A FUZZY APPROACH FOR FAULT DETECTION AND ISOLATION OF UNCERTAIN PARAMETER SYSTEMS AND COMPARISON TO BINARY LOGIC

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Abstract: This paper deals with fault detection and isolation off-line affecting sensors and actuators of uncertain parameter systems modelled by bond graph. A fuzzy approach for fault detection based on residual fuzzification is proposed. Besides, an isolation method based on fuzzy processing of the detection results is proposed. Finally binary approach and fuzzy one are compared through an illustrative example.

1 INTRODUCTION

Due to the increasing size and complexity of modern processes, their safety and their efficiency become very important. The aim of our work is to keep process in a good level of safety. Presently, fault detection and isolation is an increasingly active research domain. A process is in a defective state if causal relations between its known variables changed (Brunet J., 90). A widespread solution for fault detection and isolation is to use analytical model-based redundancy (Evsukoff A. et al., 2000).

Moreover, the system parameters are sometimes uncertain; they can also be fluctuated by a wear, or an external disturbance (Niesner C., 2004). Fuzzy reasoning is a powerful tool for modelling the uncertainty generated by models and sensor imprecision as well as vagueness of the normal behaviour limits (Evsukoff A. et al., 2000).

This paper proposes a fuzzy approach for fault detection and isolation affecting sensors and actuators of bond graph modelled uncertain parameter systems.

In section 2, two methods for fault detection are presented, one is based on a fixed threshold in order to detect fault via a crisp decision, the other, which is proposed, is based on fuzzification of residuals provided by analytical redundancy relations (ARRs).

In section 3, an isolation method based on known variables signature is presented, then an isolation method based on fuzzy processing of the detection results is proposed. Finally, in section 4, the classical and the fuzzy approach are applied to an illustrative example and results are compared in section 5.
2 FAULT DETECTION METHODS FOR UNCERTAIN PARAMETER SYSTEMS

2.1 Binary Logic Based Method

This technique consists on a test of the signal amplitude. The adjustment parameters are the thresholds regulated according to the various operating assumptions and the desired performances for detection (Brunet J., 90).

2.2 The Proposed Method Based on Fuzzy Logic

Observed residuals, written in integral form obtained when a rectangular fault affects sensors or actuators in a limited interval, have the following forms:

Fuzzy reasoning is composed of the following stages: attribute fuzzification, application of inference rules and defuzzification (Bühler H., 94).

In the Fuzzy Logic Toolbox of Matlab 7.0, there are five steps of the fuzzy inference process:

Step 1: Fuzzify inputs
It consists in taking inputs and determining the degree to which they belong to each of the appropriate fuzzy sets via membership functions. A membership function is a curve that defines how each point in the input space is mapped to a membership value or degree of membership between 0 and 1. The output is then a fuzzy degree of membership in the qualifying linguistic set.

Step 2: Apply Fuzzy Operator
Once the inputs have been fuzzified, we know the degree to which each part of the antecedent has been satisfied for each rule. If the antecedent of a given rule has more than one part, the fuzzy operator is applied to obtain one number that represents the result of the antecedent for that rule. This number will then be applied to the output function. The input to the fuzzy operator is two or more membership values from fuzzified input variables. The output is a single truth value.

Step 3: Apply Implication method
Every rule has a weight (a number between 0 and 1), which is applied to the number given by the antecedent. Once proper weighting has been assigned to each rule, the implication method is implemented. A consequent is a fuzzy set represented by a membership function, which weights appropriately the linguistic characteristics that are attributed to it. The consequent is reshaped using a function associated with the antecedent (a single number). The input for the implication process is a single number given by the antecedent, and the output is a fuzzy set. Implication is implemented for each rule. Two built-in methods are supported by fuzzy toolbox of Matlab 7.0, and they are the same functions that are used by the AND method: min (minimum), which truncates the output fuzzy set, and prod (product), which scales the output fuzzy set.

Step 4: Aggregate All Outputs.
Aggregation is the process by which the fuzzy sets that represent the outputs of each rule are combined into a single fuzzy set. The input of the aggregation process is the list of truncated output functions.
returned by the implication process for each rule. The output of the aggregation process is one fuzzy set for each output variable.

Step 5 Defuzzify

The input for the defuzzification process is a fuzzy set (the aggregate output fuzzy set) and the output is a single number. There are five built-in defuzzification methods supported by Fuzzy Toolbox of Matlab 7.0: centroid, bisector, middle of maximum (the average of the maximum value of the output set), largest of maximum, and smallest of maximum.

2.2.1.1 Fuzzification

Two set of features are used in the fuzzification stage:

1. absolute value of the residual provided by ARRs: \( r \)
2. variation \( |r - r_{fin}| : d \)

With \( r_{fin} \) is the final value of \( r \).

The linguistic set \{small, large\} is used to describe the two attributes "\( r \)" and "\( d \)" which have trapezoidal membership functions, the supports of the membership functions are the same for "\( r \)" and "\( d \)".

\[
\mu_{\text{Small}} = [0, 0, r_{\text{max}}, R_{\text{min}}] \\
\mu_{\text{Large}} = [r_{\text{max}}, R_{\text{min}}, R_{\text{max}}, R_{\text{max}}]
\]

with:

- \( r_{\text{max}} \): maximum value of \( r \) in fault free context.
- \( R_{\text{min}} \): minimal value of \( r \) when a rectangular fault is introduced in a limited interval.
- \( R_{\text{max}} \): maximum value of \( r \) when a rectangular fault is introduced in a limited interval.

Notice: \( r_{\text{max}} = d_{\text{max}}, R_{\text{min}} = D_{\text{min}} \) and \( R_{\text{max}} = D_{\text{max}} \).

![Fuzzy Partition of \( r \) and \( d \)](image)

2.2.1.2 Inference Rules

We have established a set of inference rules which are presented in the following table:

<table>
<thead>
<tr>
<th>( r )</th>
<th>( d )</th>
<th>Small</th>
<th>Medium</th>
<th>Large</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>Small</td>
<td>Small</td>
<td>Small</td>
<td>Small</td>
</tr>
<tr>
<td>Large</td>
<td>Small</td>
<td>Large</td>
<td>Large</td>
<td>Large</td>
</tr>
</tbody>
</table>

We have used the method of inference max-min, this method consists in using the operator min for "AND" and the operator max for "OR".

2.2.1.3 Defuzzication

The defuzzification consists in transforming the fuzzy information provided in the inference stage in a real value. The output of the system is called Fault-index.

Three membership classes of Fault-index are defined: Small, Medium and Large.

![Defuzzification of Fault-index](image)

The defuzzification provides a fault indicator, if Fault-index is close to 0 it means that known variables of the residual are in normal state. If Fault-index is close to 1, that indicates the presence of a fault. If Fault-index is in the interval [0.25 0.75], then there is a detection problem.

2.2.2 (C) Residuals

It is assumed that, if a known variable appears in a (c) residual, it appears also in (a) or (b) residuals.

We are interested now in a (c) residual \( r_j \), we suppose that the system has \( m \) residuals (a) or (b) having at least one common known variable with \( r_j \). These residuals are \( r_1, r_2 \ldots r_m \).

The set of features of the system are: Fault-index-1, Fault-index-2… Fault-index-m and \( r_j \)

2.2.2.1 Fuzzification

The set \{Small, Large\} is used to describe all the attributes of the system. For \( r_j \), we use the membership functions presented in figure 2. For
Fault-indexes, we use trapezoidal membership functions. The supports of the membership functions are as follows:

\[ \mu_{\text{Small}} = [0, 0, 0.25, 0.75] \]
\[ \mu_{\text{Large}} = [0.25, 0.75, 1, 1] \]

Figure 4: Fuzzification of Fault-index.

2.2.2.2 Inference Rules

The inference rules are based on the following observation: the cancellation of a fault doesn’t appear in a (c) residual because of its divergence, (see figure 1).

The inference rules are as follows:
- IF \( r_j \) is "Small" THEN Fault-index-j is "Small"
- IF \( r_j \) is "Large" and Fault-index-i is "Large" THEN Fault-index-j is "Large" (Fault-index-i is a fault indicator of \( r_i \), \( 0 < i < m+1 \))
- IF \( r_j \) is "Large" and Fault-index-1 is "Small" and… and Fault-index-m is "Small" THEN Fault-index-j is "Small". \( \{\text{Fault-index-1}... \text{Fault-index-m}\} \) is the set of Fault-indexes corresponding to the residuals \( \{r_1, r_2... r_m\} \).

2.2.2.3 Defuzzification

The defuzzification provides a fault indicator for the residual \( r_j \). The output is called Fault-index-j. The fuzzy partition of Fault-index-j is the same as the fuzzy partition of Fault-index of (a) or (b) residuals.

3 FAULT ISOLATION

3.1 A Signature-based Isolation Method

This method consists in associating each known variable with a binary vector. The terms equal to 1 indicate the presence of the variable in the corresponding residual. This binary vector is the fault signature of the variable (Tagina M., 95).

The residual processing provides a coherence binary vector which terms equal to 1 indicate the presence of a fault. For the fault isolation, the coherence binary vector must be compared to the various fault signatures as well as the normal functioning mode signature.

3.2 The Proposed Fuzzy Isolation Method

In this paragraph, we propose a fuzzy isolation method. The attributes of the system are Fault-indexes provided in the detection stage. The outputs correspond to fault indicators of known variables (Fault-j).

3.2.1 Fuzzification

The descriptive set \{Small, Large\} is used to describe the Fault-indexes. The fuzzy partition is the same as figure 4.

3.2.2 Inference Rules

We suppose that we have \( N \) residuals and \( M \) known variables.

Fault signatures can be rewritten by replacing 1 by "Large" and 0 by "Small".

This allows writing inference rules in the form: IF Fault-index-1 is ("Small"/"Large") and Fault-index-2 is ("Small"/"Large")… and Fault-index-N is ("Small"/"Large") THEN Fault-j is ("Small"/"Large"), \( j \in \{1, 2, ..., M\} \)

Notice: Fault-j is "Large" in only one rule, this is when the coherence binary vector is identical to fault signature of the known variable \( j \).

3.2.3 Defuzzification

Three membership classes of Fault-j are defined: Small, Medium and Large.

Figure 5: Fault-j fuzzy partition.

The defuzzification provides a fault indicator in known variable \( j \) (Fault-j), a value close to 0 means that the variable \( j \) is in normal state, a value close to
1 indicates the presence of a fault in variable j. If Fault-j is in the interval [0.25, 0.75], then there is an isolation problem.

4 ILLUSTRATIVE EXAMPLE

We illustrate our approach with an RLC circuit:

![RLC Circuit Diagram](image)

Figure 6: RLC circuit in sinusoidal mode.

With:

\[ R = 500\, \Omega, \quad C = 5\, \mu\text{F}, \quad L = 0.2\, \text{H}, \quad \omega = 1000\, \text{rad/s} \]
\[ e(t) = \sqrt{2} \times 10\sin(\omega t) \]

A procedure described in (Borne et al., 92) and (Dauphin-Tanguy G. et al., 95) enables us to elaborate its bond graph model shown in figure 7.

![Bond Graph Model](image)

Figure 7: Bond graph model of the RLC circuit.

We have placed two sensors in the bond graph model:

- An effort sensor \( De \)
- A flow sensor \( Df \)

Variables to be supervised (the known variables) are \( De, Df \) and \( Se \).

Analytical redundancy relations consist in finding relations between known variables of the system. We have elaborated the following analytical redundancy relations (ARRs) from the bond graph model by course of the causal paths. ARRs are written in integral form.

\[
\text{RRA1: } Df = \frac{Se - De}{Ls} \\
\text{RRA2: } \frac{Df - De}{Rs} - De = 0 \\
\text{RRA3: } \frac{Se - De}{Ls} - \frac{De}{sC} - De = 0
\]

The procedure of generating (ARRs) is described in (Tagina M., 95).

The fault signature table of the three known variables \( (Se, De, Df) \) is given by table 2:

<table>
<thead>
<tr>
<th></th>
<th>( Se )</th>
<th>( De )</th>
<th>( Df )</th>
</tr>
</thead>
<tbody>
<tr>
<td>RRA1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>RRA2</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>RRA3</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

The bond graph model is converted into a synopsis diagram which is simulated under Simulink/Matlab environment.

In normal functioning mode, the residuals have to be close to zero. The simulation of the residuals gives the following results:

![Residuals](image)

Figure 8: residuals in fault free context and without uncertainty in the components.

We verify that the three residuals are null.

Application choices:

* Uncertainty: We consider uncertainty as a white noise added on \( R, L \) and \( C \) values in the synopsis diagram (1% or 5% of each value). We have selected uncertainty of 1% and 5% on one hand because many components are given with this uncertainty; on the other hand because the
identification is possible in this case. For larger uncertainties, we can not distinguish between the fault and uncertainty.

* Fault: We have considered fault as a rectangular signal added to a known variable, the fault amplitude is equal to 15% of the variables amplitude. We have selected this amplitude because the noise-to-signal ratio should not exceed 10%.

* Fault Interval length:
Faults are applied in intervals which length are between 0.1s and 8s.

4.1 Case of 1% Uncertainty

The following results are obtained by simulation of the three residuals $r_1$, $r_2$ and $r_3$ in fault free context and when faults are introduced in intervals which lengths are between 0.1s and 8s:

Table 3: Characteristics for 1% of uncertainty.

<table>
<thead>
<tr>
<th>Residual</th>
<th>Threshold</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_1$-max</td>
<td>0.00188</td>
</tr>
<tr>
<td>$R_1$min</td>
<td>0.0307</td>
</tr>
<tr>
<td>$R_1$max</td>
<td>211.5</td>
</tr>
<tr>
<td>$r_2$-max</td>
<td>0.0001615</td>
</tr>
<tr>
<td>$R_2$min</td>
<td>0.001091</td>
</tr>
<tr>
<td>$R_2$max</td>
<td>0.3</td>
</tr>
<tr>
<td>$r_3$-max</td>
<td>19.62</td>
</tr>
<tr>
<td>$R_3$min</td>
<td>1.232e+5</td>
</tr>
<tr>
<td>$R_3$max</td>
<td>2.15e+8</td>
</tr>
</tbody>
</table>

We note that $r_1$ and $r_2$ are (a) and (b) residuals. However $r_3$ is a (c) residual.

A fault affects respectively $D_f$, $D_e$ and $S_e$ at time 4s up to 6s.

4.1.1 Fault Detection and Isolation by Binary Logic

1st case: (a) and (b) residuals ($r_1$ and $r_2$)
The fault detection algorithm is as follows:
IF ($r_1 > \text{threshold}_{-1}$ and $d_1 > \text{threshold}_{-i}$)
THEN Fault-index-i=1
ELSE Fault-index-i= 0

2nd case: (c) residuals ($r_3$)
IF ($r_3 > \text{threshold}_{-3}$ and $r_2 < \text{threshold}_{-2}$ and $r_1 < \text{threshold}_{-1}$) or ($r_3 < \text{threshold}_{-3}$)
THEN Fault-index-3 = 0
ELSE Fault-index-3 = 1

The following thresholds are used in the detection program: $r$-max, $2*r$-max and $R$min.

We note that the fault detection is perfect for the thresholds shown in table 4.

Table 4: Thresholds retained for 1% of uncertainty.

<table>
<thead>
<tr>
<th>Residual</th>
<th>Threshold</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_1$</td>
<td>$2*r_1$-max</td>
</tr>
<tr>
<td>$r_2$</td>
<td>$r_2$-max</td>
</tr>
<tr>
<td>$r_3$</td>
<td>$2*r_3$-max</td>
</tr>
</tbody>
</table>

We can observe in figure 9 that Fault-index-1, Fault-index-2 and Fault-index-3 when a fault affects $D_e$ between 4s and 6s.

4.1.2 Fault Detection and Isolation by Fuzzy Reasoning

By applying the proposed fuzzy approach to $r_1$, $r_2$ and $r_3$ in Simulink/Matlab environment, we perform good fault detection and isolation results for faults affecting $D_e$, $S_e$ and $D_f$.

Figure 10 shows detection when a fault affects $D_f$ between 4s and 6s. Fault-index-1 and Fault-index-2 are equal to 1 in this interval.

By applying the fuzzy isolation method (see figure 11), we find that Fault-$D_f$ is equal to 1, between 4s and 6s, whereas Fault-$D_e$ and Fault-$S_e$ are null in this interval, so we have perfectly isolated fault at sensor $D_f$. 
As a conclusion, in the case of 1% of uncertainty, fuzzy approach as well as classical one allows a good fault detection and isolation.

4.2 Case of 5% Uncertainty

The following results are obtained by simulation of the three residuals $r_1$, $r_2$ and $r_3$ in fault free context and when faults are introduced in intervals which lengths are between 0.1s and 8s.

Table 5: Characteristics for 5% of uncertainty.

<table>
<thead>
<tr>
<th>Residual</th>
<th>Threshold</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_1$</td>
<td>$2*r_1\text{-max}$</td>
</tr>
<tr>
<td>$r_2$</td>
<td>$r_2\text{-max}$</td>
</tr>
<tr>
<td>$r_3$</td>
<td>$2*r_3\text{-max}$</td>
</tr>
</tbody>
</table>

4.2.1 Fault Detection and Isolation by Binary Logic

We perform good fault detection and isolation results for the following thresholds:

4.2.2 Fault Detection and Isolation by Fuzzy Reasoning

In this case, we notice a small problem for faults affecting $Df$. Figure 12 and figure 13 show that Fault-index-1 and Fault-Df are disturbed in the interval [4s 6s], however, we can isolate fault at variable $Df$. Fault detection and isolation is good in case of $Se$ and $De$ faults.

We conclude in the case of 5% uncertainty that classical logic is more suitable for fault detection and isolation.
4.3 Faulty Estimated Uncertainty

In practice, this case is very frequent, that is generally due to a wear of components. Let us consider the case where uncertainty reaches 5% whereas the estimated one is equal to 1%.

4.3.1 Binary Logic

We apply the retained thresholds in case of 1% uncertainty. As shown in figure 14, in the three cases of faults, fault–index-1 is very disturbed, it passes infinitely between 0 and 1 outside the interval [4s 6s].

So binary logic does not ensure good fault detection in case of faulty estimated uncertainty.

4.3.2 Fuzzy Reasoning

By applying the fuzzy approach, the fault detection and isolation is good for faults affecting De, Se and Df. Figure 15 and figure 16 show fault detection and isolation when a fault affects Df.

5 COMPARISON

Binary logic allows performing good detection and isolation results if the threshold is correctly chosen and if the values of parametric uncertainty are known. However, when uncertainty is faulty
estimated, detection with binary logic is not suitable whereas the fuzzy proposed approach allows performing good fault detection and isolation results.

6 CONCLUSION

In this paper, we have proposed a fuzzy fault detection and an isolation method for faults affecting the sensors and the actuators off-line. A fuzzy processing of residuals provided by ARRs is followed by fuzzy processing of fault-indexes in order to isolate the fault.

We have compared binary approach to fuzzy approach through an illustrative example. We have noticed that in the case of 1% or 5% of uncertainty, binary logic allows a perfect fault detection and isolation if the threshold between normal and faulty state is correctly chosen, fuzzy approach allows also fault detection and isolation in spite of some disturbance in case of 5% uncertainty. However, in case of faulty estimated uncertainty, the proposed fuzzy approach allows good fault detection and isolation where binary approach is not suitable for fault detection and isolation.

REFERENCES


Fuzzy logic toolbox. Matlab 7.0


