A NEW PUBLIC-KEY ENCRYPTION SCHEME BASED ON NEURAL NETWORKS AND ITS SECURITY ANALYSIS

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Abstract: A new public-key Encryption scheme based on chaotic attractors of neural networks is described in the paper. There is a one-way function relationship between the chaotic attractors and their initial states in an Overstoraged Hopfield Neural Networks (OHNN), and each attractor and its corresponding domain of attraction are changed with permutation operations on the neural synaptic matrix. If the neural synaptic matrix is changed by commutative random permutation matrix, we propose a new cryptography technique according to Diffie-Hellman public-key cryptosystem. By keeping the random permutation operation of the neural synaptic matrix as the secret key, and the neural synaptic matrix after permutation as public-key, we introduce a new encryption scheme for a public-key cryptosystem. Security of the new scheme is discussed.

1 INTRODUCTION

Networking security is one of key problems in the study of IPng (Goncalves, 2000), and the encryption algorithm selected for IPSec is the core of this key problem. In order to meet the requirements of multimedia real time communications via the IPng, the encryption algorithm should have higher complexity for the security of information system, and higher processing speed for the efficiency of information encryption and decryption. So, neural networks, with the properties of nonlinear dynamics such as chaotic behavior and parallel processing, are regarded as one of good design options for encryption algorithm applied in the communications of IPng.

Since the thought of public-key cryptosystem was proposed (Diffie, 1976), the public-key scheme has been the focus of modern cryptographist's attention. Many algorithms of public-key encryption were put forward in recent years (Stallings, 2003). However, the encryption scheme of public-key based on neural networks isn't reported till now.

Although neural networks are made up of simple elements, they have complex nonlinear dynamics with chaotic attractors. Since the synchronization of chaotic system was discovered (Pecora, 1990), some symmetric encryption schemes base on the chaotic synchronization of neural networks have been proposed (Crounse, 1996, Milanovic, 1996, Donghui 1999). Here, we will introduce a new public-key encryption scheme based on the chaotic attractors of neural networks according to Diffie-Hellman public-key cryptosystem and commutative matrix theory.

2 PRINCIPLES OF THE PROPOSED SCHEME

For a discrete HNN(Hopfield,1982), if system initial state is converge to one of the system attractors by a Minimum Hamming Distance (MHD) criterion, the attractor is a stable state as an associative sample of HNN and can be stored in HNN. If the number of sample to be stored is over the capacity of HNN, the stable attractors of HNN system will be became aberrant and chaotic attractors will emerge. The capacity of networks is increased. HNN becomes overstoraged HNN (OHNN).

Guo Donghui and Chen L. M. further proved that these attractors are chaotic, and message in the attraction domain of an attractor are unpredictable related to each other. After the neural synaptic matrix *T* multiplied by random permutation matrix *H*,

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original initial state *S* and corresponding attractor S^{μ} become new initial state Ŝ and attractor $\hat{S}^{\mu}$, respectively. They are shown as follows:

$$S_i(t+1) = f\left(\sum_{j=0}^{N-1} T_{ij} S(t) + \theta_i\right) \tag{1}$$

$$\hat{T} = H * T * H' \quad \hat{S}^{\mu} = S^{\mu} * H \tag{2}$$

Where H' is the transpose of matrix of *H*.

Provided that neural synaptic matrix *T* is a $n \times n$ singular matrix, and *H* is a $n \times n$ random permutation matrix. For any given *T* and *H*, $\check{T}=H*T*H'$ is easy to compute according to matrix theory, and $\check{T}$ is a singular matrix too. Furthermore, there is a kind of special matrix, which is referred as commutative matrix, in the random permutation matrix (Chen, 2001). Suppose that H_1 and H_2 both are two of commutative matrices, and they have same order. Then, they must meet the following equation: $H_1*H_2=H_2*H_1$

According to Diffie-Hellman public-key cryptosystem, all users in a group jointly select a neural synaptic matrix T_0, which is a $n \times n$ singular matrix. Each user randomly selects a permutation matrix from a $n \times n$ commutative matrices group. i.e., user A firstly selects any nonsingular matrix H_a from this commutative matrices group, and compute $T_a=H_a*T_0*H'_a$. Secondly, he keeps T_a open as a public key, and keeps H_a secret as a private key. When user A and B in a group need secure communication, they will get a shared key $T=H_a*T_b*H'_a=H_b*T_a*H'_b$. User A or user B can easily compute the shared key using his own private key and the other's public key. However, the third can not obtain the shared key because it is computationally infeasible to calculate the shared secret key *T* given the two public values T_a and T_b when the number of neurons *n* is sufficiently large.

In order to improve the security of information during network transmission, the authenticated Diffie-Hellman key agreement protocol (Emmanuel, 2001) is proposed to adopt in the new scheme. The immunity is achieved by allowing the two parties to authenticate themselves to each other by the use of digital signatures and public-key certificate.

3 ENCRYPTION SCHEME

According to the properties of chaotic attractors in OHNN, we know that a lot of chaotic-classified attractors can be obtained as long as stored sample S^{μ} or a few of neural synaptic strength T_{ij} are modified. So we can design a new public-key encryption system with high security, as shown in Fig.1. The encryption scheme can be described as follows.

3.1 Key Generation and Distribution

As described in the previous section, all users in a group needed secure communication jointly and carefully select a neural synaptic matrix T_0. The synaptic matrix should meet the following requirements: (1) it must be a singular matrix. (2) Based on the statistical probability of MHD, if we want to obtain more unpredictable attractors, the neural network requires equal concentrations of excitatory and inhibitory synapses (Gardner, 1987). Suppose the actions between neuron *i* and neuron *j* can be excitatory ($T_{ij}=1$), inhibitory ($T_{ij}=-1$), or not direct connected ($T_{ij}=0$). Thus, in each row and each column of T_0, the number of "1", "-1" and "0" is about equal. Then, the attractor set of T_0 is computed. All users in a group randomly select a same coding matrix *M* in common.

Each user in a group randomly selects a different permutation matrix from a commutative matrix group. i.e., user *i* randomly select a permutation matrix H_i, and keep it secret as a private key. Then, he computes $T_i=H_i*T_0*H'_i$ and corresponding chaotic attractor set of T_i using Eq. (2), and keeps T_i open as a public key. Then, each user publishes his public key and corresponding chaotic attractors by placing them in a public register in the form of digital signatures and gets a public-key certificate with authenticated function from the public-key authority.

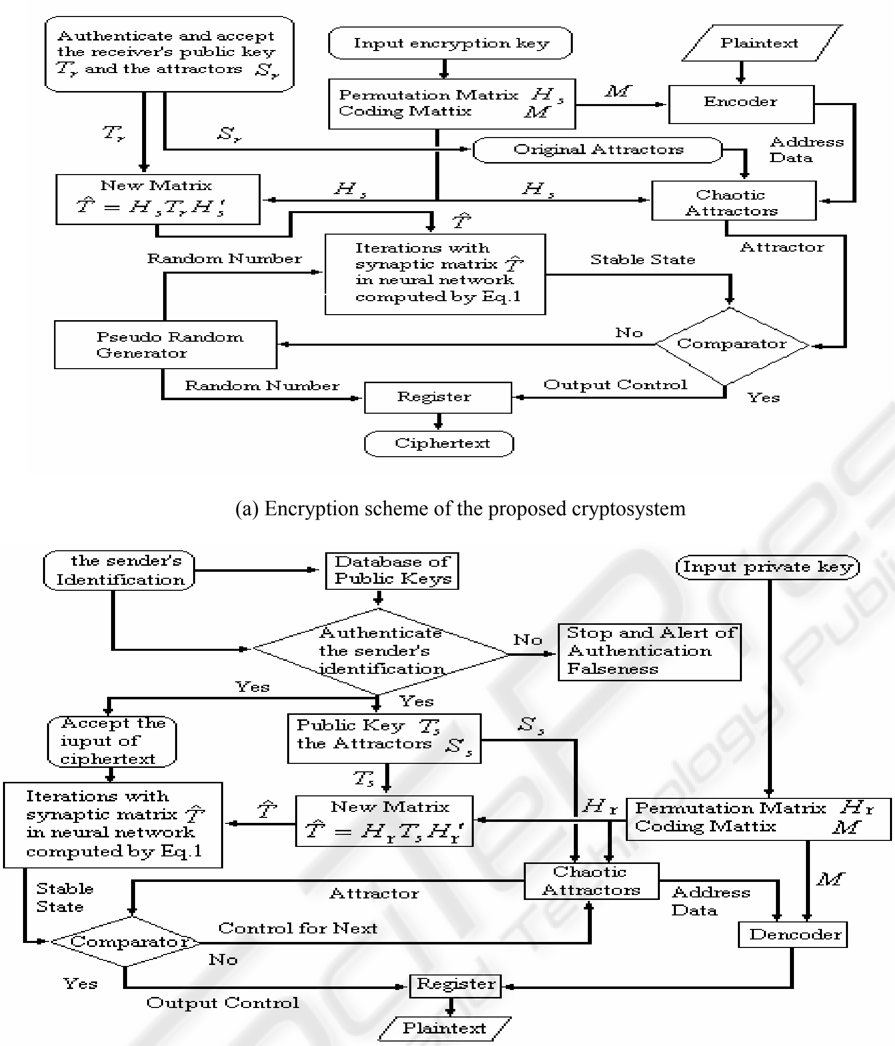
3.2 Encryption

The steps of data encryption are as follows:

(1) Shared key generation. The sender of information firstly input his own private key H_s and accepts the public key T_r and corresponding attractors S_r of T_r, which are legal by authentication of digital signatures, from an information receiver. Then, he calculates the new neural synaptic matrix $T=H_s*T_r*H'_s$ and the attractors Ŝ of *T* using Eq. (2).

(2) Coding process. Use the coding matrix *M* and attractor $\hat{S}^{\mu}$ belonging to *T* to map the plaintext *Y* onto a coded plaintext $Y_x=\{\hat{S}^{\mu}\}$.

(3) Message generation. A random number of forms $\{0, 1\}^N$ are generated by a pseudorandom number generator (PRG) as the initial state S(0) for the OHNN process based on synaptic matrix *T*. Using Eqs. (1), a steady state $S(\infty)$ is obtained and compared to the value of $\hat{S}^{\mu}$. If $S(\infty)=\hat{S}^{\mu}$, a message that belongs to the domain of attraction for attractor $\hat{S}^{\mu}$ is found, and the initial state *S (0)* is outputted as the cipher text *X* for plaintext *Y*.



(a) Encryption scheme of the proposed cryptosystem

(b) Decryption scheme of the proposed cryptosystem

Figure 1: Diagram of the proposed cryptosystem

If $S(\infty) \neq \hat{S}^{\mu}$, PRG generate a new random number as initial state *S (0)*, and step as the previous described is repeated.

3.3 Decryption

The steps of data decryption are as follows:

(1) Identification verification. The receiver firstly verifies the identification of the sender based on his public-key certificate. If the result of authentication is true, the sender will be a legal user and the receiver will accept the cipher text *X*. Otherwise, the receiver rejects to accept any information coming from the sender and gets an alert for authentication falseness.

(2) Decryption process. The receiver firstly inputs his own private key H_r, and accepts the public key T_s and the attractor set $\hat{S}_s$ of the sender. Calculate the new matrix $T=H_r*T_s*H_r^{/}$ and new attractors $\hat{S}^{\mu}$. Secondly, inputs the cipher text *X* into Eq.(1) for iterating, and the corresponding coded plaintext $Y_x=\{\hat{S}^{\mu}\}$ is obtained. Finally, using coding matrix *M*, decode $Y_x=\{\hat{S}^{\mu}\}$ into plaintext *Y*.

4 SECURITY

The security of the proposed cryptosystem is based on the difficulty of singular matrix decomposition and the chaotic-classified properties of OHNN. There are two ways of finding the private key in the proposed cryptosystem by attacking the chaotic properties of OHNN or by matrix decomposition.

As stated in the previous section, the neural synaptic matrix T_0 is a singular matrix. Thus, the matrices T_s, T_r and T all are singular matrices. For any given matrix T_0, T_s and T_r, T is relatively easy to compute according to matrix theory. However, for any given higher order matrix T_s, T_r or T, it is computationally infeasible to find permutation matrix H_s or H_r. i.e. Only Hessenberg transform of matrix in the method of conventional matrix decomposition can succeed in finding permutation matrix. However, the difficulty of computation can not be overcome. Firstly, when any square matrix is transformed a Hessenberg matrix, the complexity of computation time is $O(n^3)$, where n is the order of T. Secondly, the decomposition of Hessenberg matrix is not unique (Chen, 2001). For any n order square matrix, the number of Hessenberg decomposition is over 2^n. Thus, it is computationally infeasible to traverse the space of Hessenberg matrices for any synaptic matrix when n is larger.

As illustrated in the previous section, our cryptosystem is designed based on the chaotic-classified properties of the OHNN. It is impossible to find the private key H by using chosen-plaintext attack or known- plaintext attack at present (Guo, 1999). Furthermore, the proposed cryptosystem is uneven in the encryption and decryption process, i.e. it uses a random substitution during the encryption and auto-attraction during the decryption. Differential cryptanalysis methods cannot unfold our proposed cryptographic scheme because of these uneven processes. Only an exhaustive search based on the statistical probabilities of plaintext characters can succeed in breaking our proposed cryptosystem. However, the breaking cost of this method is very high. i.e., for N =32, some 10^{20} MIPS years would be required for a successful search, which is well above the acceptable security level of current states, i.e., 10^{12} MIPS years.

On the other hand, the necessity of our cryptosystem is that the attractors are randomly substituted by the messages in their domains of attraction to eliminate the statistical likeness of the plaintext and avoid this attack based on the statistical probabilities of plaintext characters. So that, in the encryption process, the number of messages in the domain of attraction is another key parameter for the security of our proposed cryptosystem. If the PRG for random substitutions in our cryptosystem is designed to have temporal variations, the same message in the plaintext can be encrypted to different cipher texts at different times. To break our proposed scheme using probabilistic attacks requires that one store all the information of the attractors and their domains of attraction, which is not practical even when N is reasonably large, i.e., N =32.

5 CONCLUSIONS

We propose a new public-key cryptosystem based on the chaotic attractors of neural networks. According to above discussions of the new cryptosystem, the proposed scheme has a high security, and is eminently practical in the context of modern cryptology. Neural networks rich in nonlinear complexities and parallel features are suitable for use in cryptology to meet the requirement of secure communication of IPng, as proposed here. However, we do not know whether the new public-key encryption scheme described in this paper can be kept from new types of attack. The exploration into the potential relevance of neural networks in cryptography needs be studied in detail.

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