AN IMPROVED GENETIC ALGORITHM FOR SOLVING THE VEHICLE ROUTING PROBLEM WITH TIME WINDOWS

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Abstract: Many practical transport logistics and distribution problems can be formulated as the vehicle routing problem with time windows (VRPTM). The objective is to design an optimal set of routes that services all customers and satisfies the given constraints, especially the time window constraints. The complexity of the VRPTW requires heuristic solution strategies for most real-life instances. However, the VRPTM is a combination optimization problem and is a NP-complete problem, so we can't get satisfying results when we use exact approaches and normal heuristic ones. In this paper, an improved genetic algorithm to solve the VRPTM problem is developed, which use an improved Route Crossover operator (RC') and can meet the needs for solving VRPTM problem. Computational experiments show that the GA based on RC' can obtain a general optimality for all evaluated indexes on the premise of satisfying every customer's demand and its performance is superior to the GA based on PMX or RC.

1 INTRODUCTION

Many practical transport logistics and distribution problems can be formulated as the vehicle routing problem with time windows (VRPTM) which objective is to obtain a minimum-cost route plane serving a set of customers with known demands. Each customer is assign to exactly one vehicle route and the total demand of any route must not exceed the vehicle capacity. In addition, a time window is associated with each customer defining an interval wherein the customer has to be supplied. Specific examples of VRPTM include bank deliveries, postal deliveries and situations where the customer must provide access, verification, or payment upon delivery of the product or service (Solomon and Desrosiers, 1991).

The VRPTM is a NP-complete problem (Lenstra and Rinnooy, 1981). Due to the intrinsic difficulty of the problem, search methods based on heuristics are most likely applied to solve practical size problems. Heuristic methods often obtain optimum or near optimum solutions for large problems in rational computing time. Therefore, during the last decades much effort has been devoted to the development of powerful heuristic algorithm (Savelsbergh, 1985) (Koskosidis, et. al 1992) (Desrosiers, et. al 1992) (Fisher, et. al 1997) (Kohl and Madsen, 1997) (Russel, 1995).

The Genetic Algorithm (GA) is an adaptive heuristic search method based on population genetic. The basic concepts were primarily developed by Holland (Holland, 1975). The practicality of using the GA to solve complex problem is demonstrated in (DeJong, 1980) (Grefenstette, 1986) (Goldberg, 1989). The GA tries to develop a viable solution for an optimization problem through the successive evolution of solutions that don't meet a criterion. It mimic the process of biological evolution where, over successive generations, individuals who adapt to the environment best live on and reproduce, while other individuals die off (Mitchell and Forrest, 1993). Eventually, over many generations, the group has been optimized and only individuals who adapt to the environment mostly survive.

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Many efficient methods to use GA heuristic for solving the VRPTM have been proposed recently (Bräysy, 2001). Thabgiah describes a method called GIDEON (Thangiah, et. al 1995) (Thangiah, 1995).Potvin and Bengio propose a genetic algorithm GENEROUS that applies genetic operators to solutions (Potvin and Gengio, 1996). Berger et al present a approach based on the hybridization of a GA with well-known construction heuristics (Berger, et. al 1998). Homberger and Gehring propose two evolutionary strategies for solving the VRPTM (Homberger and Gehring, 1999). Bräysy presents a comprehensive sensitivity analysis of the main components of the genetic algorithm in the VRPTW context by proposing several new crossover and mutation operators, testing different forms of genetic algorithms, selection schemes, scaling schemes, and the significance of the initial solutions (Bräysy, 1999). In (Bräysy, 2000), Bräysy describes a two-phase hybrid evolutionary algorithm based on the hybridization of a GA and an evolutionary algorithm consisting of several local search and route construction. Ombuki, Nakamura and Osamu present a hybrid search based on genetic algorithms and tabu search for vehicle routing (Ombuki, et.al 2003).

In this paper, we proposed an improved genetic algorithm based on improved Route Crossover (RC') to solve the VRPTM, which ensures to satisfy every customer's demand. its performance is superior to existing methods based on PMX or Route Crossover (RC) (Davis, 1991).

The remainders of this paper are organized as follows. In Section 2, we present the mathematical model of VRPTM. In Section 3, a heuristic method for solving VRPTM will be developed based on the principle of GA. The computational experiments will be presented in Section4, and Section 5 will conclude the paper.

2 MATHEMATICAL MODEL

The VRPTM consists of a large number of customers. Each customer has a known demand level, which must be supplied from a single depot. Delivery routes for vehicles are required, start and finish at the depot, so that all customers' demands are satisfied and each customer is visited just by one vehicle. Each vehicle capacities and its maximum distance that it can travel are given previously. A vehicle may arrive before the beginning of the time window and wait without cost until service is available. However, no vehicle may arrive past the closure of a given time interval. So the mathematical model of the VRPTM problem is formulated as follows:

VRPTM:

$$\text{Min}\ (Dis + Cov + Idt + no\_service) \quad (2.1)$$

$$Dis = \sum_{k\in V}\sum_{i\in N}\sum_{j\in N} c_{ij} X_{ijk} \quad (2.2)$$

$$Cov = \sum_{k\in V} w_k Z_k \quad (2.3)$$

$$Idt = \sum_{k\in V}\sum_{i\in N} IT_{ik} \quad (2.4)$$

Subject to:

$$\sum_{i\in C} d_i \sum_{j\in N} X_{ijk} \le q_k \quad \forall k\in V \quad (2.5)$$

$$\sum_{i\in N}\sum_{j\in N} c_{ij} X_{ijk} \le D_k \quad \forall k\in V \quad (2.6)$$

$$\sum_{k\in V}\sum_{j\in N} X_{ijk} = 1 \quad \forall i\in C \quad (2.7)$$

$$\sum_{j\in N} X_{0jk} = 1 \quad \forall k\in V \quad (2.8)$$

$$\sum_{i\in N} X_{ihk} - \sum_{j\in N} X_{hjk} = 0 \quad \forall h\in C, \forall k\in V \quad (2.9)$$

$$\sum_{i\in N} X_{i0k} = 1 \quad \forall k\in V \quad (2.10)$$

$$e_{ik} + t_{ij} - K(1 - X_{ijk}) \le s_{jk} \quad \forall i,j\in N, \forall k\in V \quad (2.11)$$

$$a_i \le s_{ik} \le b_i \quad \forall i\in N,\ \forall k\in V \quad (2.12)$$

$$Z_k,\ X_{ijk} \in \{0,1\} \quad \forall i,j\in N, \forall k\in V \quad (2.13)$$

$$X_{ijk} = \begin{cases} 1, & \text{if the vehicle k travels directly from i to j} \\ 0, & \text{otherwise} \end{cases} \quad (2.14)$$

$$Z_k = \begin{cases} 1, & \text{if vehicle k is used} \\ 0, & \text{otherwise} \end{cases} \quad (2.15)$$

where

V =a set of vehicles

C =a set of geographically dispersed customers

N =C and 0 (0 denotes the central depot)

c_{ij} =cost per unit amount transported from source i to destination j

t_{ij} =travel time between source i and destination j

q_k =maximum capacity of vehicle k

D_k =maximum travel distance permitted for vehicle k

w_k =cost of vehicle k once it is used

d_i =demander of customer i

a_i =earliest time allowed for delivery to customer i (the minimum of time window)

b_i =lastest time allowed for delivery to customer i (the maximum of time window)

IT_{ik} =waiting time of vehicle k at customer i when it arrives before a_i

no_service =number of customers not served

s_{ik} =time of vehicle k starting visiting customer i , s_{0k}=0

e_{ik}=time of vehicle k finishing visiting customer i , e_{0k}=0

T_i= e_{ik} -s_{ik}

Our objective is to minimize the total distance for each vehicle to travel, the cost used by each vehicle, the waiting time wasted to meet the demands of all the customers (2.1) without breaking the constraints of vehicle capacity, travel distance and arrival time. Thus, this problem can be treated as a multi-objective optimization problem. A feasible solution for the VRPTM is to serve all the customers without vehicle exceeding its maximum capacity (2.5) or its maximum travel distance (2.6). In addition, each customer can be served by one and only one vehicle (2.7). Constraints (2.8), (2.9) and (2.10) ensure that each vehicle starts from the depot and returns to it after finishing visit. The constraint (2.11) ensures that vehicle k can't arrive at destination j before time $e_{ik}+t_{ij}$ when it drives from source i to destination j, and K is a relative large scalar quantity. The constraint (2.12) enforces the arrival time of a vehicle at a customer site to be between the customers' earliest and latest arrival time.

3 AN IMPROVED GENETIC ALGORITHM FOR VRPTM

3.1 General Explanation

A GA is directly derived from the behavior of genes and chromosomes in nature. Each 'generation', or family of possible solutions, is made up of a set of strings or 'chromosomes'. Each chromosome is in turn made up of individual 'genes'. These genes are codes of designing variables that are used to evaluate the function being optimized. The GA calls a subroutine to compute the fitness value or normalized objective value for each chromosome in a population. These values are evaluated and compared with each other. Those chromosomes having the best values 'survive' and are passed on to the next generation. In each generation, there is a small probability for each chromosome to mutate (or change) in one or more positions in the string (gene). There is a different probability that two strings will mate or crossover to produce a child. The children and mutations are placed into the next generation. The process of mutation, crossover, evaluation and reproduction are repeated until it is convergent to a suitable 'solution' for the problem.

In following sections, we will introduce how we apply the genetic algorithm to the VRPTM.

3.2 Chromosome Representation

When we use the GA to solve VRPTM, designing a kind of chromosome representation for solution space is first problem to need solving. Here we represent each chromosome as a sequence of a set of vehicle route. A route is composed of a sequence of nodes (customers). For example, the chromosome: 1 5 2 4 3 6 8 7 is composed of three routes r1: D C1 C5 C2 D and r2: D C4 C3 C6 D and r3: D C8 C7 D (D denotes central depot, C denotes customer). The routing scheme given in reference (Ombuki, et. al 2003) is used to transform each chromosome into a set of routes.

3.3 Selection Operator

We use Roulette Wheel Selection to generate a new population for the next generation. Roulette Wheel Selection is an elite model which can ensure that the best individual can be preserved into the next generation.

3.4 Improved Crossover Operator

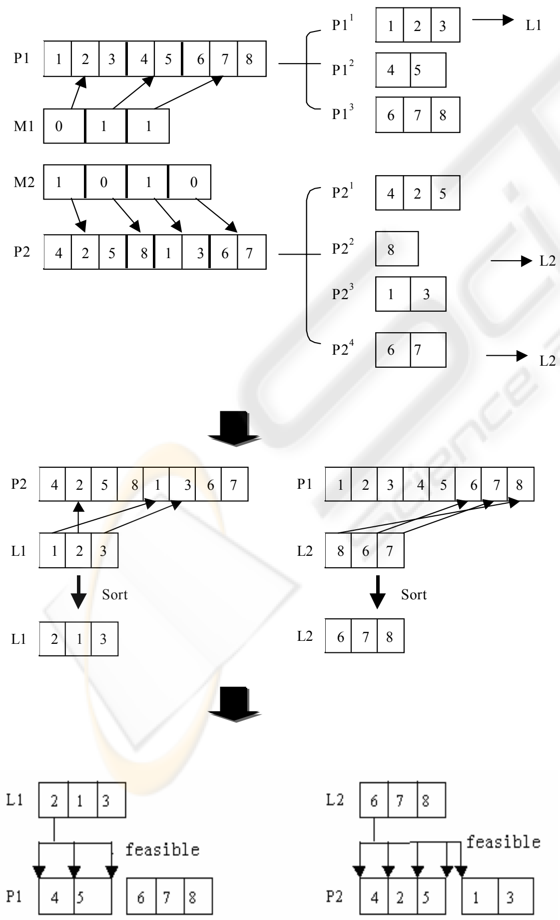
Since some customers may not be served when the Route Crossover (RC) (Davis, 1991) is used to solve VRPTM, we proposed the improved Route Crossover (RC'), which is an improvement of RC. It can optimize the problem on the premise of satisfying every customer's demand. RC' operation is presented as the following steps:

(1) Transform two parent chromosomes, P1 and P2, into two clusters of routes by the routing scheme mentioned above.

(2) A binary mask-pattern is generated randomly for each cluster of routes, whose length is equal to the number of the routes. One bit of the mask pattern corresponds to one route in the cluster of routes.

(3) Considering one parent Pi (i=1,2), the contents of the route(s) corresponding to a “1” in the mask pattern are copied directly to its offspring. The customers of the routes equivalent to “0” in the mask pattern are sorted out to form a list in the order of the appearance in its crossing parent Pj (j=1,2, j ≠ i).

(4) Considering the list in (3), each of the customers from the list is inserted according to their order of appearance to the first feasible position in the upcoming route represented by its respective offspring. The steps above are almost the same as RC, the only difference between the two operators is that if a customer can’t be inserted into any existent route, it will be declared as a non-served customer and will not be assigned to any route in RC. However in RC’, a new route will be generated and the customer will be inserted into it as well.



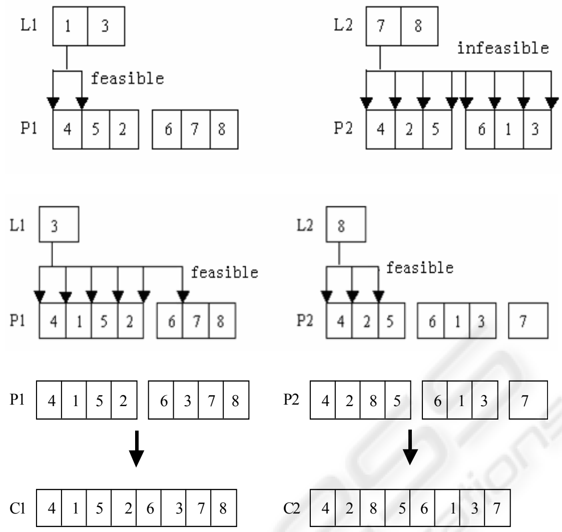


Figure 1: Improved Route Crossover(RC’) operator

3.5 Design of the fitness function

The fitness function measures how “good” a chromosome is, so that better chromosomes can be selected for mating. Because each chromosome represents a viable solution of VRPTM problem, we need to make an estimate of the results that be obtained after optimizing routes。 In this paper, the fitness function of an individual is as follows:

$$F = 1/\left(\alpha \cdot Dis + \beta \cdot Cov + \gamma \cdot Idt + \mu \cdot no\_service\right)$$

Where α,β,γ,μ are weight parameters associated with the total traveled distance, the cost used by vehicles, the waiting time and the number of customers not served respectively. The weight values of the parameters used in this function were determined empirically and set asα=0.001, β=1.0,γ=0.001,μ=1.0. The evaluation function is implemented after converting each of the chromosomes into a set of feasible routes.

4 EXPERIMENT AND ANALYSIS

The algorithm described in the previous section and HG1 proposed in (Homberger and Gehring, 1999) are coded in visual C++ 6.0, and executed using a machine with Pentium 2.0G CPU.

Considering a problem with 10 customers and a central depot, all the customers’ characteristics and demands are given in table 1. The distances between the central depot and the customers are given in table 2. Assuming a vehicle's capacity is 8 tons and speed is 50km/h, then the travel time from source i to destination j is $d_{ij}/50$(h), and the travel cost between them is assumed as d_{ij}. In first line of table 1, zero(i=0) represents the central depot, a_0 represents the earliest time to move away from central depot, b_0 represents last time restriction to return central depot.

We apply the improved genetic algorithm based on RC' and HG1 to solve the above problem, the population size is set at 20, dynamic crossover probability is used. After 100 generations, 3 vehicle routes are needed in all:

Our algorithm:

first vehicle route: 0→4→10→7→0
second vehicle route: 0→6→2→3→0
third vehicle route: 0→9→5→1→8→0
total distance: 437.0 (km)

HG1:

first vehicle route: 0→2→10→0
second vehicle route: 0→9→5→1→8→0
third vehicle route: 0→6→7→4→3→0
total distance: 734.0 (km)

Considering all the customers' characteristics and demands are given in table 3, distance and other situation is kept constant. After 100 generations, the experimental results of our algorithm and HG1 are same. 5 vehicle routes are needed in all:

first vehicle route: 0→5→2→3→0
second vehicle route: 0→6→8→0
third vehicle route: 0→4→9→0
fourth vehicle route: 0→1→7→0
fifth vehicle route: 0→10→0
total distance: 658.0 (km)

As can seen from results above, the improved genetic algorithm based on RC' was better than HG1 in some situation.

Apparently, this is a feasible solution to this problem, which satisfies both the constraints and every customer's demand. We also implement the genetic algorithm based on RC for the same problem. Though we still need 3 vehicles, customer 2 never gets service. It indicates that RC' operator is superior to RC. In addition, under the same circumstance, we traced the processes of the GAs based on PMX, RC and RC' respectively and found that the best fitness and average fitness of RC' were better than those of PMX and RC, the results are illustrated in Fig.2 and Fig.3. Considering two kinds of population size of the problems, the results of advanced experiment show that RC' is still superior to PMX and RC in other performances, which is given in table 4.

first vehicle route: 0→4→10→7→0
second vehicle route: 0→6→8→3→0
third vehicle route: 0→9→5→1→0

Table 1: Customers' Characteristics and Demands

Customer i	D i(ton)	T_i (hour)	$[a_i , b_i]$ (time)
0	0	0	[0 , 100]
1	3	2	[1.52 , 7]
2	4	2	[1.78 , 7]
3	3	1	[1.78 , 14]
4	3	1	[0.28 , 15]
5	2	2	[2.6 , 13]
6	1	2	[0.78 , 3]
7	1	2	[3.6 , 6]
8	3	2	[3.7 , 11]
9	1	2	[1.36 , 5]
10	4	1	[0.6 , 13]

Table 2: Distances between the central depot and the customers (km)

d_{ij}	0	1	2	3	4	5	6	7	8	9	10
0	0	76	89	89	14	130	39	180	185	68	30
1	157	0	82	176	35	113	142	80	105	94	137
2	185	140	0	40	178	138	180	66	171	126	18
3	1	200	43	0	83	129	142	43	196	66	107
4	190	121	85	120	0	15	159	182	34	65	1
5	128	2	1	46	103	0	142	31	75	45	28
6	141	117	16	16	170	30	0	45	29	150	193
7	1	14	194	80	85	85	164	0	60	187	4
8	85	6	48	152	107	79	4	82	0	113	91
9	21	30	80	127	53	31	32	110	49	0	185
10	46	162	123	14	87	35	132	34	10	106	0

Table 3: Customers' Characteristics and Demands

Customer i	D i(ton)	T_i (hour)	$[a_i , b_i]$ (time)
0	0	0	[0 , 100]
1	3	2	[1.52 , 7]
2	4	2	[1.78 , 7]
3	3	1	[1.78 , 14]
4	3	1	[0.28 , 15]
5	2	2	[2.6 , 13]
6	1	2	[0.78 , 3]
7	1	2	[3.6 , 6]
8	3	2	[3.7 , 11]
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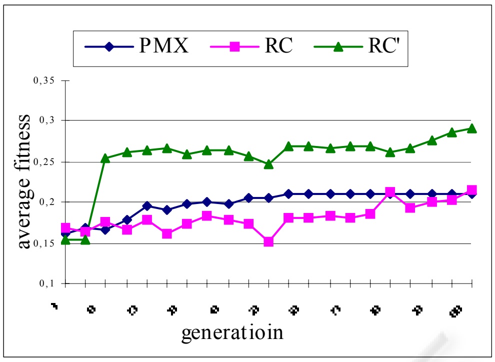

Figure 2: Comparing average

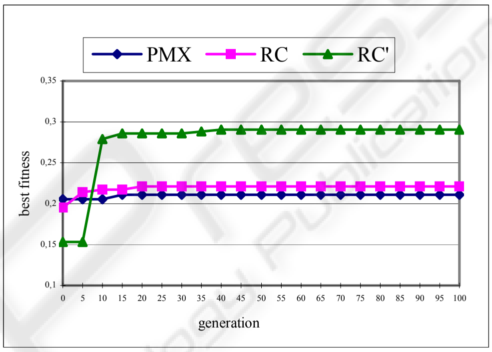

Figure 3: Comparing the best fitness

Table 4: Comparing the searching results using methods based on PMX, RC, RC' respectively

Problem size (population size , generation)	crossover operator	best fitness	average fitness	distance	number of cars	sparing time	no_ service
10 (20,100)	PMX	0.211238	0.211238	734.0	4	0.00	0
	RC	0.220753	0.215167	529.0	3	0.96	1
	RC′	0.290899	0.290899	437.0	3	0.620	0
50 (100,200)	PMX	0.043584	0.043584	4944.0	18	0.30	0
	RC	0.049735	0.049735	5104.0	15	2.50	0
	RC′	0.086816	0.086268	2518.0	9	0.660	0
100 (70,200)	PMX	0.019266	0.018823	10903.0	41	2.940	0
	RC	0.022225	0.022081	9994.0	32	0.60	3
	RC′	0.034420	0.032051	7051.0	22	2.280	0

5 CONCLUSION

Based on genetic algorithm, we constructed an improved genetic algorithm using RC' operator to solve VRPTM problem. Experimental results show that with the increasing size of a problem, this algorithm can obtain general optimality for all evaluated indexes on the premise of satisfying every customer's demand and its performance outperforms PMX and RC. Compared with HG1, its performance and speed are more effective.

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Table 4. Comparing the searching results using methods based on PMX, RC, RC' respectively