MOTION-EMBEDDED COG JACOBIAN FOR A REAL-TIME HUMANOID MOTION GENERATION

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Abstract: For a legged robot such as a humanoid, balancing its body during a given motion is natural but the most important problem. Recently, a motion given to a humanoid is more and more complicated, and thus the balancing problem becomes much more critical. This paper suggests a real-time motion generation algorithm that guarantees a humanoid to be balanced during implementing a given motion. A desired motion of each arm and/or leg is planned by the conventional motion planning method without considering the balancing problem. In order to balance a humanoid, all the given motions are embedded into the COG Jacobian. The COG Jacobian is modified to include the desired motions and, as a result, dimension of the COG Jacobian is drastically reduced. With the motion-embedded COG Jacobian, balancing and performing a task is completed simultaneously, without changing any other parameters related to the control or planning. Validity and efficiency of the proposed motion-embedded COG Jacobian is simulated in the paper.

1 INTRODUCTION

A high mobility of a humanoid makes it difficult to generate a motion and to interact with environment in real time. Many previous works in motion generation of a humanoid is to replay a pre-defined joint motion and modify it little by a certain control method in real situation. (Yamaguchi et al., 1999; Nagasaka et al., 2004) For a complicated, smooth and agile motion, it is necessary to develop a real time motion generating method. In addition, it is desired to include dynamics to improve the stability of a humanoid. Dynamics, however, requires a large amount of computation. The center of gravity(COG) can be a simple alternative of full dynamics, since it can be treated as a usual kinematics which needs less computation than that of dynamics, and ,furthermore, it reflects dynamic properties such as the mass and the mass center. Thus, the COG is the important property which relates to the stability, motion, and dynamics.

In order to use the COG relation in motion generation, the COG Jacobian is needed. The COG Jacobian is firstly proposed by Kagami, et al, in 2000, but they developed a numerical method(Kagami et al., 2000). An analytic formulation of the COG Jacobian is proposed by Sugihara, et al, in 2002(Sugihara et al., 2002, Sugihara and Nakamura, 2002). They used the COG Jacobian as follows: At first, the ZMP trajectory is predefined and it is used as a desired motion of the COG Jacobian. Motions of some limbs are used as constraints. An optimization problem that satisfies the COG Jacobian and given constraints simultaneously, are solved to generate joint motion. As an application, Sugihara, et al, used the COG Jacobian for whole body cooperative balancing(Sugihara and Nakamura, 2002), but the method needs large computation in optimization.

The method proposed in this paper is that all the given motions are embedded in the COG Jacobian to generate joint motions in real time. In order to embed motions into the COG Jacobian, a humanoid is divided into several sections. A motion of each section is considered independently, and relations of each independent motion are combined with the motion-embedded COG Jacobian. The dimension of the modified COG Jacobian and the number of constraints are reduced drastically, since motions are considered independently. In addition, the modified COG Jacobian guarantees the balancing a humanoid and performing a task.

This paper is organized as follows: Section 2 describes the overall system. Section 3 introduces...
the COG Jacobian suggested by Sugihara, et al., in 2002 (Sugihara et al., 2002; Sugihara and Nakamura, 2002). Section 4 derives the motion-embedded COG Jacobian. Section 5 simulates the suggested algorithm, and section 6 concludes the paper.

2 OVERALL SYSTEM

For a human, if arms do a certain task, they focus on the task not on the balancing. The remaining parts of the human balance its body regardless of the task. The proposed method, i.e., the motion-embedded COG Jacobian, implements the fact. Consequently, when we plan a motion, balancing problem does not need to be considered, and furthermore, a conventional motion planning method can be applied without any modification.

The overall control scheme can be divided into two major categories: 1) COG and ZMP related algorithms and 2) motion related algorithms. The COG-ZMP related algorithms are based on the inverted pendulum model\(^1\) and the motion related algorithms focus on how to distribute the COG-ZMP profile and given task motion to each joint with guaranteeing the balancing and performing the given task motion. Fig. 1 shows the overall control flow.

For the stability of a humanoid, the COG-ZMP profile is predefined for several basic motion, for example, forward/backward movement, side movement and turning. The COG-ZMP profile is derived from the inverted pendulum model, which gives relatively simple relation. COG is calculated from the internal joint information, thus it represents the pose of a humanoid. ZMP is an dynamic property that represents the stability whether a humanoid is to be rollover or not. Two properties are tightly related, and therefore, the predefined COG-ZMP are controlled simultaneously with real COG and ZMP, in order to guarantee the pose and the stability.

\(^1\)In this paper, we don’t include the planning of COG and/or ZMP. You can find this subject in references (Hirai et al., 1998; Choi et al., 2004).

Before developing the motion-embedded COG Jacobian, a humanoid is divided into several parts. A humanoid has four limbs attached at the main body. Hereafter, each limb and the main body will be called a section. For a human, a task is done with some sections, usually two arms, and balance the body with other sections, usually two legs. It means that not all sections are dedicated into one task, but they do their own tasks, for example, balancing or a given task. Thus the motion of each section can be given independently. According to the existence of a given desired motion, the section is classified into the idle section and the busy section: the idle section has no given motion and the busy section has a given motion. If a section has zero motion, i.e., it is fixed at current position, it can be considered as an idle section or a busy section. In most case, a supporting section which is attached on the ground are considered as a busy section, since they have a role of balancing the body. A given motion of each limb or a main body can be given as a joint motion and/or a Cartesian motion. If a busy section has a given motion in the Cartesian space, it will be called C-busy section.
and a section with a given motion in the joint space is J-busy section. Motion control step in Fig. 1 is done for a C-busy section. A J-busy section will be controlled at the joint control step.

The given motion and corresponding COG-ZMP profile are distributed to joints of a humanoid by using the COG Jacobian, i.e., the motion of the inverted pendulum model is transformed into the motion of a complicated multi-body model. By using the motion-embedded COG Jacobian, the stability and a given motion are guaranteed simultaneously.

The algorithm suggested in this paper is applied to the humanoid, Mahru, developed at IRRC (Intelligent Robotics Research Center), KIST in 2004 shown in Fig. 2. Mahru has the height of about 150cm, and the weight is about 67kg. It has 6-dof for each legs and arms, 1-dof for the waist, 2-dof for the neck, and each hand has 4-dof. In total, it has 35-dof. A stereo camera is equipped.

3 COG JACOBIAN\(^2\)

Let us consider a \(n\)-DOF humanoid. There are two referential frames to describe a humanoid as shown in Fig. 3. The world coordinate frame is fixed somewhere in the world and represents the global motion of a humanoid. The body center frame is fixed on a humanoid to describe the local motion. Almost all the properties of a humanoid is described in the body center frame. The leading superscript "("(\(\cdot\)) implies that the elements are represented in the body center frame, and without it, a value is based on the world coordinate frame.

The COG, \(p_G\), of a humanoid is a function of joint angle vector, \(q\), i.e., \(p_G = f(q)\). Thus, there exists a Jacobian \(J_G\) which can relate \(\dot{q}\) to \(\dot{p}_G\): \(p_G = J_G \dot{q}\) (1)

where the \((3 \times n)\) matrix \(J_G\) is defined by \(J_G \equiv \frac{\partial p_G}{\partial q}\) (2)

We call \(J_G\) the COG Jacobian hereafter. \(p_G\) is a quite complex function with multiple arguments. Kagami, et al., proposed the numerical method to calculate it(Kagami et al., 2000), which unfortunately needs a large amount of computation. Sugihara, et al., developed a fast and accurate calculation method of \(J_G\) with the following approach(Sugihara et al., 2002).

\[ q = (q_1, \ldots, q_n) \]

Firstly, the relative COG velocity with respect to the body center frame, \(\dot{p}_G\), can be expressed as:

\[ \dot{p}_G = \sum_{i=0}^{n-1} m_i \dot{r}_G, \quad \dot{q} = \sum_{i=0}^{n-1} m_i \dot{J}_G, \dot{q} \]

where \(m_i\) is the mass of \(i\), \(\dot{r}_G\) is the position of the center of mass of \(i\) with respect to the body center frame, and \(\dot{J}_G\) \((3 \times n)\) is defined by

\[ \dot{J}_G \equiv \frac{\partial \dot{r}_G}{\partial \dot{q}} \]

Therefore, Jacobian \(\dot{J}_G\) which relates \(\dot{q}\) to \(\dot{p}_G\) is

\[ \dot{J}_G = \frac{\sum_{i=0}^{n-1} m_i \dot{J}_G}{\sum_{i=0}^{n-1} m_i} \]

Secondly, suppose link 1 is the base section, which is fixed in the world frame (for example, when the right leg is the supporting leg, the right leg is fixed on the ground and becomes the base section), the COG velocity with respect to the world coordinates frame, \(\dot{p}_G\) is

\[ \dot{p}_G = \dot{p}_o + \omega_o \times R_o \dot{p}_G + R_o \dot{J}_G \dot{q} \]

\[ = R_o \dot{p}_1 + (\dot{p}_G - \dot{p}_1) \times \omega_o \]

\[ = R_o \dot{J}_G \dot{q} + [\dot{p}_G - \dot{p}_1] \times \omega_o \]

where \(p_o\) is the position of the body center, \(\omega_o\) is the angular velocity of the body center, and \(R_o\) is the attitude matrix of the body center with respect to the

\[ \text{Figure 3: Coordinate System of Mahru} \]
world frame. \( \omega_1 \) is the position of the base section, \( \omega_i \) is the angular velocity of the base section, \( \mathbf{J}_{v1} \) is the linear velocity part of the base Jacobian and \( \mathbf{J}_{\omega_j} \) is the angular velocity part of the base Jacobian with respect to the body center frame. \([v \times]\) means outer-product matrix of a vector \( v \) (3 x 1). \( \dot{q}_i \) is the joint velocity of section \( i \). Note that the base section can be any section that is fixed on the ground, but here, we assigned the base section with the number 1 without loss of generality.

In order to use Eq. (6) in the following section, it is rewritten here as:

\[
\mathbf{p}_G = \sum_{i=1}^{m} \mathbf{R}_o \mathbf{J}_{G, i} \hat{q}_i = \mathbf{R}_o \{ -\mathbf{J}_{v1} + [\mathbf{p}_G - \mathbf{p}_1] \times \mathbf{J}_{\omega_1} \} \dot{q}_1
\]  

(7)

where \( m \) is the number of sections.

Now, let us derive the motion-embedded COG Jacobian from Eq. (7).

4 MOTION-EMBEDDED COG JACOBIAN

4.1 Embedment of J-Busy Section

It is easy to embed a joint motion into the COG Jacobian, since the joint motion can be directly replaced \( \dot{q}_j \) in Eq. (7).

If section \( j \) is a J-busy section, Eq. (7) becomes

\[
\mathbf{p}_G = \sum_{i=1, i \neq j}^{m} \mathbf{R}_o \mathbf{J}_{G, i} \dot{q}_i = \mathbf{R}_o \{ -\mathbf{J}_{v1} + [\mathbf{p}_G - \mathbf{p}_1] \times \mathbf{J}_{\omega_1} \} \dot{q}_1
\]  

(8)

The second term of the left hand side compensates the motion of the \( j \)-th section. Therefore, the other sections shown in the right hand side can generate a joint motion with the compensated COG motion.

If at least one section, \( i.e. \), the base section, is an idle section, then Eq. (8) can compensate motions of the other sections. If all sections are the J-busy section, there is no section to compensate given motions. In this case, an optimization method needs to be applied.

4.2 Embedment of C-Busy Section

Let us derive the motion-embedded COG Jacobian for the C-busy section. Each section of a humanoid is considered as an independent section, \( i.e. \), any section can have its own motion independently without considering other sections. In general, the \( i \)-th section has the following relation:

\[
\mathbf{x}_i = \mathbf{J}_i \dot{q}_i
\]  

(9)

where \( \mathbf{x}_i = [\mathbf{p}_i^T, \mathbf{\omega}_i^T]^T \) is the end point velocity of the section, \( \mathbf{p}_i \) and \( \mathbf{\omega}_i \) are the linear and the angular velocity, respectively. \( \dot{q}_i \) is the joint velocity, and \( \mathbf{J}_i \) is the usual Jacobian matrix represented in the body center frame.

In our case, the body center is floating, and thus the end point motion about the world coordinate frame is written as follows:

\[
\dot{x}_i = \mathbf{X}_i^{-1} \dot{x}_o + \mathbf{X}_o \mathbf{J}_i \dot{q}_i
\]  

(10)

where \( \dot{x}_o = [\mathbf{p}_o^T, \mathbf{\omega}_o^T]^T \) is the velocity of the body center represented in the world coordinate system, and

\[
\mathbf{X}_i = \begin{bmatrix} \mathbf{I}_3 & 0_3 \mathbf{R}_o \mathbf{r}_i \times \mathbf{I}_3 \end{bmatrix}
\]  

(11)

is a (6 x 6) matrix which relates the body center velocity and the end point velocity of the \( i \)-th section. \( \mathbf{I}_3 \) and \( 0_3 \) are the (3 x 3) identity and zero matrix, respectively. \( \mathbf{R}_o \) is the orientation of the body center based on the world coordinate system. \( \mathbf{r}_i \) is the position vector from the body center to the end of the \( i \)-th section based on the body center frame. The transformation matrix \( \mathbf{X}_o \) is

\[
\mathbf{X}_o = \begin{bmatrix} \mathbf{R}_o & 0_3 \\ 0_3 & \mathbf{I}_3 \end{bmatrix}
\]  

(12)

By combining Eq. (9) and Eq. (10), the end point velocity based on the world coordinate system is

\[
\dot{x}_i = \mathbf{X}_i^{-1} \dot{x}_o + \mathbf{X}_o \mathbf{J}_i \dot{q}_i
\]  

(13)

For simplicity, we will use the relation \( \mathbf{J}_i = \mathbf{X}_o \mathbf{J}_i \mathbf{X}_o^{-1} \) hereafter.

From Eq. (13), we can see that all sections should satisfy the compatibility condition, \( i.e. \), the body center velocity, \( \dot{x}_o \), in Eq. (13) for each section is the same, so that sections are connected without being broken, and thus arbitrary two sections, for example, the \( i \)-th and \( j \)-th section should satisfy the following relation:

\[
\mathbf{X}_i (\mathbf{x}_i - \mathbf{J}_i \dot{q}_i) = \mathbf{X}_j (\dot{x}_j - \mathbf{J}_j \dot{q}_j)
\]  

(14)

From Eq. (14), the joint velocity of any section can be represented by the joint velocity of any other section. However, all sections will be represented by the base section, since the motion of the body center is represented by the base section, as shown in Eq. (7). The base section can be the supporting leg in the single supporting phase or one of both legs in the double supporting phase if a humanoid is standing. Let us
express the base section with the subscript 1, then the joint velocity of any section is expressed as:

\[
\dot{q}_i = J_i^{-1} \dot{x}_i - J_i^{-1} X_{i1}(x_1 - J_1 \dot{q}_1), \tag{15}
\]

for \( i = 2, \ldots, m \), where \( m \) is the number of sections. Here,

\[
X_{i1} = \begin{bmatrix} I_1 & [R_o(\rho_{1i} - \rho_1) \times] \\ 0_3 & I_3 \end{bmatrix}. \tag{16}
\]

Note that if a section is a redundant system, any null space optimization scheme can be added in Eq. (15), and \( J_i^{-1} \) becomes a generalized inverse.

By substituting Eq. (15) into Eq. (7), the motion-embedded COG Jacobian relation becomes

\[
\dot{p}_G = R_o(-\omega J_{V_i} + \{[\rho p_G - \rho_1] \times \} \omega \omega_i) \dot{q}_i
+ \sum_{i=1}^m R_o \omega J_{G1} J_i^{-1}(\dot{x}_i - X_{i1} \dot{x}_1)
+ \sum_{i=1}^m R_o \omega J_{G1} J_i^{-1} X_{i1} J_i \dot{q}_i \tag{17}
\]

where \( J_{V_i} = R_o \omega J_{V1} \) and \( J_{\omega i} = R_o \omega J_{\omega1} \) are the linear and angular velocity part of the base section Jacobian. Note that if \( i = 1 \), \( \dot{x}_1 - X_{11} \dot{x}_1 = 0 \) and \( R_o \omega J_{G1} J_1^{-1} X_{11} J_1 \dot{q}_1 = R_o \omega J_{G1} \dot{q}_1 \).

Finally, all desired motions, \( \dot{x}_i \), are embedded in the modified COG Jacobian. Thus the effect of the COG movement generated by the desired motion is compensated by the base section. Eq. (17) can be rewritten like the usual kinematic Jacobian of the base section:

\[
\dot{p}_{meG} = J_{meG} \dot{q}_1, \tag{18}
\]

where

\[
\dot{p}_{meG} = \dot{p}_G - \sum_{i=1}^m R_o \omega J_{G1} J_i^{-1}(\dot{x}_i - X_{i1} \dot{x}_1), \tag{19}
\]

\[
J_{meG} = R_o(-\omega J_{V1} + \{[\rho p_G - \rho_1] \times \} \omega \omega_1)
+ \sum_{i=1}^m R_o \omega J_{G1} J_i^{-1} X_{i1} J_i. \tag{20}
\]

The modified COG motion, \( \dot{p}_{meG} \), consists of two relations: a desired COG motion (the first term) and the relative effect of motions of each section (the second term). The modified COG Jacobian, \( J_{meG} \), also consists of two relations: the effect of the body center (the first term) and the effect of motions of each section (the second term).

The modified COG Jacobian \( J_{meG} \) is a \((3 \times n_1)\) matrix where \( n_1 \) is the dimension of the base section, which is smaller than that of the original COG Jacobian. For example, Mahru in Fig. 2 has a 6-dof leg, and thus \( n_1 = 6 \) if the leg is the base section. After solving Eq. (18), the joint motion of the base section is obtained. The resulting base section motion balances a humanoid robot automatically during the movement of all other sections. With the resulting joint motion of the base section, the joint motion of all other sections are obtained by Eq. (15). The resulting motion follows the desired motion, regardless of the balancing motion of the base section.

5 SIMULATION

In order to show the validity of the suggested algorithm, a dynamic simulator developed at IRRC, KIST is used. The simulator can simulate a humanoid Mahru shown in Fig. 2.

To show the effect of the J-busy and C-busy section, simultaneously, two arms are J-busy and two legs are C-busy. The left and right foot is moved up and down, in turn, with a sinusoidal function. Two motions for arms are given: 1) both arms are stretched forward as shown in Fig. 4 and 2) the right arm is raised and waves it shown in Fig. 5. The desired arm motions are obtained by the motion captured data (Kim et al., 2005). In order to move the COG to the center of the supporting foot, a predefined COG-ZMP trajectory is applied.

The first motion, stretching two arms forward, can make the humanoid fall down since the mass center is moved in front of the humanoid. By using the motion-embedded COG Jacobian, the desired motion of arms are compensated by the supporting leg, and thus the leg moves the humanoid body back automatically as shown in Fig. 4(b), the side view.

The second motion, raising and waving right hand, makes a large disturbance to the humanoid since the fast waving motion changes the mass center quickly to the left and right. Thus if the right hand goes to the right side when the right foot is off the ground, it is difficult to move the COG and ZMP to the stable region, i.e., the left foot. However, the left leg compensates the motions of arms by the motion-embedded COG Jacobian, and consequently, the COG and ZMP are still in the stable region as shown in Fig. 5.

The overall motions are complicated and it is easy to make the humanoid unstable. However, by using the motion-embedded COG Jacobian, any parameters such as control gain, COG/ZMP trajectory, and etc. do not need to be changed to stabilize or balance the humanoid.

6 CONCLUDING REMARK

In this paper, a real-time motion generation method for a humanoid is suggested. A desired motion of each section is obtained by the conventional motion
planning method without considering the balancing problem.

The method can balance a humanoid by using the motion-embedded COG Jacobian, which reduces the computation time to generate a motion and satisfies the given motion and the balance, simultaneously.

Validity and efficiency of the suggested method is examined by the simulation. Two motions are applied to the humanoid, and without changing any parameters on the control or planning, the humanoid accomplishes the given motions and it also balanced its body.

In the paper, we assume that all the sections have their own tasks. If a section does not have a desired motion, i.e., an idle section, there are two possible cases:

1. the idle section is considered as a busy section with a zero motion, i.e., it is fixed at the current position. In this case, the section is considered as a C-busy section, and the method suggested in this paper can be applied.

2. with some optimization scheme, the idle section is used actively to balance the humanoid. Since the idle section has no desired motion, by changing its position actively, the humanoid can balance the body more easily and naturally. The optimization of the idle section is still under development.

REFERENCES


Figure 4: example 1: both arms move forward

Figure 5: example 2: right arm is raised and waves