

Towards a formalization of model conformance in Model Driven Engineering

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Abstract. The principle of “*everything is an object*” basically supported by two fundamental relationships *inheritance* and *instantiation* has helped much in driving the object technology in the direction of simplicity, generality and power of integration. Similarly in the Model Driven Engineering (MDE) today, the basic principle that “*everything is a model*” has many interesting properties. The two relations *representation* and *conformance* are suggested [2] to be the two basic relations in the MDE. This paper tends to support this ideas by investigating some concrete examples of the *conformance* relation concerning three technological spaces (TS) [10]: Abstract/Concrete Syntax TS, XML TS and Object-Oriented Modeling (OOM) TS. To go further in this direction we try to formalize this relation in the OOM TS by using the category theory – a very young and abstract but powerful branch of mathematics. The OCL language is (partially) reused in this scheme to provide a potentially useful environment supporting MDE in a very general way.

1 Introduction

Model Driven Engineering (MDE) today does not limit itself to the OOM Technological Space (TS) but many other TSs such as AS TS, XML TS ... [10]. This means explicitly that its principles must be very general and not only restricted to OOM TS. Today, the principle « Everything is a model » as suggested by many authors such as [3] becomes the main principle of the MDE similarly to the principle « Everything is an object » in object technology. Conformance is one of the fundamental relations supporting this principle in MDE. This paper investigates the conformance relation in some well-known Technological Spaces such as Abstract/Concrete Syntax, XML and OOM technological spaces.

The paper is organized as follow: section 1 presents the context of our work; section 2 presents some ideas about the notion of conformance in several well-known TSs; section 3 presents a formalization of the conformance relation in the OOM TS using category theory and the OCL language. The practical usage of this formalization will be discussed in the section 4. Some related works are briefly introduced in the section 5. Some conclusions will be provided in the section 6.

2 Conformance in some Technological Spaces

We begin our discussion with a simple example coming from Regular Expression. It is not difficult to see that there is a mapping from a string $S = \text{accdd}$ to a regular expression $E = a(b|c^*)d?$ when the string S matches the expression E . This mapping is illustrated in the Fig.1.

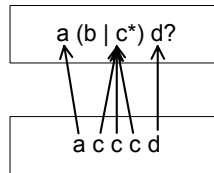


Fig. 1. A very simple form of conformance – a string matches a regular expression

The regular expression E defines characters that may appear in a string conforming to E : $\{a,b,c,d\}$ and how these characters are structured using several constructions:

- *alternation* with a vertical bar such as $b | c$ specify the choice of b or c .
- *quantification* with a quantifier $(+,?,*)$ that following a character specifies how often that character is allowed to occur.
- *grouping* with brackets to define the scope and precedence of the other operators.

If the guiding principle of the MDE:

“Everything is a model” [P0]

is accepted, we have the following two models: the string S and its definition E (is also a string) with their characters as model elements. It can be said that S is defined by E or S conforms to E .

“A model conforms to its definition, this definition is also a model called meta-model of the first one” [P1]

From our first observation, we propose the following principle:

“Every element of a model finds an unique definition in a meta-model that the model conforms to” [P2]

We have also the following comments:

- The order of elements in S must respect to the order of elements defined in E . [C1]
- The group of elements in S must respect to the group definition in E . [C2]
- The number of occurrences of elements in S must respect to quantification definitions in E . [C3]

Now we move to an illustrative example in the Abstract/Concrete Syntax TS. Let’s consider a well-known *HelloWorld* program written in the Pascal programming language. This program is considered to be a syntactically correct with respect to the grammar of the Pascal programming language. In this example, the *HelloWorld* program is a model and the grammar of the Pascal programming language is the meta-model defining the former. The principle [P2] is applicable in this case and is illustrated in the Fig.2. A part of the grammar is represented in the flowchart form extracted from [9]. Every symbol of this program finds a unique definition in the grammar. The three comments [C1, C2, C3] are also correct in this case.

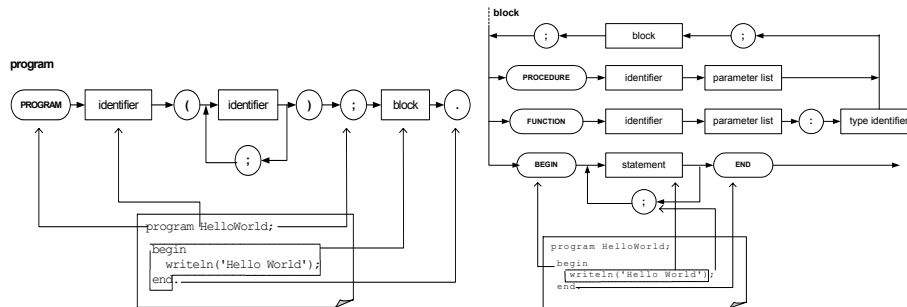


Fig. 2. A Pascal program conforms to the grammar of the Pascal programming language

In the XML TS, we find the following definition [6]: « *An XML document is valid if it has an associated document type declaration and if the document complies with the constraints expressed in it* ». This means explicitly that a valid XML document must conform to a DTD. DTDs specify two kinds of constraints as classified in [5]: *structural* constraints given by element declaration rules and *attribute* constraints given by attribute declaration rules. Also following [5], « *the structural constraints of DTD are abstracted as extended context free grammars, that is, context free grammars where the right hand side of each production contains a regular expression. An XML document is valid with respect to the structural constraints of a DTD if its abstraction as a tree represents a derivation tree of the extended CFG corresponding to that DTD* ». Attribute constraints deal with the values of attribute nodes while structural constraints deal with the labels of nodes in the XML tree.

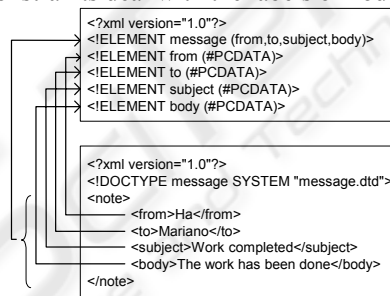


Fig. 3. An XML document conforms to a DTD.

Let's consider an example that illustrates the relation between an XML document and a DTD. In this case, the model is the XML document and the meta-model defining this model is the DTD. The XML document has (element and attribute) nodes as its elements. The principle [P2] and the three comments [C1, C2, C3] are also applicable in this case.

We have analyzed the conformance relation in the case of regular expression, Abstract/Concrete syntax and XML. The principle [P2] is also applicable in Object-Oriented modeling.

In the left of the Fig.4 is an UML diagram represented in a case tool such as Rose. This model is an *instance-of* of the UML meta-model as simplified in Fig.4. Every elements of this model finds its unique definition in the meta-model.

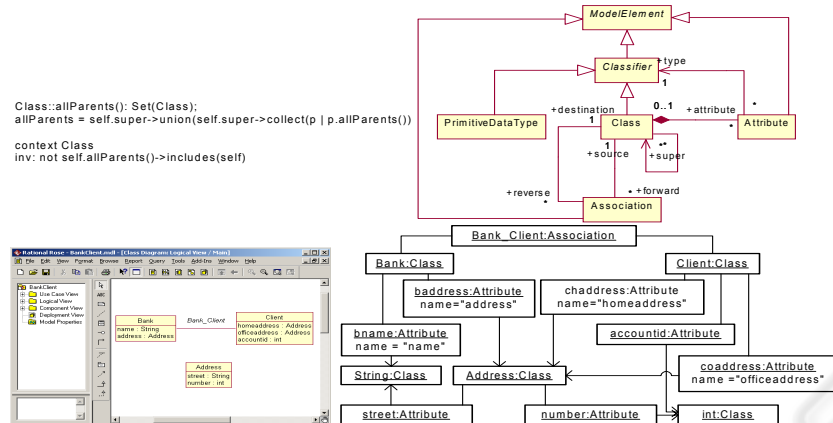


Fig. 4. An illustrative example: a model UML conforms to its meta-model

An UML model conforms to the UML meta-model must also satisfied all the well-formedness rules defined with the meta-model. The multiplicity in the meta-model can also be expressed as constraints associated to the meta-model [16]. Furthermore, we have the following principle:

- Every link in the model finds a unique definition in the meta-model. [P3]

This principle is so important as the [P2] principle for a model UML and also for the conformance relation between a model and a meta-model defining it in meta-modeling. These two principles [P2, P3] are also applicable in the “strict meta-modeling” approach in which the OMG’s MOF is an example: “Every element of an M_n level model is an *instance_of* exactly one element of an M_{n+1} level model” [1].

3 A formalization of the conformance relation in the OOM TS

In a very general way, a model can be viewed as containing:

- A set of model elements (character in a string or regular expression, symbols and terminals in a grammar, element or attribute nodes in XML, model elements in modeling)
- Some of those elements are associated to some sorts of literal (integer, real, string...)
- A set of links that associates elements (link is directed). Those links forms a navigation network among model elements.
- To make sense, each model must be associated with a meta-model defining it.
- Every model element finds its unique definition in the meta-model.
- Every model link finds its unique definition in the meta-model.

The fact that there is a mapping from a model (the defined artifact) and its meta-model (the defining artifact) is one of the *necessary conditions* for the model to conform to its meta-model. This mapping includes model elements mapping and model links mapping and is then a structural mapping. Together with this structural mapping the model must satisfy constraints associated to the meta-model. Those

constraints can be evaluated based on structural mapping and literal values associated to model elements.

Before taking into details of the formalization, we put some words about the category theory. Category theory originally arose in mathematics out of the need of formalism to describe the passage from one type of mathematical structure to another [7]. Category theory has been used in diverse branches of software engineering and computer science as pointed out by Goguen [8], in object-oriented software evolution [11] and recently the formalization of UML [14] and MOF [4] etc. In category theory there are structures called categories that contain objects and morphisms. Those morphisms can be composed and the composition of morphisms is associative. Functor is a structure-preserving mapping between two categories. Definitions of category, functor and other notion of category theory can be found at [15], [7]. A computational aspect of category theory can be found in [12].

The next topic is the proposed formalization of the conformance relation between a model and its meta-model in the OOM TS. The OOM TS bases on OMG's technology (MOF, UML, QVT...), which is originally based on object models. Adapted from [13], an object model is a tuple

$$\mu = (\text{CLASS}, \text{ATT}_c, \text{OP}_c, \text{ASSOC}, \text{associates}, \text{roles}, \text{multiplicities}, <, \text{PRIMITIVETYPE})$$

such that

- i. CLASS is a set of classes.
- ii. ATT_c is a set of operation signatures for functions mapping an object of class c to an associated attribute value.
- iii. OP_c is a set of signatures for user-defined operations of a class c .
- iv. ASSOC is a set of association names.
 - a. *associates* is a function mapping each association name to a list of participating classes.
 - b. *roles* is a function assigning each end of an association a role name.
 - c. *multiplicities* is a function assigning each end of an association a multiplicity specification.
- v. $<$ is a partial order on CLASS reflecting the generalization hierarchy of classes.
- vi. PRIMITIVETYPE is a set of primitive data types used in the object model = {STRING, INTEGER, REAL }.

In our formalization, model navigation plays an important role. We proposed the concept of *navigation morphism* which is represented by a tuple

$$\text{nav} = (e_s, L, E_t)$$

such that

- i. e_s is the model element that is the source of the navigation morphism
- ii. L is a sequence of navigation label
- iii. E_t is a sequence of elements that is orderly located in the navigation from the source element e_s to the target element. The last element of this sequence is the target of the navigation morphism.

Now, from every object model μ , there is a derived category C_μ :

$$C_{\mu} = (Ob_c, Mor_c, dom, cod, id, composition)$$

such that

- i. $Ob_c = CLASS \cup PRIMITIVETYPE$
- ii. $PRIMITIVETYPE$ is the set of primitive types used in the object model
- iii. $Mor_c = Mor_{c_1} \cup Mor_{c_2}$
- iv. Mor_{c_1} is the set of all navigation morphisms $(e_s, [role\ name], [e_t])$ representing a navigation from e_s to e_t ($e_s, e_t \in CLASS$) through the “role name” role. Mor_{c_1} can be calculated from $CLASS, ASSOC, associates$ and roles.
- v. Mor_{c_2} is the set of all navigation morphisms $(e_s, [attribute\ name], [e_t])$ representing a navigation from e_s ($e_s \in CLASS$) to e_t ($e_s \in PRIMITIVITES$) through the “attribute name” attribute. Mor_{c_2} can be calculated from $CLASS, ATT_c, PRIMITIVETYPE$.
- vi. $dom: Mor_c \rightarrow Ob_c$ is a function that takes a navigation morphism as argument and gives the source of that navigation morphism as result. This function can be calculated from $CLASS, ATT_c, ASSOC, associates, roles$ and $<$.
- vii. $cod: Mor_c \rightarrow Ob_c$ is a function that takes a navigation morphism as argument and gives the target of that navigation morphism as result. This function can be calculated from $CLASS, ATT_c, ASSOC, associates, roles$ and $<$.
- viii. id is an identity function that takes a model element e as its argument and give a navigation morphism $(e, [], [e])$ as result. i.e this function returns a navigation morphism from the element e to itself (there is no navigation label)
- ix. $composition$ is a function that takes two navigation morphisms $nmor1 = (e_{s_1}, L_1, E_{t_1})$ and $nmor2 = (e_{s_2}, L_2, E_{t_2})$ as its arguments and give a composite navigation morphism $nmor = (e_{s_1}, L_1 \text{ concat } L_2, E_{t_1} \text{ concat } E_{t_2})$ when $cod(nmor1) = dom(nmor2)$

Once the model μ is promoted as a meta-model (M_2 level), any model of this meta-model can be represented by a category :

$$C_{model} = (Ob_c, Mor_c, dom, cod, id, composition)$$

such that

- i. $Ob_c = OBJECT \cup LITERAL$
- ii. $OBJECT$ is the set of objects in the selected model
- iii. $LITERAL$ is the set of objects associated to a primitive value used in the selected model
- iv. $Mor_c = Mor_{c_1} \cup Mor_{c_2}$
- v. Mor_{c_1} is the set of all navigation morphisms $(e_s, [role\ name], [e_t])$ representing a navigation from e_s to e_t ($e_s, e_t \in OBJECT$) through the “role name” role. Mor_{c_1} can be calculated from the selected model.

- vi. Mor_{c_2} is the set of all navigation morphisms $(e_s, [attribute\ name], [e_t])$ representing a navigation from e_s ($e_s \in OBJECT$) to e_t ($e_s \in LITERAL$) through the "attribute name" attribute. Mor_{c_1} can be calculated from the selected model.
- vii. $dom: Mor_c \rightarrow Ob_c$ is a function that takes a navigation morphism as argument and gives the source of that navigation morphism as result. This function can be calculated from the selected model.
- viii. $cod: Mor_c \rightarrow Ob_c$ is a function that takes a navigation morphism as argument and gives the target of that navigation morphism as result. This function can be calculated from the selected model.
- ix. id is an identity function that takes a model element e as its argument and give a navigation morphism $(e, [], [e])$ as result. i.e this function returns a navigation morphism from the element e to itself (there is no navigation label)
- x. $composition$ is a function that takes two navigation morphisms $nmor1 = (e_{s1}, L_1, E_{t1})$ and $nmor2 = (e_{s2}, L_2, E_{t2})$ as its arguments and give a composite navigation morphism $nmor = (e_{s1}, L_1 \text{ concat } L_2, E_{t1} \text{ concat } E_{t2})$ when $cod(nmor1) = dom(nmor2)$

An example: BankClient model conforms to SimpleUML model

The simplified meta-model UML and the Bank_Client model (Fig.4) are illustrated partially in the categorical form in the Fig. 5. Model elements and model links of these two models is provided in the Table.1.

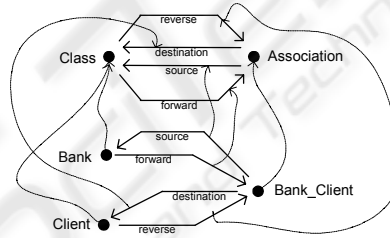


Fig. 5. A partial view of mapping from BankClient (model) to SimpleUML (meta-model)

The mapping from Bank_Client model to SimpleUML model illustrated in the Table.2 can be expressed by a functor $F: C_{Bank_Client} \rightarrow C_{SimpleUML}$ that contains:

- A model element mapping
 $F_{element} = Bank \rightarrow Class ; Client \rightarrow Class ; Bank_Client \rightarrow Association$
- A model link mapping $F_{navigation} =$
 $(Bank, [forward], [Bank_Client]) \rightarrow (Class, [forward], [Association]) ;$
 $(Client, [reverse], [Bank_Client]) \rightarrow (Class, [reverse], [Association]) ;$
 $(Bank_Client, [source], [Bank]) \rightarrow (Association, [source], [Class]) ;$
 $(Bank_Client, [destination], [Client]) \rightarrow (Association, [destination], [Class])$

Table 1. Model elements and model links of Bank_Client and SimpleUML model

	C_{Bank_Client}	$C_{SimpleUML}$
elements	$\{Bank_Client, Bank_Client\}$	$\{Class, Association\}$
links/ basic navigations	$\{(Bank, [forward], [Bank_Client]), (Client, [reverse], [Bank_Client]), (Bank_Client, [source], [Bank]), (Bank_Client, [destination], [Client])\}$	$\{(Class, [forward], [Association]), (Class, [reverse], [Association]), (Association, [source], [Class]), (Association, [destination], [Class])\}$

Table 2. Navigation mapping and mapping of a composition

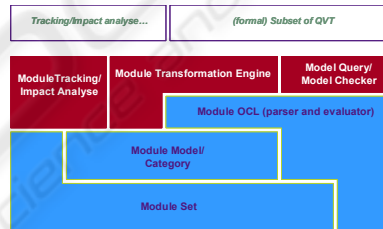
From Bank_Client	To SimpleUML
$(Bank, [forward], [Bank_Client])$	$(Class, [forward], [Association])$
$(Bank_Client, [destination], [Client])$	$(Association, [destination], [Class])$
$(Bank, [forward], [Bank_Client])^\circ$	$(Class, [forward], [Association])^\circ$
$(Bank_Client, [destination], [Client]) = (Bank, [forward, destination], [Bank_Client, Client])$	$(Association, [destination], [Class]) = (Class, [forward, destination], [Association, Class])$

Remarks. The mapping of the composition of two navigations is the composition of the mappings of the two navigations. This is an important property of the structural mapping and is called structure-preserving mapping in the category theory.

4 Exploiting the formalization

In order to demonstrate the benefits of the proposed formalization, we have developed a prototype of an MDE environment in which different kind of data such as models, meta-models, mapping specifications, conformance relationships and more generally, any structure-preserving relationship can be represented in a unified manner (using categories and functors).

The developed prototype having architecture depicted in Fig.6 contains an OCL evaluator that exploits categorical representations of models and conformance mapping to navigate through model elements. The implementation of this evaluator is well facilitated since model navigation – an important part of the language is made explicit in the categorical representation of (meta-)models.

**Fig. 6.** The MDE environment prototype

The developed prototype has allowed us to point out several potential usages of the formalization presented in the previous sections. Some of these usages are provided below:

- *Verifying for model conformance*: the input and output model of a transformation can be respectively verified if each model conforms to its meta-model due to the OCL evaluator.
- *Model query*: models can be queried with the OCL language.

- *Model transformation execution*: a set of model transformations (structure preserving transformation) can be executed due to the transformation engine.
- *Systematic traceability*: the traceability information is stored as categorical functors and is produced as explicit result of transformation together with output model.
- *Tracking for multi-step transformations*: since traceability information is stored in the form of functors, those functors can be composed in the case of successive transformation.
- *Help to the analysis of impacts*: since the structural relation between input and output model is captured by a functor (this functor is also the traceability information), it is possible to ask some kind of questions about transformation executed such as: *if a model element (or model link) in the input model is removed then which parts of the output model will change? Or in the inverse direction: if I want to make some change in the output model, which parts of the input model need to be changed?* These kind of questions can be answered without making real change and re-execute transformations and is very useful in an interactive environment where model transformation is an interactive computer aided tool to the development or may be in the specification phase of model transformation when debugging facility is a requirement.
- *Analysis for (structural) completeness of model transformations*: with the traceability information we can easily verify which parts of the input model do not take part in the generation of any model element in the output model, this *may* be the case in that the specification of model transformation is not complete.

5 Related works

Category theory has been used to formalize UML [14] and recently MOF [4]. These formalizations based on Slang, a language supporting category theory of the Kestrel Institute [14]. Our formalization uses directly the graph representation (interpreted as categories) of models, functors to describe conformance mapping and OCL to describe constraints. In our work, functor is also used to represent relation between models at different levels of abstraction of the same system.

6 Conclusions

The work presented in this paper bases on a categorical abstraction of model and OCL to formalize the conformance relation of a model to its meta-model in the Object-Oriented Modeling TS. This relation can be expressed by a conformance mapping from the model to its meta-model and a set of constraints associated to the meta-model. These constraints must be satisfied when being evaluated over the model, the meta-model and the conformance mapping between them. We believe that the same kind of formalization can be used to other TSs due to the conformance mapping from a model to its definition (meta-model) in OOM TS or from a XML document to its DTD (or XML Schema), etc. The main advantage of this formalization is that it is

very abstract and can be applied to any kind of (meta-)models. This formalization is also a first step in defining a model transformation formalism in which traceability and analysis of impacts is fully supported.

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