TRANSMISSION OF A MESSAGE DURING LIMITED TIME WITH THE HELP OF TRANSPORT CODING

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Abstract: A fundamental characteristic of the majority of communications networks is the mean message delay. In a packet-switching network, the mean packet and message delay may differ considerably from each other, and their distribution will often take different forms. The mean message delay depends both on the mean packet delay and on the dispersion of the packet delay. Obviously, by reducing the mean packet delay one can also reduce the message delay. However, it is not always possible to decrease the mean packet delay in the network. A proposed method of transmitting data in a network is based on the use of error-correcting coding, which reduces the dispersion of the packet delay with some increase in the mean packet delay. The conditions were obtained for which an increase in the mean packet delay with simultaneous reduction in the dispersion leads to a reduction in the mean message delay. In many real time networks there are exist some restrictions on the message delay. Use of the transport coding makes it possible to deliver the messages over the network during some limited time with high probability.

1 INTRODUCTION

In this paper we will consider the applications of error correcting codes to the data network. It is well known, that using of error controlling codes adapted to typical errors in a defended system is the universal method of error protection. However in modern data networks error correcting (or controlling) codes are used only as means of increasing the reliability of information during the data transmission over the different channels; and no correlation between coding and other network procedures is considered. The application of coding not only to the physical layer but also to the procedures at higher layers (e.g. transport layer) gives us some unexpected results indicating that coding in network helps not only to increase the reliability of the transmitted information, but also can be used to improve such important characteristic of a network, as the mean message delay. In this chapter we will consider mostly the packet switching network with datagram routing (or in datagram mode). *Packet switching* is switching in which messages are broken into packets and one packet at a time is transmitted on a communication link. Thus, when a packet arrives at a switching node on its path to the destination site, it waits in a queue for its turn to be transmitted on the next link in its path. The *datagram routing* is packet switching in which each packet finds its own path through the network according to the current information available at the nodes visited (Bertsekas, Gallager, 1992). To be more precise, there is only one restriction on the considered network model, which is essential for the exposition of this chapter. This is the possibility to get the packets out of order at the destination node. It is shown in (Bertsekas, Gallager, 1992) that not only the datagram networks but also the virtual circuit networks have this feature as well. However, for simplicity we assume that we are dealing with a datagram network and that packets can get out of order arbitrarily on the network consider a packet switching network.

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The outline of the paper is as follows. Section 2 describes the concept of transport coding. Section 3 analyses the possibility of using transport coding for improving the probability of message delivery during limited time. Section 4 discusses an interpretation of the results.

2 DECREASING THE MESSAGE DELAY WITH THE HELP OF TRANSPORT CODING

The one of the most important measure of the effectiveness of a data network is the information delay. The mean packet delay has been subject to many studies, for example (Kleinrock, 1975), (Kleinrock, 1964), (Kleinrock, Naylor, 1974). However, in a packet-switching network, the parameter of interest is not the delay of a separate packet but the delay of a message as a whole. And the mean message delay can differ from the mean packet delay, as the assembly of a message at a destination node can be delayed due to the absence of a small number of packets (for example one). This section deals with an analysis of the method of decreasing the mean message delay with the help of error-correcting code at the transport level of network. This method was suggested in (Kabatianskii, Krouk, 1993) and generalized in (Krouk, Semenov, 2002). The possibility of using error-correcting code in a bipolar network was described in (Maxemchuk, 1975).

Let us consider a model of a network having M channels, in which the capacity of the ith channel is C_i. The time taken to transmit a packet over a channel has an exponential distribution with the expectation $1/\mu$. When the servicing device is busy, the packet may be placed in a queue. Each message, arriving in the network, is divided into K similar packets. The length of each packet is s bits. The traffic arriving in the network from external sources forms a Poisson process with the intensity γ (packets per second). We will denote the mean number of packets passing through the ith channel per second by λ_i. The total network traffic is then

$$\lambda = \sum_{i=1}^{M} \lambda_i \cdot, \qquad (1)$$

If the packets arrive to a node via different routes, we can assume that the dependence between packet delays is negligible. Hence, the model of the network turns out to be close to the Kleinrock model, for which the Kleinrock «assumption of independence» holds (Kleinrock, 1975), (Kleinrock, 1964). According to this assumption, the packet delays can be regarded as independent random variables. This statement was proved in (Vvedenskaya, 1998) for some network types. Then the ith channel can be represented in the form of a queuing system with a Poisson flow of intensity λ_i at the input and an exponential servicing time with mean $\frac{1}{\mu \cdot C_i}$. In this case we can assume that the packet delays in the network have an exponential distribution with the expectation $\bar{t}(\lambda, \mu)$, where

$$\bar{t}(\lambda,\mu) = \sum_{i=1}^{M} \frac{\lambda_i}{\gamma} \cdot \frac{1}{\mu C_i - \lambda_i} \cdot, \qquad (2)$$

If we consider a case where all M channels have the same carrying capacity while the external traffic is uniformly distributed between the channels (so that the intensity of the packet flow for all channels is the same), expression (2) can be written as follows:

$$\bar{t}(\lambda,\mu) = \frac{\bar{l}}{\mu C} \cdot \frac{1}{1-\rho}, \qquad (3)$$

where $\bar{l} = \frac{\lambda}{\gamma} = \frac{M \cdot \lambda_i}{\gamma}$ is the mean path length traversed by a packet along the network, $\rho = \frac{\lambda}{\mu \cdot C}$ is the network load, and $C = \sum_{i=1}^{M} C_i$ is the overall capacity of the network channels. The value of the network load in this case is identical with the $\rho_i = \frac{\lambda_i}{\mu \cdot C_i}$, the load of a single channel. In fact, as it will be shown later, all needed assumptions are as follows: the packet delays are independent random variables with the exponential distribution and with expectation of form $\frac{a}{1-\rho}$, where ρ is the network load and a is the constant for the given network.

The delay T of an uncoded message in the network is determined by the maximum delay among the K packets of the given message

$$T = \max\{t_1,...,t_K\},$$

where t_i is the delay of the ith packet of the message; i.e., the message delay is equal to the delay of the packet which arrives last. If we redenote the packet delays of the message in increasing order $t_{1:K} \le t_{2:K} \le ... \le t_{K:K}$, we have

$$T = t_{K:K}.$$

We can apply now the coding at the transport level of the network and to encode the message, which consists of K packets with the help of an MDS (N, K) code (for example Reed-Solomon code). In case of using the Reed-Solomon code each of K packets is considered an element of a field $GF(2^s)$ (s is the packet length), and after encoding the original message consisting of K packets is replaced by a message consisting of N packets. When the encoded messages are transmitting over the network, the traffic increases by a factor of $1/R$ ($R=K/N$ is the rate of the code used). This naturally leads to an increase in the mean packet delay in the network. However, at the node-addressee, to reconstruct the message (in view of properties of MDS codes), only K packets need to be received, as against all N packets. We will show that with some restrictions on the operation of the network this method leads to a decrease of the mean message delay. We will denote this method further on as *transport coding*.

In case of using transport coding, the delay of the encoded message is

$$T_{cod} = t_{K:N}.$$

Using the apparatus of order statistics (David, 1981), the mathematical expectation of the delay of the ith packet (for an overall number of packets N) can be written as follows:

$$E\left[t_{i:N}\right] = B_i \cdot \int_{-\infty}^{\infty} t \cdot \left[P(t)\right]^{i-1} \cdot \left[1 - P(t)\right]^{N-i} dP(t) \tag{4}$$

or

$$E\left[t_{i:N}\right] = B_i \cdot \int_0^1 P^{-1}(u) \cdot u^{i-1} \cdot \left[1 - u\right]^{N-i} du, \tag{5}$$

where $B_i = N \cdot \binom{N-1}{i-1}$, $P(t)$ is the distribution function of packet delay and $P^{-1}(u)$ is the inverse function of $P(t)$. In the case of an exponential distribution of the packet delay in the network equations (4), (5) can be rewritten as follows:

$$E[t_{i:N}] = \bar{t} \cdot \sum_{j=N-i+1}^{N} j^{-1} \tag{6}$$

where $\bar{t}$ is the mean packet delay in the network (depends on λ and μ). The mean delay of the uncoded message in the network is then

$$\overline{T}_1 = E[t_{K:K}] = \bar{t}(\lambda,\mu) \cdot \sum_{j=1}^{K} j^{-1} \tag{7}$$

where $\bar{t}(\lambda,\mu)$ is defined by (3). The sum on the right-hand side of (7) can be represented as follows:

$$\sum_{j=1}^{K} j^{-1} = \varepsilon + \ln K + \frac{1}{2K} - \sum_{i=2}^{\infty} \frac{A_i}{K \cdot (K-1) \cdot ... \cdot (K+i-1)},$$

$$A_i = \frac{1}{i} \cdot \int_0^1 x \cdot (1-x) \cdot ... \cdot (i-1-x) dx,$$

where $\varepsilon = 0.577...$ is Euler's constant. Hence we obtain the following estimate for the mean delay of the uncoded message $\overline{T}_1$:

$$\overline{T}_1 \ge \bar{t}(\lambda,\mu) \cdot (\varepsilon + \ln K) = \frac{\bar{l}}{\mu C} \cdot \frac{1}{1-\rho} \cdot (\varepsilon + \ln K) \tag{8}$$

We can write the mean delay of the coded message $\overline{T}_2$ for given N, in accordance with (6), as follows:

$$\overline{T}_2 = E[t_{K:N}] = \bar{t}(\lambda/R,\mu) \cdot \sum_{j=N-K+1}^{N} j^{-1}, \tag{9}$$

where $\bar{t}(\lambda/R,\mu)$ is the mean packet delay for traffic that has been increased as a result of using coded messages by a factor of $1/R$; $R = K/N$ is the rate of the code used. The sum on the right-hand side of (9) can be represented as follows:

$$\sum_{j=N-K+1}^{N} j^{-1} = \sum_{j=1}^{N} j^{-1} - \sum_{j=1}^{N-K} j^{-1} = \ln\left(\frac{N}{N-K}\right) + \frac{1}{2N} - \frac{1}{2(N-K)} +$$

$$+ \sum_{i=2}^{\infty}\left(\frac{A_i}{(N-K)...(N-K+i-1)} - \frac{A_i}{N...(N+i-1)}\right) \le$$

$$\le \ln\left(\frac{N}{N-K}\right) \tag{10}$$

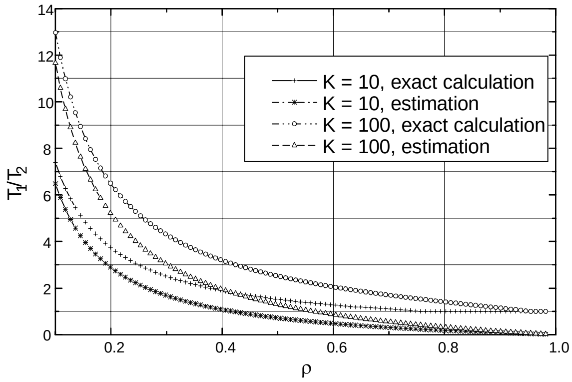

Figure 1: A gain of transport coding vs. network load.

It is possible to choose the code rate R in such a way as to minimize the mean message delay $\overline{T}_2$. For the best-chosen code, we obtain

$$\overline{T}_2 = \min_R \left\{ \bar{t}(\lambda / R, \mu) \cdot \sum_{j=K/R-K+1}^{K/R} j^{-1} \right\}. \tag{11}$$

Then, with the help of (10) we can estimate (9) as

$$\overline{T}_2 \le \min_R \left\{ \bar{t}(\lambda / R, \mu) \cdot \ln\left(\frac{1}{1-R} \right) \right\}. \tag{12}$$

The mean packet delay for traffic which has been increased by a factor of $1/R$ can be written in accordance with (3), as

$$\bar{t}(\lambda / R, \mu) = \frac{\bar{l}}{\mu C} \cdot \frac{R}{R-\rho}, \tag{13}$$

where $\bar{l} = \dfrac{\lambda/R}{\gamma/R} = \dfrac{\lambda}{\gamma}$ is the mean path length traversed by a packet along the network and $\rho = \dfrac{\lambda}{\mu \cdot C}$ is the load of the network when using the uncoded messages. Minimizing (12) with respect to R, obtain

$$\overline{T}_2 \le \frac{\bar{l}}{\mu C} \cdot \frac{4\rho}{(1-\rho)^2}. \tag{14}$$

The gain of using coding at the transport level of the network can be obtained when the following condition is satisfied:

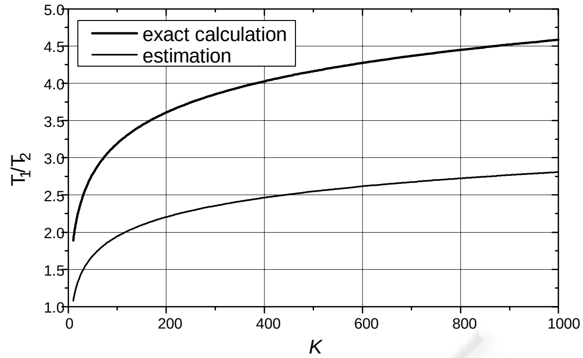

Figure 2: A gain of transport coding vs. number of information packets in message.

$$\overline{T}_1 - \overline{T}_2 > 0. \tag{15}$$

Substituting (8) and (14) into (15), we obtain the following condition for gain of using coding at transport level

$$\varepsilon + \ln K > \frac{4\rho}{1-\rho}. \tag{16}$$

The plots of gain of transport coding (in the sense of decrease the mean message delay) are shown in Fig. 1, 2.

As one can see from Fig. 1. exact calculation shows that gain of transport coding can be obtained with wider range of network load than it follows from condition (16). However, the estimation reflects the proper tend of changing gain versus network load. The plot in Fig. 2 shows that increase of number of information packets in message leads to the gain increase that has logarithmic behaviour.

3 MESSAGE DELIVERY DURING LIMITED TIME

For many data networks the probability $P(T_0)$ of message delivery during the time no more than T_0 has the same importance as the mean message delay. Let us show that in this case transport coding also can be used to increase $P(T_0)$. Let $p = \Pr\{t \le T_0\}$ denote the probability of message delivery during the time no more than T_0 and let p_R denote the same probability for the encoded message having regard to the increased network load. Then for the uncoded messages we have

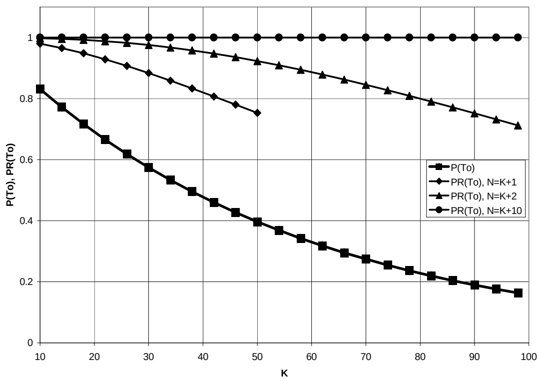

Figure 3: A probability of message delivery during time T_0 vs. number of information packets in message, a = 4, ρ = 0.2.

$$P(T_0) = p^K \tag{17}$$

and in case of using the encoded messages (code length is N) the probability $P^{(R)}(T_0)$ that the encoded message is delivered during the time no more than T_o can be written as follows

$$P^{(R)}(T_0) = \sum_{i=0}^{N-K} \binom{N}{i} \cdot \left(1 - p_R\right)^i \cdot p_R^{N-i} \tag{18}$$

Now if we will use the same assumptions as in Section 1 about exponential distribution of packet delay and about dependence of the mean packet delay on the network load (3) we obtain the following expression for p:

$$p = 1 - e^{-T_0/\bar{t}(\lambda,\mu)} \tag{19}$$

where $\bar{t}(\lambda, \mu)$ is the mean packet delay defined by (3). Let denote as a the ratio $T_0 / \bar{t}(\lambda, \mu)$ and as ξ the ratio of the mean packet delay in the ordinary network without transport coding to the mean packet delay in the network with transport coding, $\xi = \bar{t}(\lambda, \mu)/\bar{t}(\lambda / R, \mu) = (1 - \rho R^{-1})/(1 - \rho)$. Then formulas (17) and (18) can be rewritten as follows:

$$P(T_0) = \left(1 - e^{-a}\right)^K \tag{20}$$

$$P^{(R)}(T_0) = \sum_{i=0}^{N-K} \binom{N}{i} \cdot \left(1 - e^{-a\xi}\right)^{N-i} \cdot e^{-ia\xi} \tag{21}$$

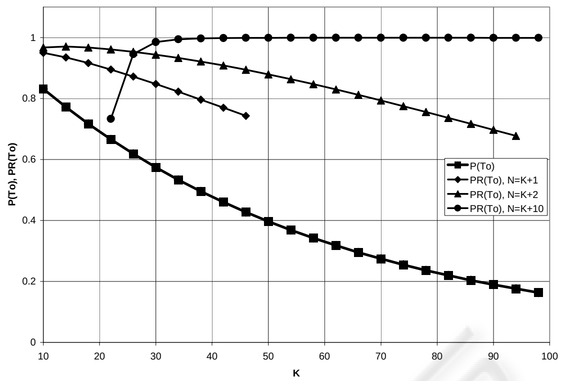

Figure 4: A probability of message delivery during time T_0 vs. number of information packets in message, a = 4, ρ = 0.6.

It is easy to verify that $P(T_0) \to 0$ with increasing the number of packets in the message K. From the other hand, in case $(1 - R) > e^{-a\xi}$, $P^{(R)}(T_0) \to c$, $(0 < c \leq 1)$ with increasing K. The condition $(1 - R) > e^{-a\xi}$ with the restriction $R > \rho$ can be written as the following inequality

$$\rho < R < 1 - e^{-a\frac{1-\rho/R}{1-\rho}}. \tag{22}$$

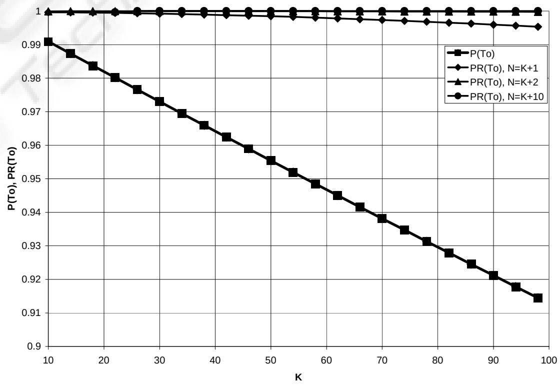

Figure 5: A probability of message delivery during time T_0 vs. number of information packets in message, a = 7, ρ = 0.6.

Thus, for any R satisfying (22) the addition of N-K redundant packets to the message leads to the fact that the probability $P^{(R)}(T_0)$ tends to the constant greater than zero, whilst the probability $P(T_0)$ tends to zero with increasing K. The plots of $P(T_0)$ and $P^{(R)}(T_0)$ against K are represented in Fig. 3 - 5.

4 CONCLUDING REMARKS

The given estimates of mean message delays in the network are rather rough. However, even these estimations show that transport coding leads to a significant decrease of mean message delay under conditions of moderate network load. Moreover, it is possible to use transport coding not only to decrease the mean message delay, but also to increase the reliability of message delivery during limited time, which also is a question of great interest. All given estimations and exact formulas given are based on the assumption that packet delay in a network has an exponential distribution. Although this assumption has some grounding in (Kleinrock, 1975), (Vvedenskaya, 1998) for many kinds of networks, it is possible that in some networks the distribution of the packet delay differs from the exponential one. However, this assumption was used only for the simplification of calculations. It is necessary to note that an exponential distribution of packet delay is not the best case for use of transport coding because the probability of a high packet delay is very small. We can therefore expect that the gain of transport coding could be more significant for another distribution of packet delay.

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