

DISCRETE TOOLS FOR VIRTUAL SCULPTURE

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Abstract: In a virtual sculpture project, we represent the material to be sculpted as a set of volume elements (voxels). Sculpture operations of subtraction and addition are applied on these voxels with tools with various shapes and sizes. A major advantage of our system is that sculpted objects can then be used as new tools, because the same model is used for both objects and tools. This is a multiresolution model based on a 3D wavelet transform. We take advantage of the levels of detail to speed up display and sculpture. However, using discrete models for objects and tools leads to three problems: important computation time, aliasing when tools are rotated, and how to perform sculpture operations between discrete objects and tools with different orientations and sizes. In this paper, we describe our model and then propose solutions to these problems that allow real-time performance.

1 INTRODUCTION

1.1 Presentation

We present in this paper a multiresolution model based on 3D wavelets to represent a 3D objects as a discrete set of volume elements (voxels). Such a discrete representation is of great use in a virtual sculpture context as it allows to simulate easily sculpture operations such as subtraction or addition of material by simply adding or removing voxels. A great improvement of our model is its multiresolution nature given by the use of wavelets, that allows to accelerate display, interaction and sculpture operations by the use of levels of detail.

Another major advantage of our system is that we use this model to represent both objects and tools, so we can let the user design his or her own tools, by sculpting them from basic objects. This is an original approach to virtual sculpture, where most of other approaches use different models for objects and tools. However, if other approaches generally use implicit tools (such as ellipsoids), it is because the use of discrete tools is a difficult topic, due to three problems:

- First, this is computationally expensive, especially when the tool has to be rotated in its discrete grid of voxels when the tool orientation is changed.
- Second, there is a degradation of the shape of

the tool due to aliasing if angles of rotation are not multiple of 90 degrees.

- Third, it is unclear how to perform sculpture operations such as subtraction or addition between discrete objects and tools with different orientations and sizes.

1.2 Previous Works

In this paper, we tackle the problem of virtual sculpture of 3D objects with tools, both represented with spatial enumerations. Such a spatial enumeration is a set of volume elements called voxels, obtained by sampling the volume of a 3D object. It can be seen as a 3D image composed of voxels, where a 2D image is an array composed of pixels. To make a spatial enumeration from a 3D object, several methods have already been suggested. The simplest way is a uniform spatial enumeration, by regularly sampling the 3D object into voxels with the same size. However, a major drawback of this representation is the large number of voxels needed to represent large objects with detailed features. This entails three main problems. The first one is the important memory cost to store this uniform spatial enumeration. The second one is that the display of these objects becomes slower. Finally, operations on these objects such as sculpture actions or displacements become less and less interactive.

To prevent these inconveniences, adaptive sampling methods have been developed. Libes (Libes, 1991) uses an octree to gather groups of adjacent voxels having same values to reduce the number of elements stored in memory. It's very simple to use and to implement this method. Ferley (Ferley, 2002) also works on a n-tree where each cell can be divided in 27 ones. This method looks like an octree and allows to reach a high level of detail. However, for an object with small details, the subdivision level of an octree or n-tree will be very high. So the processes (construction and use) will be slow.

Several other sampling methods used in collision detection propose to modify properties of the voxels, such as the size, the orientation or even the shape.

With the AABB method (Axis Aligned Bounding Boxes), Bergen (Bergen, 1997) suggests to use voxels with different sizes. Gottschalk (Gottschalk, 1996) proposes to modify not only the size of the voxels but also their orientation, with the OBB method (Oriented Bounding Boxes). Thanks to these two methods, the object rendering is optimized because the original object shape can be approached with less voxels than with a simple uniform spatial enumeration. The modeling is finer with OBB tree than AABB tree for a same number of bounding volumes. However, AABB tree uses less memory storage than OBB tree for a same number of bounding volumes. Indeed, an OBB is represented by using 15 scalars (9 scalars for the orientation, 6 scalars for position and extent), whereas an AABB only requires 6 scalars (for position and extent). Moreover, to optimize the modeling of the 3D object, these two methods suggest to reduce overlaps between bounding boxes and to increase their filling by the object, with the less possible boxes. This optimization is expensive in processing time, so we prefer using a uniform spatial enumeration or an octree, because they are faster than AABB and OBB methods.

Liu (Liu, 1988) and Hubbard (Hubbard, 1995) propose to replace cubes by spheres in an octree to form a spheres tree, because spheres accelerate collision detection between objects. Later, Hubbard (Hubbard, 1995) (Hubbard, 1996) and Bradshaw (Bradshaw, 2004) suggest a finer modeling using spheres tree thanks to an approximated medial-axis of the 3D object, but this method is slower and more complicated than an octree.

To further improve the use of spatial enumeration, several methods of multiresolution representations have been proposed. So, processing and display times are adapted with the desired level

of detail. Among these methods, there are octree and wavelet decomposition.

An octree can also be seen as a hierarchical representation of 3D object. The maximum level of subdivision of the octree defines the maximum level of detail of a multiresolution representation. Boada (Boada, 2001) defines a section in an octree that determines the displayed nodes for a defined level of detail. This method is extended to a n-tree by Ferley (Ferley, 2002).

The second multiresolution method uses wavelet decomposition. Wavelets are a mathematical tool for representing functions hierarchically. In our case, these functions are discrete 3D functions that define a set of voxels. More information about wavelets will be given in the section 2.1. Muraki (Muraki, 1992) (Muraki, 1993) shows the use of 3D Haar wavelets to represent a 3D object. Pinnamaneni (Pinnamaneni, 2002) builds a 3D Haar wavelet decomposition from a sequence of 1D Haar wavelet decomposition in each direction of the 3D voxels grid. Wavelet decomposition allows to display a 3D object faster according to the level of detail. It also permits to drastically cut down the memory cost, because high compression ratio can be achieved on wavelets coefficients, especially if lossy compression schemes are used.

The previously cited methods are about discrete representation of 3D objects. Different methods have already been proposed to sculpt these kinds of objects.

Ayasse (Ayasse, 2001) performs sculpture operations by the use of CSG (Constructive Solid Geometry). Complex objects are created by successive modifications of the material with a tool according to simple operations such as difference, union or intersection. However, the object and the tool are represented by simple uniform spatial enumerations. Moreover, voxels are limited to binary values (full or empty). Ayasse proposes to reduce the computation time for each sculpture operation by using only the effective voxels of a movement. This method can be useful because the computation times are reduced. However, it doesn't use a multiresolution representation that could improve the display performance or computation.

In the Kizamu project, Frisken (Frisken, 2001) uses ADFs (Adaptively sampled Distance Fields) to model and to sculpt the material. A 3D object is sampled adaptively with a 3D grid according to the details of the object. Each grid cell contains a scalar specifying the minimum distance to the object shape. This distance is signed to distinguish between the inside and outside of the shape.

To represent an object to be sculpted, Ferley (Ferley, 2002) also uses distance fields, stored in a “n-tree” hierarchical representation where the sampling rate depends on object’s details. The tool is an ellipsoid defined by an implicit function that is discretized to perform sculpture actions on the object.

Bærentzen (Bærentzen, 2002) proposes the Level-Set method to deform the material. This method stores distance fields around the exterior of a 3D object. The tool is a blob represented by an implicit function.

Raffin (Raffin, 2004) proposes a model of virtual sculpture based on a multiresolution representation: the octree. The tools are defined as voxels sets, but they remain parallel to the axis.

1.3 Original Contribution

The main contribution of this paper is to propose a discrete multiresolution model for virtual sculpture. We take benefit of levels of detail to accelerate display, interaction and sculpture operations. As the same model is used to represent both objects and tools, a major advantage of our system over existing methods is that the user can create his or her own tools from a previous sculpture. Furthermore, we provide solutions to problems inherent to the use of discrete objects, such as computation cost, aliasing and operations between discrete tools and objects.

The remainder of the paper is organized as follows: in section 2, we describe our discrete multiresolution model. In section 3, we present the tools, with original solutions for computation cost, aliasing and sculpture operations. Then, we conclude in section 4 and present future work in section 5.

2 OUR MODEL

In a virtual sculpture project, we represent the 3D objects to be sculpted as a discrete set of voxels to easily handle subtraction or addition of material by tools. However, a uniform spatial enumeration is expensive in processing and display times. Thus, we propose a multiresolution model based on 3D Haar wavelets.

2.1 About Wavelets

On the following example, we explain Haar wavelet decomposition on a 1D case. First, consider a

sequence of p values, where p is a power of two (here, $p = 4 = 2^2$):

$$X^0 = [9, 7, 3, 5]$$

Then, by applying Haar wavelet transform, we can represent this sequence in terms of a low-resolution sequence X^1 and a set of detail coefficients Y^1 :

$$X^1 = \left[\frac{9+7}{2}, \frac{3+5}{2} \right] = [8, 4]$$

$$Y^1 = \left[\frac{9-7}{2}, \frac{3-5}{2} \right] = [1, -1]$$

So, by repeating these operations, we obtain several sets of coefficients corresponding to different levels of detail, as shown on the following decomposition table:

level of detail #	low-resolution coefficients	detail coefficients
0	[9 7 3 5]	
1	[8 4]	[1 -1]
2	[6]	[2]

Thus, the higher the number of the level of detail, the less detailed the sequence. The sequence obtained by Haar wavelet decomposition has the same size as the original sequence. Its coefficients are the low-resolution coefficients of the last level of detail and the different detail coefficients:

$$\text{Original sequence: [9 7 3 5]}$$

$$\text{Final sequence: [6 2 1 -1]}$$

The extraction of the original sequence from the final sequence uses the inverse wavelet transform:

$$X^1 = [6 + 2, 6 - 2] = [8, 4]$$

$$X^0 = [8 + 1, 8 - 1, 4 + (-1), 4 - (-1)] = [9, 7, 3, 5]$$

2.2 Definition of our Model

Similarly, we can use this wavelet transform in a 3D case. First, the 3D discrete object is defined by a uniform spatial enumeration. Then, by using the wavelet transform we build a hierarchical structure that stores the coefficients of each level of detail of this 3D object.

We use the hierarchical structure proposed by Pinnamaneni [Pin02]. For each level of detail, the 1D Haar Wavelet transformation is applied in x-, y- and z-direction successively (figure 1).

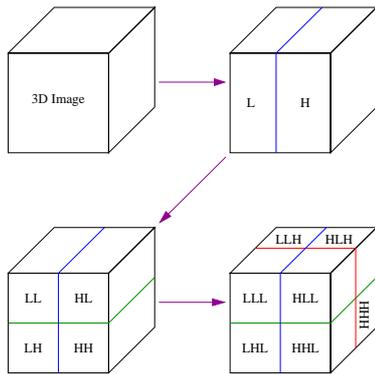


Figure 1: 3D Haar wavelet decomposition.

For each transformation step, we obtain a bloc ‘L’ with low-resolution coefficients obtained by a low-pass filter, and a bloc ‘H’ with detail coefficients obtained by a high-pass filter.

The figure 2 shows a 3D Haar wavelet decomposition for a sphere with 6 levels of detail. Each voxel contains a density value coded on one byte (from 0 for an empty voxel to 255 for a full one).

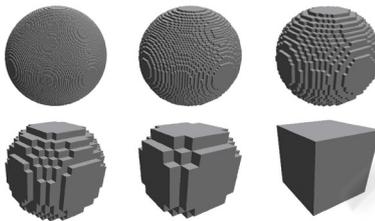


Figure 2: 3D wavelet enumeration for a sphere in 128x128x128 with 6 levels of detail (from 0 to 5, from left to right, and from top to bottom).

The building time (for wavelet decomposition of the 3D image) and the extracting time (for extraction of a level of detail from wavelet enumeration) do not depend on the kind of 3D object. These times only depend on the number of voxels of the initial uniform spatial enumeration and on the desired levels of detail of wavelet enumeration. Measured times are reported on Table 1. Note that an improvement will be described in section 2.3 to reduce these times. The results given in this paper have been obtained on a PC with an AMD 3GHz, 1GB of RAM and a NVIDIA Geforce FX 5200 with 128MB video memory.

Table 1: Building and extracting times for a 3D Haar wavelet enumeration with a most detailed level of 128x128x128 voxels.

Building (5 levels)	Extracting			
	Level 0 (128 ³)	Level 1 (64 ³)	Level 2 (32 ³)	Level 3 (16 ³)
0.6975 s	0.7529 s	0.0627 s	0.0019 s	0.0002 s

2.3 Display

We display this discrete object with the marching cubes algorithm (Lorensen, 1987) that provides a smooth surface instead of a set of blocky voxels. During display, we take advantage of the multiresolution nature of our model given by the 3D wavelets to display the more appropriate level of detail according to the situation (Distance between the object and the point of view, needed frame rate). Moreover, we implemented a data cache that improves performances by managing the triangles generated for each level of detail by the marching cubes. Thus, there are two big advantages. First, it is not necessary to extract a level of detail from the wavelets if it is already present in the cache, thus alleviating the computation times presented in Table 1. Second, during sculpture actions, it is only necessary to recompute the surface for the parts of the object modified by the tool.

In our virtual sculpture system, we use this same model to represent both objects and tools.

3 THE TOOLS

As stated in the introduction of this paper, the use of discrete objects and discrete tools leads to problems of computation time, aliasing and sculpture operations. We will provide solutions to these three problems in the following points.

3.1 Tools Orientation

In order to allow any orientation of a tool over the object to be sculpted, we have to face the problem of discrete rotation of the tool in its 3D matrix of voxels. Note that we always apply the rotation to a reference object, not to a previously rotated object, because it would result in more and more deteriorated object.

3.1.1 Computation of the Rotated Tool Bounding Box

Before performing the rotation of this 3D image of voxels, we have to determine its size after rotation. We first compute the rotation of its 8 corners. The bounding box of the 8 rotated corners gives the final size of the image after rotation. Thus, we ensure that the rotated tool will be totally enclosed in the final bounding box.

3.1.2 Direct Method

In order to perform the rotation of the 3D image of voxels, the first idea would be to apply the rotation to the voxels of the source image to fill the destination image. Thus, for each voxel of the source image, we multiply its position by the rotation matrix, thus obtaining a rotated position. As the coordinates of this position are real numbers, they are then truncated to integers. The value of the source voxel is then affected to the voxel at this rotated position in the destination image.

However, as we can see on figure 3, this results in holes in the final image as many voxels from the source can be projected to the same voxel in the destination due to truncation error to integers. Consequently, a direct rotation of the source image is inadequate.

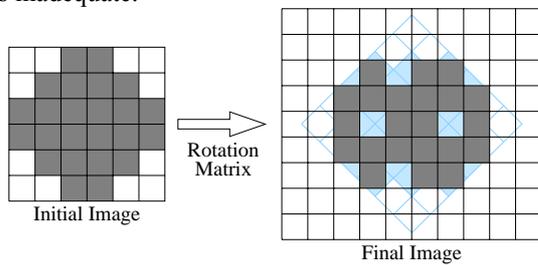


Figure 3: The use of the direct method to compute the rotation of a discrete image results in holes in the final image.

3.1.3 Inverse Method

The solution is to perform the inverse mapping. We consider each voxel in the final image and map backwards to find the closest voxel in the initial image by applying the inverse rotation matrix. In this way, every voxel in the final image is found, without hole (figure 4).

However, there are still two problems:

- First, the computation time of this “brute force” rotation is high, as we multiply each voxel position of the final image by the inverse rotation matrix (Of course, this one is only computed once). Computation time increases with the size of the 3D image, i.e. with the number of voxels.

- Second, there is important aliasing in the final image.zasing.

The aliasing problem will be tackled in the following section. In order to reduce computation time to allow real-time tool rotation, we propose an optimised rotation method with three improvements over the “brute force” method.

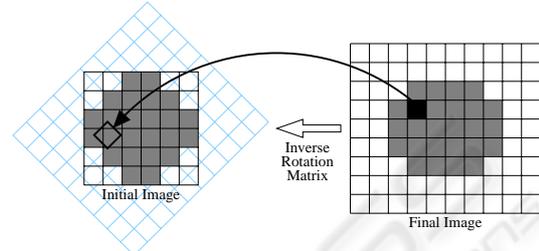


Figure 4: With the inverse rotation method, no hole is obtained in the final image.

First, we do not compute the inverse transformation for all the voxels of the final image, but only for 4 voxels defining 3 orthogonal axes. Positions of all other voxels are computed by simple interpolations from these 4 positions (algorithm 1). Computation times have been reported on table 2. Moreover, comparisons between computation times with brute force method and optimised method have been reported on graph 1, for a tool in 16x16x16. We can see that with this optimisation the rotation time is cut down by 75% compared to the “brute force” method. We can see that the computation times depend on angle of rotation because the number of voxels needed to store the rotated tool changes with the angle.

Second, we drastically cut down the computation time by only performing the rotation for the voxels of the tool that are in contact with the object to be sculpted. Indeed, this rotation is only required for internal needs to perform sculpture operations. For the display of the tool, we do not compute the discrete rotation of the voxels, but we simply use OpenGL rotation capabilities with the `glRotate()` function. Thus, we can display tools with good rendering quality, without distortion due to discrete rotation.

Third, in order to reduce furthermore the computation times, we store the already rotated

Table 2: Computation times obtained with different methods for rotation of a discrete tool for different resolutions (averaged times for various rotation angles).

	8x8x8	16x16x16	32x32x32	64x64x64
Brute force	0.1826 ms	1.321 ms	10.17 ms	82.3 ms
Optimised	0.0456 ms	0.322 ms	2.49 ms	18.1 ms
Brute force with antialiasing	0.2303 ms	1.628 ms	12.45 ms	102.1 ms
Optimised with antialiasing	0.0494 ms	0.351 ms	2.55 ms	18.7 ms

voxels in a cache structure. Thus, we avoid to compute them again when the contact area between the tool and the object has changed. We only compute the rotation of the voxels of the tool newly entered in the area.

Algorithm 1: Discrete rotation of the tool.

```

P = voxel in final image
M = point in initial image
M = Inverse_Rotation(P)

b = (i,j,k) = final basis
B = (I,J,K) = initial basis
B = Inverse_Rotation(b)

dim = voxels number along an axis
in final image

FOR P.x=0 TO dim.X DO
  FOR P.y=0 TO dim.Y DO
    FOR P.z=0 TO dim.Z DO
      // Get the value of the
      // voxel M in initial image
      P = GetValue(M)

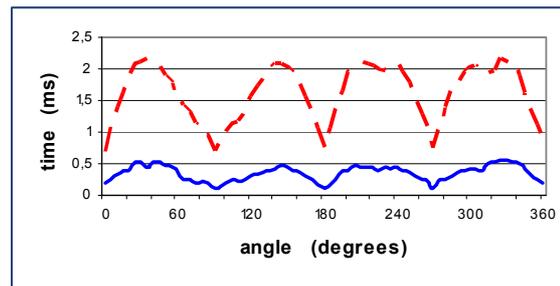
      // Next point M
      M.x = M.x + K.x
      M.y = M.y + K.y
      M.z = M.z + K.z
    END FOR

    // Return along axis K in
    // initial basis.
    // M is incremented with J
    M.x = -dim.Z * K.x + J.x
    M.y = -dim.Z * K.y + J.y
    M.z = -dim.Z * K.z + J.z
  END FOR

  // Return along axis J in
  // initial basis.
  // M is incremented with I
  M.x = -dim.Y * J.x + I.x
  M.y = -dim.Y * J.y + I.y
  M.z = -dim.Y * J.z + I.z
END FOR

```

Please note that the computation times reported on table 2, graph 1 and graph 2 are the times observed in the worst cases, when all the voxels of the tool have to be rotated (all voxels of the tool in contact with the object and no voxel present in the cache). In practical cases, the times are much lower.



Graph 1: Comparison between computation times for brute force method (dashed line) and optimised method (solid line) for various rotation angles.

3.2 Antialiasing

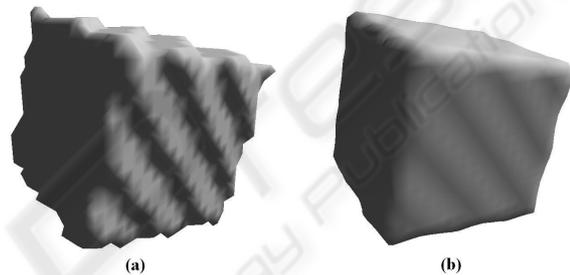
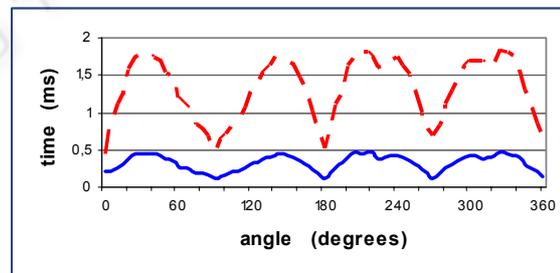


Figure 5: A rotated cube without (a) and with antialiasing by trilinear interpolation (b).

A major drawback of the rotation of a discrete image is aliasing. The lower the image resolution, the more important the aliasing (figure 5a).



Graph 2: Comparison between computation times for brute force method (dashed line) and optimised method (solid line) with antialiasing for various rotation angles.

In order to reduce aliasing, we compute the value of a rotated voxel from a trilinear interpolation of its 8 neighbours in the original image. Much better results are obtained (figure 5b) to the price of slightly higher computation times (On average, +25% for the brute force rotation, +5% for the optimised rotation as reported on Table 2). As for graph1, we can see on graph 2 that the computation times depend on angle of rotation.

3.3 Sculpture Operations

Thanks to the representation of the material to be sculpted as a set of volume elements, we can easily handle sculpture actions such as subtraction or addition of material. Other sculpture actions will be studied in future work. A major advantage of our method is that the tool used for virtual sculpture has the same representation than the material. So, the user can create his or her own tools to sculpt another 3D object.

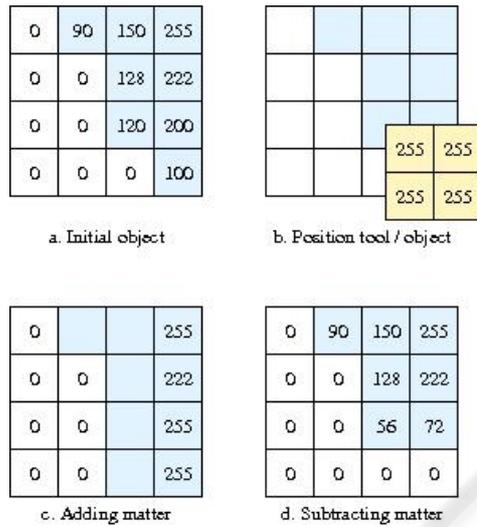


Figure 6: Adding or subtracting material to an object with a tool.

During the sculpture operations, a collision test is first made between the bounding boxes of the tool and the material. If there is a collision, the following operations are performed:

- Find the voxels of the material and the tool which are in the collision zone.
- Extract the value of each voxel of the tool in the collision zone by using the discrete rotation.
- Find which voxel of the tool intersects which voxel of the material in this zone.

If there is intersection between voxels of the material and the tool in the collision zone, we compute the filling percentage of the voxel of the material by the one of the tool. The values of the voxels of the material are then modified according to the sculpture mode (illustrated in 2D on figure 6):

- In the “Adding material” mode, if the voxel of the tool isn’t null, the filling percentage is added to the value of the voxel of the material. If this value becomes greater than 255, it is put to 255.

- In the “Subtracting material” mode, if the voxel of the tool isn’t null, the filling percentage is subtracted to the value of the voxel of the material. If this value becomes negative, it is put to 0.

For each level of detail, the triangulated surface is rebuilt by marching cubes only for the modified parts of the 3D object to improve the computation time.

Examples of sculptures produced by our system are shown on figures 7 and 8.

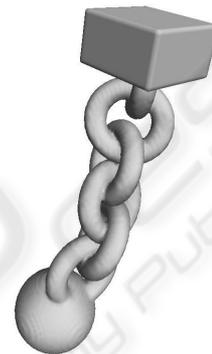


Figure 7: Chain with a ball, in 256x256x256, sculpted with a spherical tool, a cubic tool and a ring tool in less than 10 minutes.

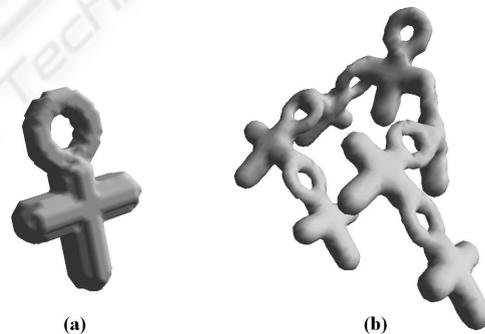


Figure 8: Design of a new tool (a) using a ring tool and a spherical tool. A more complicated object (b) sculpted in less than 5 minutes using the previously designed tool.

4 CONCLUSION

We have presented in this paper a model for virtual sculpture of 3D objects with tools. Both objects and tools are represented by 3D Haar wavelets. This multiresolution model permits to avoid speed and memory issues inherent to a representation based on voxels. The discrete representation as volume elements allows to handle easily sculpture operations such as subtraction or addition of

material. Unlike other existing virtual sculpture methods, a major advantage of our model is that the tools can freely be created by the user.

In order to allow any orientation of a tool over the object to be sculpted, we have developed an algorithm of discrete rotation of the tool in its 3D matrix of voxels. In order to enhance real-time performance, this algorithm is applied only for the voxels of the tool that are in contact with the object to be sculpted. Aliasing problems inherent to discrete rotation are reduced thanks to a trilinear interpolation to the cost of slightly higher computation time.

To verify the applicability of our sculpting system, we have conducted many sculpting sessions which have resulted in numerous interesting sculptures. Some sculptures examples are shown on figures 7 and 8, and several other examples can be seen on <http://www.iut-arles.univ-mrs.fr/thon/>.

5 FUTURE WORK

Many improvements of our sculpture system are possible, by investigating open issues such as interaction with the object or computation time.

Concerning interaction, we plan to improve the realism of sculpture actions, by adding parameters to the voxels to imitate physical behaviour. Enhanced sculpture actions will then be possible.

Interactive computation times will always be a challenging issue. In order to accelerate the sculpture actions, we plan to take more advantage of the levels of detail of the 3D Haar wavelet. We will also investigate the use of graphics hardware to speed up many parts of our system, such as voxels rotation or sculpture actions.

REFERENCES

- Ayasse, J., Müller, H., 2001. Interactive Manipulation of Voxel Volumes with Free-formed Voxel Tools. In *Proceedings of the Vision Modeling and Visualization Conference 2001*, 359-366.
- Bærentzen, J.A., Christensen, N.J., 2002. Volume sculpting using the Level-Set method. In *Shape Modelling International 2002*. IEEE Computer Society, 175-182.
- Bergen, G.V.D., 1997. Efficient collision detection of complex deformable models using AABB trees. In *Journal of Graphic Tools*, 2(4), 1-13.
- Boada, I., Navazo, I., Scopigno, R., 2001. Multiresolution volume visualization with a texture-based octree. In *Visual Computer*, 17, 185-197.
- Bradshaw, G., O'Sullivan, C., 2004. Adaptive medial-axis approximation for sphere-tree construction. In *ACM Transactions on Graphics*, 23(1), 1-26.
- Ferley, E., 2002. Sculpture virtuelle. Ph.D. thesis, Institut National Polytechnique de Grenoble.
- Frisken, S.F., Perry, R.N., 2001. Kizamu: a system for sculpting digital characters. In *Proceedings of ACM SIGGRAPH 2001*, 47-56.
- Gottschalk, S., Lin, M.C., Manocha D., 1996. OBB-Tree: A hierarchical structure for rapid interference detection. In *Proceedings of ACM SIGGRAPH'96*, 171-180.
- Hubbard, P., 1995. Collision detection for interactive graphics applications. Ph.D. Thesis, Dept. of Computer Science, Brown University.
- Hubbard, P., 1996. Approximating polyhedra with spheres for time-critical collision detection. In *ACM Transactions on Graphics*, 15(3), 179-210.
- Libes, D., 1991. Modeling dynamic surfaces with octrees. In *Computer & Graphics*, 15(3).
- Liu, Y., Noborio, J., Arimoto, S., 1988. Hierarchical sphere model (HSM) and its application for checking an interference between moving robots. In *Proceedings of the IEEE International Workshop on Intelligent Robots and Systems*, 801-806.
- Lorensen, W.E., Cline, H.E., 1987. Marching Cubes: a high-resolution 3D surface construction algorithm. In *Computer Graphics*, 21(4), 163-169.
- Muraki, S., 1992. Approximation and rendering of volume data using wavelet transforms. In *Proceedings of Visualization '92*, Boston, 21-28.
- Muraki, S., 1993. Volume data and wavelet transforms. In *IEEE Computer Graphics and Applications*, 13(4), 50-56.
- Pinnamaneni, P., Meyer, J., Saladi, S., 2002. Remote transformation and local 3-D reconstruction and visualization of biomedical data sets in Java3D. In *Proceedings of Electronic Imaging Science & Technology Visualization and Data Analysis Conference*, San Jose, CA, 44-54.
- Raffin, R., Gesquière, G., Remy, E., Thon, S., 2004. VirSculpt: a virtual sculpting environment. In *GraphiCon '04 Proceedings*, 184-187.
- Szeliski, R., Tonnesen, D., 1992. Surface modeling with oriented particle systems. In *Computer Graphics*, 26(2), 185-194.