Incorporating Context into Recommender Systems
Using Multidimensional Rating Estimation Methods

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Abstract. Traditionally recommendation technologies have been focusing on recommending items to users (or users to items) and typically do not consider additional contextual information, such as time or location. In this paper we discuss a multidimensional approach to recommender systems that supports additional dimensions capturing the context in which recommendations are made. One of the most important questions in recommender systems research is how to estimate unknown ratings, and in the paper we address this issue for the multidimensional recommendation space. We present the classification of multidimensional rating estimation methods, discuss how to extend traditional two-dimensional recommendation approaches to the multidimensional space, and identify research directions for the multidimensional rating estimation problem.

Keywords. Context-aware recommender systems, collaborative filtering, rating estimation techniques.

1 Introduction and Motivation

Recommender systems became an important and popular research area in the mid-1990’s, when researchers started focusing on recommendation problems that explicitly rely on the notion of ratings [11,14]. Subsequently there has been a significant amount of research done on developing different recommendation techniques over the past decade [1,2,3,4,5,6,7,9,12,13,15]. These technologies are based on a broad range of different approaches and feature a variety of methods from such disciplines as statistics, data mining, machine learning and information retrieval. In its most common formulation, the recommendation problem is reduced to the problem of estimating ratings for the items that have not been seen by a user. This estimation is usually based on the ratings given by the user to other items, ratings given to the item by other users, and possibly on some other user and item information as well (e.g., user demographics, item characteristics). Naturally, once the ratings are estimated for the yet unrated items, the recommendations can be made to each user by presenting the
Note that, while a substantial amount of research has been performed in the area of recommender systems, the vast majority of the existing approaches focus on recommending items to users or users to items based on the ratings information and do not take into the consideration any additional contextual information, such as time, place, the company of other people (e.g., for watching movies). In other words, traditionally recommender systems have dealt with applications that have two types of entities, users and items. In such cases, the recommendation process starts with the specification of the initial set of ratings that is either explicitly provided by the users or is implicitly inferred by the system. For example, in case of a movie recommender system, John Doe may assign a rating of 7 (out of 10) for the movie “Gladiator,” i.e., \( R_{\text{movie}}(\text{John Doe}, \text{Gladiator}) = 7 \). Once these initial ratings are specified, a recommender system tries to estimate the rating function \( R \) for the \((user, item)\) pairs that have not been rated yet by the users. Here \( Rating \) is a totally ordered set (e.g., non-negative integers or real numbers within a certain range). Once function \( R \) is estimated for the whole \( User \times Item \) domain, a recommender system can recommend the highest-rated item (or \( k \) highest-rated items) for each user.

While the current generation of recommendation technologies performs well in several applications, including the ones for recommending books, CDs, and news articles [10,13], the traditional two-dimensional (2D) framework described in Equation (1) is restrictive and is not sufficient to capture the intricacies of more complex applications, such as recommending vacations or financial services, and needs to be extended in order to overcome its inherent limitations. For this purpose, in the prior research a multidimensional (MD) approach to recommendations has been proposed, where the traditional recommendation framework (1) is extended to support additional dimensions capturing the context in which recommendations are made [1]. This multidimensional approach explores the synergies between recommender systems and the multidimensional data model used for data warehousing and On-Line Analytical Processing (OLAP) applications in databases [8] and provides recommendations not only over the two traditional dimensions (i.e., \( User \) and \( Item \)), as the classical (2D) recommender systems do, but over several additional dimensions, such as Time, Place, etc.

More formally, given dimensions \( D_1, D_2, \ldots, D_n \), the recommendation space for these dimensions is defined as a Cartesian product \( S = D_1 \times D_2 \times \ldots \times D_n \). Moreover, let \( Rating \) be a rating domain representing the ordered set of all possible rating values. In order to take into consideration the contextual information (e.g., the date, time, and the companion in movie recommendations), the utility function (or rating function) \( R \) over a multidimensional space \( D_1 \times D_2 \times \ldots \times D_n \) (as opposed to the traditional 2-dimensional \( User \times Item \) space) is defined as:

\[
R: D_1 \times D_2 \times \ldots \times D_n \rightarrow Rating
\]

An important research question is how to estimate unknown ratings in a multidimensional recommendation space. As in traditional recommender systems, the key problem in multidimensional systems is the extrapolation of the rating function from
a (usually small) subset of ratings that are specified by the users. Since there has been much work done on estimating these ratings for the 2D case, it would be advantageous to leverage this work. One natural approach would be to reduce the multidimensional rating estimation problem to the 2D case. This approach was proposed in [1], where it was call the reduction-based approach. Alternatively to the reduction-based approach, it may be possible to extend some existing 2D methods to the multidimensional case. Since the 2D methods are often broadly classified into heuristic- and model-based approaches [6], we follow this classification when considering the extension of 2D methods to the MD recommendation space. Therefore, combining the reduction- and the extension-based approaches, we can classify multidimensional rating estimation methods into: (a) reduction-based, (b) heuristic-based, and (c) model-based.

In the rest of this paper we discuss the above three classes of multidimensional rating estimation (in Sections 2, 3, and 4, respectively), provide examples of possible estimation methods, and outline several directions for future research. The contributions and the conclusions of the paper are presented Section 5.

2 Reduction-Based Approach

The reduction-based approach [1] is arguably the most straightforward way to apply traditional two-dimensional rating estimation methods for multidimensional recommendations. Using this approach, the problem of multidimensional recommendations is reduced to the traditional two-dimensional User×Item recommendation space, and, therefore, any available 2D rating estimation method can be applied directly in this case. The reduction can be done in several ways. Consider the multidimensional recommender system with three dimensions User, Item, and Time, and assume that \( R^D: \text{User} \times \text{Item} \rightarrow \text{Rating} \) is a two-dimensional rating estimation function that, given existing ratings \( D \) (i.e., \( D \) contains records \(<\text{user}, \text{item}, \text{rating}>\) for each of the user-specified ratings), can calculate a prediction for any rating, e.g., \( R^D(\text{"John"}, \text{"Dow-JonesReport"}) = 5 \). A 3-dimensional rating prediction function supporting time can be defined similarly as \( R^D: \text{User} \times \text{Item} \times \text{Time} \rightarrow \text{Rating} \), where \( D \) contains records \(<\text{user}, \text{item}, \text{time}, \text{rating}>\) for the user-specified ratings. Then the 3-dimensional prediction function can be expressed through a 2-dimensional prediction function as follows:

\[
\forall (u,i,t) \in \text{User} \times \text{Item} \times \text{Time}, \quad R^D(u,i,t) = R^D(\text{Time}=t)(\text{User}, \text{Item}, \text{Rating})(u,i) \tag{3}
\]

where \( D(\text{Time}=t)(\text{User},\text{Item},\text{Rating}) \) denotes a rating set obtained from \( D \) by selecting only the records where \( \text{Time} \) dimension has value \( t \) and keeping only the values for \( \text{User} \) and \( \text{Item} \) dimensions as well as the value of the rating itself. In other words, if we treat a set of 3-dimensional ratings \( D \) as a relation, then \( D(\text{Time}=t)(\text{User},\text{Item},\text{Rating}) \) is simply another relation obtained from \( D \) by performing two relational operations: selection and, subsequently, projection. “Time = \( t \)” is called a contextual segment for this recommendation application.

For example, in order to predict how John would like the Dow Jones Report in the morning, i.e., in order to calculate \( R^D(\text{"John"}, \text{"DowJonesReport"}, \text{"Morning"}) \), the
reduction-based approach would proceed as follows. First, it would eliminate the
Time dimension by selecting only the morning ratings from the set of all ratings D.
As a result, the problem is reduced to the standard User×Item case on the set of morning ratings. Then, using any traditional 2D rating estimation technique [2], we can calculate how John likes the Dow Jones Report based on the set of these morning ratings. In other words, this approach would use the two-dimensional function \( R^{D[\text{Time}=\text{morning}]}(\text{User, Item, Rating}) \) to estimate ratings for the User×Item domain. The intuition behind this approach is simple: if we want to predict a “morning” rating for a certain user and a certain content item, we should consider only the previously specified “morning” ratings for the rating estimation purposes.

Note, that in this example we chose to eliminate the Time dimension and left the traditional User and Item dimensions for the two-dimensional estimation of ratings. However, one could choose to eliminate other dimensions instead of Time, e.g., User or Item. Which dimension(s) should be eliminated in the reduction-based approach constitutes an interesting problem for future research. Furthermore, the reduction-based algorithm [1] uses the same two-dimensional recommendation method (e.g., traditional memory-based collaborative filtering [6]) for all discovered contextual segments. This approach can be extended to incorporate several two-dimensional recommendation methods, since potentially different recommendation algorithms can perform the best on different contextual segments. Finally, another way to improve the reduction-based approach is by “extending” the notion of contextual segments to include not only the segments of contextual dimensions, such as Time and Place, but also the segments defined by “slicing” the main dimensions, such as User and Item. For example, segment Frequent-Moviegoers, consisting of the users who usually watch a lot of movies, may be very interesting from the marketing perspective and should deserve a special treatment. While it is not strictly a contextual segment since it is defined in terms of the user characteristics and not in terms of the contextual dimensions Time, Place, and Companion, the definition of contextual segments can be straightforwardly extended to include such segments.

The main benefit of the reduction-based approach is that all the previous research on two-dimensional recommender systems is directly applicable in the multidimensional case. On the other hand, only a part of data (i.e., two-dimensional “slices” of the multidimensional space) is actively involved in the rating prediction process.

3 Heuristic-Based Approaches

As mentioned before, the reduction-based approach is not the only one for estimating multidimensional ratings. In fact, several traditional two-dimensional recommendation methods could be directly extended to the multidimensional case.

A large number of commonly used recommender systems techniques are two-dimensional heuristic-based collaborative filtering methods [6,9,11,12,14] and some of them may be extended to multiple dimensions. More specifically, in many traditional collaborative filtering methods the prediction of rating \( r_{ui} \) (i.e., the rating how user \( u \) would like item \( i \)) is computed as a weighted sum of the ratings given to the same item \( i \) by similar users \( u' \):
\[ r_{u,i} = k \sum_{u' \in \text{User}} \text{sim}(u,u') \times r_{u',i} \]  

(4)

where \( k \) is a normalizing factor. In other words, it constitutes a “nearest neighbors” problem, where we need to discover the “closest” neighbors among those who rated a given item.

Furthermore, if we look at the traditional two-dimensional content-based recommender systems \([4,10]\), some of the heuristic algorithms used in such systems also employ the nearest neighbor approach, although in a different manner. In particular, the prediction of rating \( r_{u,i} \) is computed as an aggregation (e.g., a weighted sum) of the ratings given by the same user \( u \) to similar items \( i' \). Therefore, the prediction of a particular rating \( r_{u,i} \) in many heuristic-based approaches in two-dimensional recommender systems (both collaborative and content-based) can be represented by Fig. 1, which can be interpreted as follows.

Fig. 1. Data used by nearest neighbor-based approaches in the traditional two-dimensional heuristic-based recommender systems

Let’s assume that we want to predict rating \( r_{u,i} \), which is depicted as the origin point in the figure. The horizontal axis represents all the users \( u' \) arranged according to their similarity to user \( u \) using some distance measure, and the vertical axis depicts all the items \( i' \) according to their similarity to item \( i \). All the known data (i.e., all user/item pairs with a known rating) can be described as points in the resulting two-dimensional space. Furthermore, as mentioned earlier, many two-dimensional heuristic-based collaborative filtering systems would use the ratings on the horizontal axis (i.e., how users that are similar to \( u \) rated item \( i \)) to predict rating \( r_{u,i} \). Similarly, the content-based systems usually use the ratings on the vertical axis (i.e., how items that are similar to item \( i \) were rated by user \( u \)) to predict rating \( r_{u,i} \). The hybrid heuristic-based approaches typically combine the collaborative and content-based ideas \([2,4,7]\). However, virtually all of them use some combination of the information represented
by the two axes. Therefore, user-specified ratings $r_{u,i'}$ (where $u' \neq u$ and $i' \neq i$) lying
outside of the two axes are usually not used for the prediction of $r_{u,i}$ (e.g., in the
weighted sum (4)) in the traditional heuristic-based recommendation approaches.1

Based on this discussion, we can generalize this two-dimensional heuristic
approach to multiple dimensions as follows. First, we can generalize this approach by
introducing a two-dimensional distance metric between two arbitrary rating points
$(u,i)$ and $(u',i')$ in the entire User×Item space. By using the entire two-dimensional
space in the prediction process instead of just the data on the two axes (as shown in
Fig. 1) we will be able to (a) incorporate collaborative and content-based approaches
as special cases and (b) identify additional nearest neighbors that lie outside of the
User and Item axes and that were not even considered by collaborative and content-
based approaches. Arguably, by identifying extra nearest neighbors that were not
considered before, we should increase the predictive accuracy of recommendations.
The choice of a specific distance metric (Euclidian, etc.) depends largely on a specific
application domain. For example, one metric may work better for recommending
movies and another metric for news articles. Identification of appropriate metrics for
different applications constitutes an interesting problem for future research.

Second, we can extend the two-dimensional nearest neighbor approach described
above to multidimensional recommendation spaces (i.e., that include contextual in-
formation) in a straightforward manner by using an $n$-dimensional distance metric
instead of a two-dimensional metric mentioned above. To see how this is done, con-
sider an example of the User×Item×Time recommendation space. Following the
traditional nearest neighbor heuristic that is based on the weighted sum, the prediction
of a specific rating $r_{u,i,t}$ in this example can be expressed as:

$$r_{u,i,t} = k \sum_{(u',i',t') \in N_{u,i,t}} W((u,i,t),(u',i',t')) r_{u',i',t'}$$  \hspace{1cm} (5)

where $W((u,i,t),(u',i',t'))$ describes the “weight” rating $r_{u',i',t'}$ carries in the prediction of
$r_{u,i,t}$, and $k$ is a normalizing factor. Weight $W((u,i,t),(u',i',t'))$ is typically inversely
related to the distance between points $(u,i,t)$ and $(u',i',t')$ in multidimensional space,
i.e., $\text{dist}((u,i,t),(u',i',t'))$. In other words, the closer the two points are (i.e., the smaller
the distance between them), the more weight $r_{u',i',t'}$ carries in the weighted sum (5).

One example of such relationship would be $W((u,i,t),(u',i',t')) = 1 / \text{dist}((u,i,t),(u',i',t'))$, but many alternative specifications are also possible. As before,
the choice of the specific distance metric $\text{dist}$ is likely to depend on a specific application.
This idea is depicted in Fig. 2, where we have a three-dimensional recom-

mender system with User, Item, and Time dimensions, and where ratings, equidistant
from the rating to be predicted are schematically represented with concentric spheres.

The distance function $\text{dist}$ can be defined in various ways. One of the simplest
ways to define a multidimensional $\text{dist}$ function is by using the reduction-like ap-
proach (similar to the one described in Section 2), by taking into account only the
points with the same contextual information, i.e.,

1 In collaborative filtering systems, $r_{u,i'}$ may be used for computing the similarity between two
users, but it is usually absent in the weighted sum that represents the predicted rating.
This distance function makes $r_{u,i,t}$ depend only on the ratings from the segment of points having the same values of time $t$. Therefore, this case is reduced to the standard 2-dimensional rating estimation on the segment of ratings having the same context $t$ as point $(u,i,t)$. Furthermore, if we further refine function $\text{dist}[(u,i),(u',i')]$ in (6) so that it depends only on the distance between users when $i = i'$, then we would obtain a method that is similar to the reduction-based approach described in Section 2. Moreover, this approach easily extends to an arbitrary $n$-dimensional case by setting the distance $d$ between 2 rating points to $\text{dist}[(u,i),(u',i')]$ if and only if the contexts of these two points are the same.

![Fig. 2. Data used by nearest neighbor-based approaches in the multidimensional heuristic-based recommender systems](image)

Other ways to define the distance function would be to use the weighted Manhattan distance metric, i.e.,

$$
\text{dist}[(u,i,t),(u',i',t')] = w_1d_1(u,u') + w_2d_2(i,i') + w_3d_3(t,t'),
$$

or the weighted Euclidean distance metric, i.e.,

$$
\text{dist}[(u,i,t),(u',i',t')] = \sqrt{w_1d_1^2(u,u') + w_2d_2^2(i,i') + w_3d_3^2(t,t')} ,
$$

where $d_1$, $d_2$, and $d_3$ are distance functions defined for dimensions User, Item, and Time respectively, and $w_1$, $w_2$, and $w_3$ are the weights assigned for each of these dimensions. In summary, distance function $\text{dist}[(u,i,t),(u',i',t')]$ can be defined in many different ways, and it constitutes an interesting research problem to identify various ways to define this distance and compare these different ways in terms of predictive performance.
4 Model-Based Approaches

There have been several model-based recommendation techniques proposed in recommender systems literature for the traditional two-dimensional recommendation model [3,5,6,15]. Some of these methods can be directly extended to the multidimensional case, such as the method proposed in [3], who show that their 2D technique outperforms some of the previously known collaborative filtering methods.

The method proposed by [3] combines the information about users and items into a single hierarchical regression-based Bayesian preference model that uses Markov Chain Monte Carlo (MCMC) techniques for exact estimation and prediction. In particular, this Bayesian preference model allows statistical integration of the following types of information useful for making recommendations of items to users: a person’s expressed preferences (ratings), preferences of other consumers, expert evaluations, item characteristics, and characteristics of individuals. For example, for recommending movies this information may include known movie ratings, gender and age of users, movie genres, and movie reviews by critics. Formally, [3] propose a linear model of rating estimation:

\[
\begin{align*}
    r_{ui} &= x_{ui}'\mu + z_{ui}'\gamma_i + w_i'\lambda_u + e_u, \\
    e_u &\sim N(0, \sigma^2), \\
    \lambda_u &\sim N(0, \Lambda), \\
    \gamma_i &\sim N(0, \Gamma)
\end{align*}
\]

where \(r_{ui}\) represents the rating of user \(u\) (movie-goer) for item \(i\) (which will be used interchangeably with “movie”). The vector \(x_{ui}\) represents all the observed parameters for user \(u\), item \(i\), and their interactions. Observed parameters would be the parameters explicitly recorded in the profiles of the users, such as gender, age, and movie preferences, and in the profiles of items, such as genre, year and expert ratings of a movie. Interaction terms represent second-order interaction effects between observed user and item attributes. For example, there can be an interaction term in \(x_{ui}\) representing second-order effects between gender and genre (e.g., even though overall romantic movies appear to get higher ratings than action movies, men might rate action movies higher than romantic movies). The coefficient of \(x_{ui}\) is \(\mu\) which represents the fixed effects in the regression. The term \(z_{ui}\) contains the observed attributes of user \(u\), such as gender, age, and movie preferences, and \(w_i\) represents the observed attributes of item \(i\) such as genre, year and expert ratings. The unobserved effects of the movies, that are not explicitly recorded in the movie profiles, such as direction, music, acting, etc., are captured by the vector of random variables \(\gamma_i\) that is normally distributed as \(\gamma_i \sim N(0, \Gamma)\). Unobserved user-effects are represented by \(\lambda_u \sim N(0, \Lambda)\).

The error term \(e_u\) belongs to a normal distribution with mean zero and standard deviation \(\sigma\). \(x_{ui}'\), \(z_{ui}'\) and \(w_i'\) denote the transposes of the corresponding vectors. The parameters \(\mu, \sigma^2, \Lambda\) and \(\Gamma\) of model (7) are estimated from the data of already known ratings using MCMC methods.

While the approach presented in [3] is described in the context of traditional two-dimensional recommender systems, it can be directly extended to combine more dimensions (i.e., the contextual information) in addition to users and items. For example, assume that we have a third dimension Time that is defined by the following two attributes (variables): (a) Boolean variable weekend specifying whether a movie was
seen on a weekend or not, and (b) a positive integer variable numdays indicating the number of days after the release when the movie was seen.

In such a case, the (3) model can be extended to the third (Time) dimension as

\[ r_{uit} = x_{uit}' \mu + p_{ui}' \delta_i + q_{ui}' \gamma_i + r_{ui}' \theta_t + w_{ui}' \pi_u + y_{ui}' \sigma_i + e_{uit} \]  

(8)

where \( e_{uit} \sim N(0, \sigma^2) \), \( \gamma_i \sim N(0, \Gamma) \), \( \lambda_u \sim N(0, \Lambda) \), \( \theta_t \sim N(0, \Theta) \), \( \delta_i \sim N(0, \Delta) \), \( \pi_u \sim N(0, \Pi) \), and \( \sigma_{ui} \sim N(0, \Sigma) \).

This model encompasses the effects of observed and unobserved user-, item- and temporal-variables and their interactions on rating \( r_{uit} \) of user \( u \) for movie \( i \) seen at time \( t \). The variables \( z_u \), \( w_i \), and \( y_t \) stand for the observed attributes of users (e.g. demographics), movies (e.g. genre) and time dimension (e.g. weekend, numdays). The vector \( x_{uit} \) represents the observed parameters of users, movies and time, and their interaction effects, and its coefficient \( \mu \) represents the fixed effects in the regression. The vectors \( \lambda_u \), \( \gamma_i \) and \( \theta_t \) are random effects that stand for the unobserved sources of heterogeneity of users (e.g. their ethnic background), movies (e.g. the story, screenplay, etc.) and temporal effects (e.g. was the movies seen on a holiday or not, the season when it was released, etc). The vector \( p_{ui} \) represents the interaction of the observed user and item variables, and likewise \( q_{ui} \) and \( r_{ui} \). The vector \( \sigma_{ui} \) represents the interaction of the unobserved user and item attributes, and likewise \( \pi_{ui} \) and \( \delta_{ui} \). Finally, the parameters \( \mu, \sigma^2, \Lambda, \Gamma, \Theta, \Delta, \Pi \) and \( \Sigma \) of the model can be estimated from the data of already known ratings using Markov Chain Monte Carlo (MCMC) methods as was done in (7). [3] assume that the features (attributes) in vectors \( z_u \), \( w_i \) and \( y_t \) are already specified. In more general cases, these features need to be selected, and various machine learning methods can be used for this purpose.

Finally, note that the number of parameters in the above model that would need to be estimated rises with the number of dimensions and, therefore, the sparsity of known ratings may become a problem. Therefore, one of the research challenges is to make such models scalable for the sparse contexts with a large number of dimensions.

5 Conclusions

Traditionally, the vast majority of recommender systems technologies has been focusing on recommending items to users or users to items and typically do not take into the consideration any additional contextual information, such as time or place. In this paper we discuss a multidimensional approach to recommendations which supports additional dimensions capturing the context in which recommendations are made. One of the most important questions in recommender systems research is how to estimate unknown ratings, and in the paper we address this issue for a multidimensional recommendation space.

The contributions of this paper include: (a) the classification of multidimensional rating estimation methods into reduction-, heuristic-, and model-based approaches, (b) the discussion on how to extend the traditional two-dimensional heuristic- and model-based recommendation approaches to the multidimensional recommendation space, and (c) the identification of various research directions for the multidimensional rating estimation problem.
We believe that our classification of multidimensional rating estimation methods represents the natural evolution of research in this area. The area of multidimensional recommender systems is very young and there have been no real-life implementations of multidimensional heuristic- or model-based methods. Therefore, the obvious choice at this point is to reduce the multidimensional rating estimation problem to the standard two-dimensional one, for which there is a large number of estimation algorithms available. Not surprisingly, the only implementation of multidimensional recommender systems has been reduction-based [1], which already demonstrated performance improvements over the traditional two-dimensional recommender systems. Eventually, multidimensional heuristic-based rating estimation methods will be developed and implemented, and they are expected to outperform reduction-based approaches, since they will be able to take into account the contextual dimensions directly (unlike the reduction-based approaches operating with 2D slices of the recommendation space). Finally, since the model-based rating estimation methods typically outperform heuristic-based approaches in the traditional 2D recommendation space [6], we expect that the same would happen in the MD case, i.e., the model-based approaches would eventually outperform the heuristics, especially when they address the sparsity and scalability issues, as mentioned in Section 4.

In summary, we believe that context-aware recommender systems constitute an important and problem-rich research area and that the multidimensional rating estimation methods will play a significant role in their development.

References