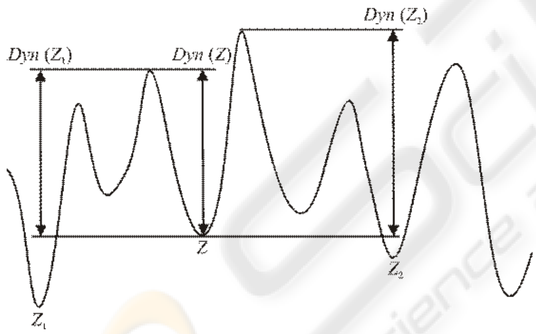


Figure 2: Dynamics of a Regional Minimum.

Dynamics computation can be implemented by using tree of critical lakes (Meyer, 1996) or based on flooding simulations algorithms (Grimaud, 1992).

Given the dynamics of a regional minimum Z, some metrics can be used to evaluate such minimum (da Silva, 2001):

1. depth of the catchment basin which the minimum is contained (given by the dynamics of the minimum itself) (Fig. 3 (a));
2. area of the catchment basin (Fig. 3 (b));
3. volume of the catchment basin (Fig. 3 (c));

Let us denote by $Dyn_i(f)(Z)$ the function that computes to Z from f an value given by the metric $i \in \{1, 2, 3\}$ introduced above. $Dyn_i(f)(Z)$ will be used to evaluate the significant distributions in the histogram, as will be explained below.

Note that two catchment basin which have the same depth may have different volume or area measurements. Classification of regional minima in an image can be achieved by application of such metrics.

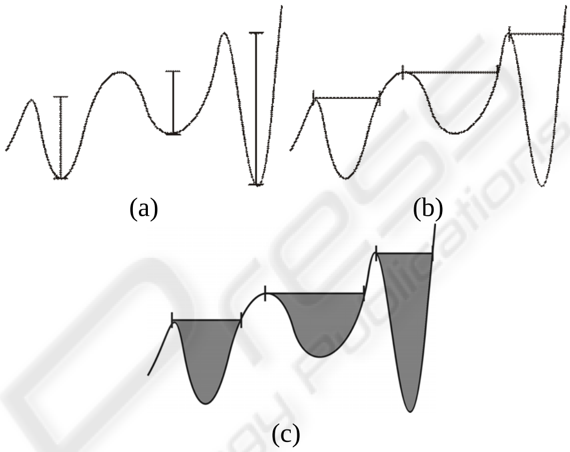


Figure 3: Dynamics: (a) Depth. (b) Area. (c) Volume.

4 THE PROPOSED TECHNIQUE

Let us introduce a new technique applied to connected filtering. One characteristic of the proposed filter is that, despite its connecting property, it does not require a connectivity parameter (4-connect or 8-connect) because all processing is done in the histogram of image.

As a consequence of such processing we have a reduction in the graylevels appearing in the image. In other words, the proposed filter is a mapping $\psi : Fun[E, K_1] \rightarrow Fun[E, K_2]$, where $|K_2| < |K_1|$.

Let $f \in Fun[E, K]$. Let us consider the histogram of the image as a function $h_f : K \rightarrow \mathbb{Z}_+$ (Fig. 4). Despite the domain of h_f, the morphological operators used in the graylevel classification were applied to in the same manner. In other words, we consider K as a subset of E.

Let $f \in Fun[E, K]$ and $h_f \in Fun[K, \mathbb{Z}_+]$ its histogram. Let $\max(h_f) = \max\{h_f(x) : x \in K\}$. Let $\kappa : Fun[K, \mathbb{Z}_+] \rightarrow Fun[K, \mathbb{Z}_+]$ be the mapping, given by, $\forall x \in K$,

$$\kappa(h_f)(x) = \begin{cases} h_f(x), \text{ if } \mu_B^{\max}(h_f)(x) = 1 \\ 0, \text{otherwise} \end{cases},$$

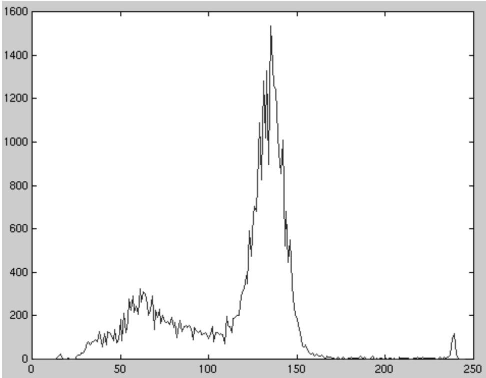


Figure 4: Histogram.

where $B \subset K$ is the structuring element (Heijmans, 1994). The mapping κ gives a function with only the regional maxima of h_f. Figure 5 shows the regional maxima of the histogram shown in Fig. 4.

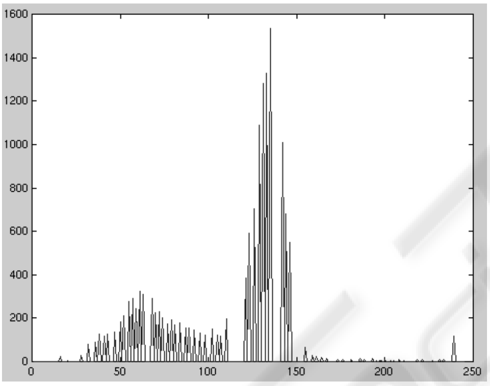


Figure 5: Histogram regional maxima.

Let $\pi : Fun[K, \mathbb{Z}_+] \rightarrow Fun[K, \mathbb{Z}_+]$ be the mapping, given by, $\forall x \in K$,

$$\pi(h_f)(x) = \begin{cases} \max{(h_f)}, \text{ if } \kappa(h_f)(x) = 0 \\ 0, \text{ otherwise} \end{cases} .$$

Let $\eta : Fun[K, \mathbb{Z}_+] \rightarrow Fun[K, \mathbb{Z}_+]$ be the sup-reconstruction of κ, denoted by,

$$\eta = \rho^*_{B,\kappa}(\pi),$$

where $B \subset K$ is the structuring element.

If exists in $\kappa(h_f)$ a sequence of increasing regional maxima followed by a sequence of decreasing regional maxima, its regional maxima must compose a curve whose regional maximum is the maximum among them. The operator η is applied to the function $\kappa(h_f)$, in order to preserve the regional maxima among the set of regional maxima of $\kappa(h_f)$ and construct the curves with the remaining regional maxima (Fig. 6).

The reason of the processing of the graylevel distributions in the histogram is that each object in the image is represented by a significative graylevel distribution. The idea is to filter the histogram in order to get new distributions where the objects are simplified and well represented. These new distributions are used to get a meaningful classification of graylevels by application of watershed operator introduced below.

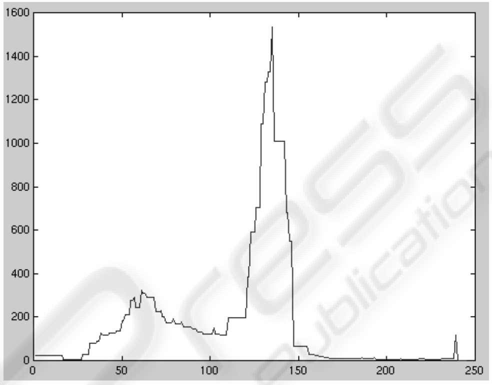


Figure 6: Reconstruction by application of η operator.

Let $\omega : Fun[K, \mathbb{Z}_+] \rightarrow Fun[K, [0, 1]]$ be the watershed operator (Beucher and Meyer, 1992; Vincent and Soille, 1991). Let $\nu : Fun[K, \mathbb{Z}_+] \rightarrow Fun[K, \mathbb{Z}_+]$ be the negation operator.

Let $\lambda : Fun[K, \mathbb{Z}_+] \rightarrow Fun[K, \mathbb{Z}_+]$ be the mapping, given by, $\forall x \in K$,

$$\lambda(x) = \begin{cases} \max{(h_f)}, \text{ if } \omega(\nu(\eta))(x) = 1 \\ 0, \text{ otherwise} \end{cases} .$$

The mapping λ gives a preliminary classification; the graylevel classes are separeted but not labeled (Fig. 7). The labeling of classes is given by the application of next steps.

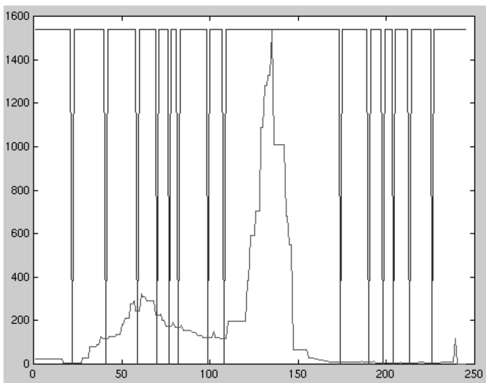


Figure 7: Pre-classification.

Let $\varrho : K \to \mathbb{Z}_+$, such that $\varrho(x) = x$, $\forall x \in K$. Let $\theta : Fun[K, \mathbb{Z}_+] \to Fun[K, \mathbb{Z}_+]$ be the mapping, given by, $\forall x \in K$,

$$\theta(x) = \begin{cases} \max(h_f), \text{ if } \mu_B^{\max}(\eta)(x) = 1 \\ 0, \text{ otherwise} \end{cases}.$$

Let $\beta : Fun[K, \mathbb{Z}_+] \to Fun[K, \mathbb{Z}_+]$ be the mapping, given by,

$$\beta = \varrho \wedge \theta.$$

The mapping β gives a function where each point of the regional maxima of $\eta(\cdot)$ is labeled by its corresponding graylevel. Its reconstruction conditioned to $\lambda(\cdot)$ gives the labeling of all graylevel classes. Figure 8 shows a composition of $\lambda(\cdot)$ (pre-classified graylevels) and $\beta(\cdot)$ (the labeled peaks inside each pre-class).

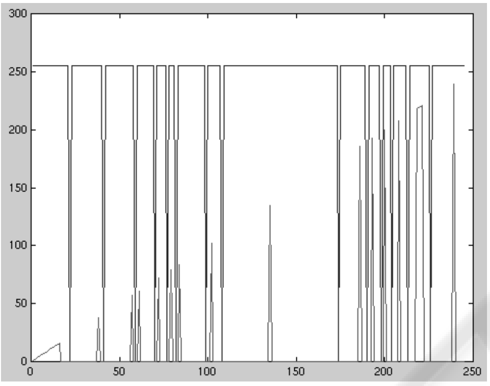


Figure 8: Function β assigning labels to each pre-class.

Let $\zeta : Fun[K, \mathbb{Z}_+] \to Fun[K, K]$.

$$\zeta = \delta_B(\rho_{B,\lambda}(\beta)),$$

where $B \subset K$ is the structuring element, and $\delta_B(\cdot)$ is the dilation operator (Serra, 1982; Heijmans, 1994).

Let $|K|$ be the number of distinct graylevels in K. We can say that the mapping ζ is a graylevel classificator. Given the histogram h_f, $f \in Fun[E, K]$, the classificator gives a new set of graylevels G, where $|G| < |K|$. The processed histogram is used as a look-up table in order to reduce the graylevels.

Definition 5 *Let $f \in Fun[E, K_1]$, $K_1 = [0, k_1]$, and $h_f \in Fun[K_1, \mathbb{Z}_+]$ the histogram of f. The* ***graylevel reducer by classification of regional maxima*** *is a mapping $\psi : Fun[E, K_1] \to Fun[E, K_2]$, where K_2, $|K_2| < |K_1|$, is given by,*

$$K_2 = \zeta(h_f).$$

Since there is a reduction in the number of graylevels in the image, and it may causes the union of two or more flat zones, without split any flat zone, it is a connected operator. Figure 9 shows the graylevel classification provided by ζ, given the histogram shown in Fig. 4.

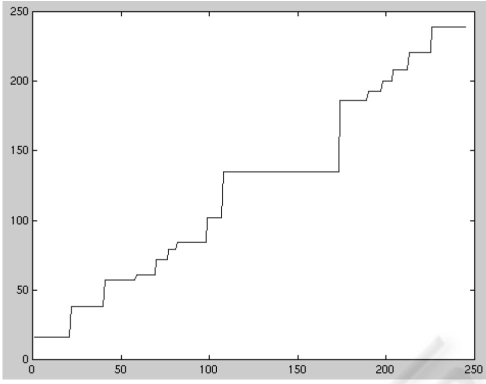


Figure 9: Classified graylevels.

4.1 Reduction to n Graylevels by Classification of Region Maxima by Dynamics Metrics

When the operator ψ is applied to an image f, it reduces the graylevels appearing in f to the number of regional maxima of $\eta(\cdot)$. However, it is possible to reduce the graylevels to a smaller number, by adding a parameter n which gives the number of graylevels to appear in $\psi(f)$. In this section we will present a way to select the n most significant regional maxima of $\eta(\cdot)$ by application of dynamics.

We will denote by $\psi_n : Fun[E, K_1] \to Fun[E, K_2]$, $|K_2| < |K_1|$, $|K_2| = n$, the operator which performs the reduction of the graylevels in the image to n graylevels.

Remember that the result of η operator is the original histogram filtered in a way that the highest regional maximum among a set of regiona maxima belonging to the same distribution is preserved. Let $D_i : Fun[K, \mathbb{Z}_+] \to Fun[K, \mathbb{Z}_+]$ be the function given by,

$$D_i(\eta)(x) = \begin{cases} Dyn_i(\nu(\eta))(Z) : x \in Z, \text{ if } \mu_B^{\max}(\eta)(x) = 1 \\ 0, \text{ otherwise} \end{cases},$$

where Z is one of the regional minima of the negation of $\eta(\cdot)$. I.e., if x belongs to a regional maximum in $\eta(\cdot)$, $D_i(\eta)(x)$ will be equal to the dynamics (see section 3) of the regional minimum where x is located in the negation of $\eta(\cdot)$. i is the criterium chosen to evaluate $\eta(\cdot)$: depth (1), area (2) or volume (3).

Let m be the number of regional maxima in $\eta(\cdot)$. Let Q be the set defined by

$$Q = \{q_i \in K : D_i(\eta)(q_i) > 0 \text{ and } D_i(\eta)(q_i) \geq D_i(\eta)(q_{i+1})), i = 1, \cdots, m-1\}.$$

Let $\sigma_n : Fun[K, \mathbb{Z}_+] \times n \to Fun[K, \mathbb{Z}_+]$ be the mapping, given by, $\forall x \in K$, $\forall n \in \mathbb{Z}_+$,

$$\sigma_n(x) = \begin{cases} \max(h_f), \text{ if } x \in Q \\ 0, \text{ otherwise} \end{cases}.$$

Let $\eta_n : Fun[K, \mathbb{Z}_+] \times n \to Fun[K, \mathbb{Z}_+]$ be the mapping, given by, $\forall x \in K$, $\forall n \in \mathbb{Z}_+$,

$$\eta_n = \nu(\rho^*_{B,\nu(\eta)}(\sigma_n)).$$

By applying the operator η_n, the n regional maxima of $\eta(\cdot)$ which have the highest dynamics are selected. The function $\eta_n(\cdot)$ contains just n regional maxima and they are responsible for the classification of n classes (given by application of watershed operator). The remaining peaks are removed.

The method proposed in section 4 can be now extended to reduce an image to n graylevels, by adding the dynamics step introduced in this subsection to the framework, before the application of the watershed operator.

5 EXPERIMENTAL RESULTS

In this section, it will be presented two experimental results. In both presented experiments, it was applied the proposed method to graylevel reduction and it were observed the simplification of the image in terms of flat zones and the quality of the obtained images. Four method were compared in both experiments: the choice of n highest peaks selection (Flores and Lotufo, 2001) and the three dynamics proposed in this paper (depth, area and volume).

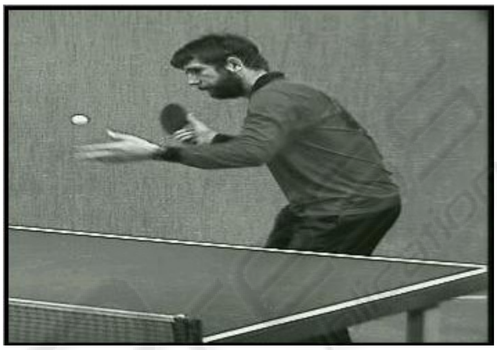
In the first experiment, it was aimed to simplify the *table tennis player* (Fig. 10 (a)), an image with 66574 flat zones and 254 graylevels, to an image with 8 graylevels. It were applied the highest peaks method and the four dynamics to reduce the image to the desired amount of graylevels. Table 1 shows the amount of graylevel reduction given by the application of each criterion and Fig. 10 (b) and Fig. 11 (a-c) show their respective results.

Note that the results given by the highest peaks and the depth dynamics are close, both in flat zones reduction and the quality of image, and the depth dynamics presented a slight better visual result. Area and volume dynamics provided the best results, both in the simplification of the image and in the quality of the resulting image.

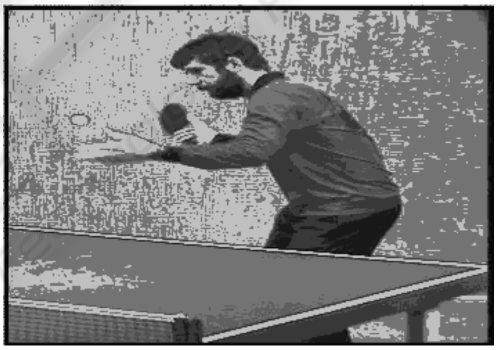
The goal in the second experiment was to simplify the *foreman* image (Fig. 12 (a)) to 10 graylevels. This original image has 69301 flat zones and 252 graylevels. In this experiment, it was compared the highest peaks to the volume dynamics. Figure 12 (b) and (c), show, respectively the quantization provided by application of both metrics. The highest peaks provided a reduction to 6575 in the number of flat zones and the volume dynamics provided a reduction to 4757 flat zones. Note that the result provided by the volume dynamics is also visually better: note that the changes given by the dynamics metric is smoother than the result given by the highest peaks criterion.

Table 1: Flat Zones Reduction.

Criterion	Flat Zones
Original Image	66574
Highest Peaks	4375
Depth Dynamics	4595
Area Dynamics	2138
Volume Dynamics	1994



(a)



(b)

Figure 10: (a) Original Image (66574 Flat Zones). (b) Highest Peaks (4375 Flat Zones).

6 CONCLUSION

This paper proposes an extension to a method applied to graylevel quantization, proposed in a previous paper (Flores and Lotufo, 2001). The method consists in the classification of graylevels by application of morphological operators to the histogram. Its result is a new image with a fewer regions and a simplified colormap, compared to the original image.

In the previous paper, it was proposed a way to compute the reduction of graylevels to n levels, consisting in the choice of the highest peaks and discarding the remaining ones. The extension of this method in given by the application of dynamics as a criterion to the choice significant distributions in the histogram, instead to choose the highest ones.

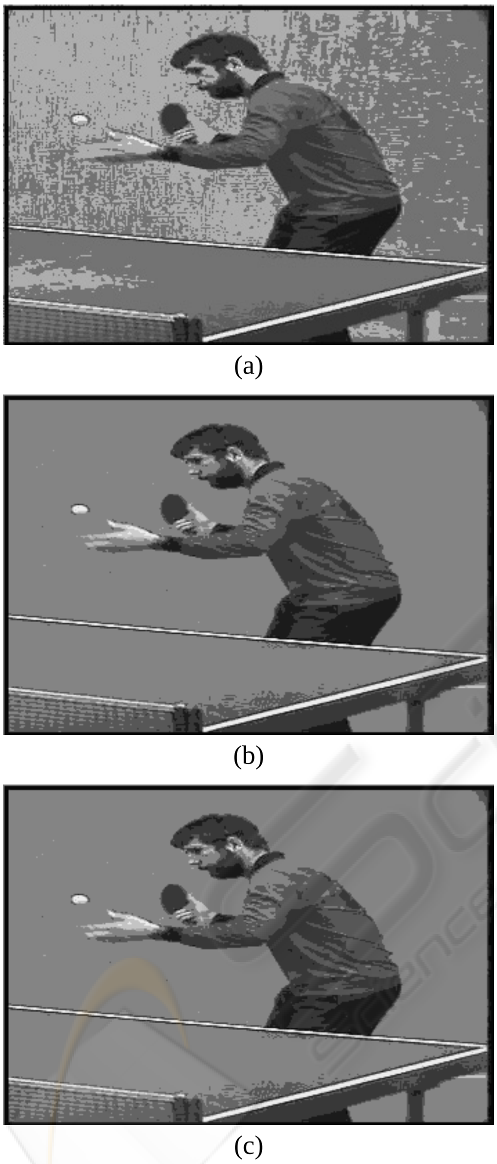


Figure 11: (a) Depth Dynamics (4595 Flat Zones). (b) Area Dynamics (2138 Flat Zones). (c) Volume Dynamics (1994 Flat Zones).

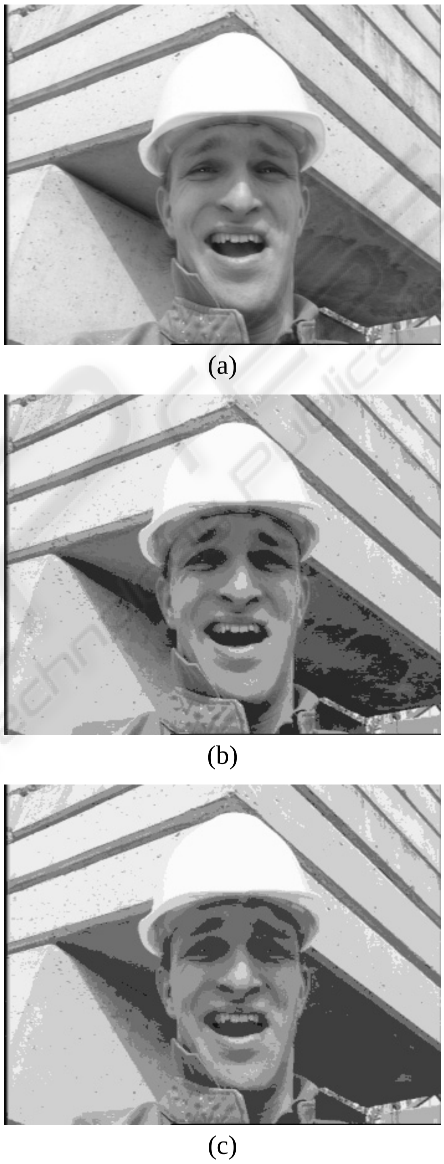


Figure 12: (a) Original Image (69301 Flat Zones). (b) Highest Peaks (6575 Flat Zones) (c) Volume Dynamics (4757 Flat Zones).

Some experiments were done in order to compare the highest peaks criterion to the dynamics metrics. The goal in both experiments presented here was to analyze the flat zone reduction provides by the metrics and the visual quality of the resulting images. The two experiments showed better results provided by the dynamics, especially the results given by the volume dynamics.

Further work includes the investigation of alternative metrics to choose the distributions in the histogram and the extension of the method proposed in this paper to color images.

REFERENCES

Beucher, S. and Meyer, F. (1992). *Mathematical Morphology in Image Processing*, chapter 12. The Morphological Approach to Segmentation: The Watershed Transformation, pages 433–481. Marcel Dekker.

Crespo, J., Schafer, R. W., Serra, J., Gratind, C., and Meyer, F. (1997). The flat zone approach: A general low-level region merging segmentation method. *Signal Processing*, 62(1):37–60.

da Silva, W. D. F. (2001). *Marcadore Mínimos Usando Watershed*. PhD thesis, School of Electrical and Computer Engineering, State University of Campinas.

F. C. Flores, S. M. P. and Zuben, F. J. V. (2002). Automatic Design of W-Operators using LVQ: Application to Morphological Image Segmentation. In *IEEE Proceedings of International Joint Conference on Neural Networks (IJCNN2002)*, pages 1930–1935, Honolulu, Hawaii.

Flores, F. C. (2000). Segmentação de Seqüências de Imagens por Morfologia Matemática. Dissertação de Mestrado, Instituto de Matemática e Estatística - Universidade de São Paulo.

Flores, F. C., Hirata Jr., R., Barrera, J., Lotufo, R. A., and Meyer, F. (2000). Morphological Operators for Segmentation of Color Sequences. In *IEEE Proceedings of SIBGRAPI'2000*, pages 300–307, Gramado, Brazil.

Flores, F. C. and Lotufo, R. A. (2001). Connected Filtering by Graylevel Classification Through Morphological Histogram Processing. In *IEEE Proceedings of SIBGRAPI'2001*, pages 120–127, Florianopolis, Brazil.

Gomes, J. and Velho, L. (1994). *Computação Gráfica : Imagem*. IMPA/SBM, Boston.

Gonzalez, R. C. and Woods, R. E. (1992). *Digital Image Processing*. Addison-Wesley Publishing Company.

Grimaud, M. (1992). A New Measure of Contrast: the Dynamics. In SPIE, editor, *Image Algebra and Morphological Image Processing III*, volume 1769, pages 292–305.

Heckbert, P. (1982). Color Image Quantization for Frame Buffer Display. *Computer Graphics*, pages 297–307.

Heijmans, H. J. A. M. (1994). *Morphological Image Operators*. Academic Press, Boston.

Heijmans, H. J. A. M. (1999). Introduction to Connected Operators. In Dougherty, E. R. and Astola, J. T., editors, *Nonlinear Filters for Image Processing*, pages 207–235. SPIE–The International Society for Optical Engineering,.

Hirata Jr., R. (1997). Segmentação de Imagens por Morfologia Matemática. Dissertação de Mestrado, Instituto de Matemática e Estatística - USP.

Hirata Jr., R., Barrera, J., Flores, F. C., and Lotufo, R. A. (1999). Automatic Design of Morphological Operators for Motion Segmentation. In Stolfi, J. and Tozzi, C. L., editors, *IEEE Proc. of Sibgrapi'99*, pages 283–292, Campinas, SP, Brazil.

Meyer, F. (1996). The Dynamics of Minima and Contours. In P. Maragos, R. S. Butt, M., editor, *ISMM 3rd. Computational Imaging and Vision*, pages 329–336.

Meyer, F. (1998). From Connected Operators to Levelings. In Heijmans, H. and Roerdink, J., editors, *Mathematical Morphology and its Applications to Image and Signal Processing*, Proc. ISMM'98, pages 191–198. Kluwer Academic Publishers.

Meyer, F. and Beucher, S. (1990). Morphological Segmentation. *Journal of Visual Communication and Image Representation*, 1(1):21–46.

Salembier, P. and Serra, J. (1995). Flat Zones Filtering, Connected Operators, and Filters by Reconstruction. *IEEE Transactions on Image Processing*, 4(8):1153–1160.

Serra, J. (1982). *Image Analysis and Mathematical Morphology*. Academic Press.

Soille, P. (1996). Morphological Partitioning of Multiespectral Images. *Electronic Imaging*, 5(3):252–265.

Vincent, L. and Soille, P. (1991). Watersheds in Digital Spaces: An Efficient Algorithm Based on Immersion Simulations. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 13(6):583–598.