MULTI-OBJECTIVE PREDICTIVE CONTROL: APPLICATION FOR AN UNCERTAIN PROCESS

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Abstract: This paper deals with the application of the Multi Objective Generalized Predictive Control (MOGPC) to level control in a laboratory process. The major characteristic of the considered plant is that the manual draining vane can take many positions causing changes in plant dynamics and strong disturbances in the process. The controller is based on a set of Controlled Auto Regressive Integrated Moving Average (CARIMA) model. The Recursive Least Squares (RLS) algorithm is used to estimate each model parameters. The control law is obtained by minimizing a multi objective optimization problem. The weighting sum approach is considered to formulate the control problem as a single criterion optimisation one. The real time control system implementation confirms the opportunity of using the MOGPC scheme to an uncertainty system.

1 INTRODUCTION

The Generalized Predictive Control (GPC) principle consists in calculating the control input by the minimization of a cost function over a future time horizon under certain process constraints (Clarke *et al*. 1987). Since the constraints on the input and the output signals can be explicitly taken in account by the GPC, this approach of control has attracted the attention of many control researchers and industrials (Boucher and Dumur 1996, Ben Abdennour *et al*. 2001).

Dynamics of industrial plants are usually not completely known and are subject to change from time to time. The complexity of industrial process makes difficult their representation by only one model. Consequently, the strategy which consists to characterize the system with several models, every model possessed its own validity domain, has been developed (Brian and Bequette 2001). The strategy of multi model control suffers from the difficulty of determination model's validity especially in noisy systems. Another technique can be used to handle nonlinear systems is the robust control design.

Robust controllers explicitly consider the parametric variation in the process model for calculating the control law (Gutierrez and Camacho 1995, Oliveira *et al*. 2000, Brdys and Chang, 2002). The introduction of the uncertainty parameters leads to the resolution of a min-max optimisation problem which is hard to solve (Ramirez *et al.* 2002).

This paper presents the application of multi objective predictive controller to an uncertain plant. The major characteristic of the considered plant is that the manual draining vane can take many positions causing changes in plant dynamics and strong disturbances in the process. Each operating region can be modelled with a CARIMA model. The Recursive Least Squares (RLS) algorithm is used to estimate the model parameters. The control law is obtained by minimizing a multi objective optimization problem. The weighting sum approach is considered to formulate the control problem as a single criterion optimisation one.

This paper is organized as follows. Section 2 presents the description of the process. Section 3 is reserved to the multi criteria generalized predictive control based on a set of CARIMA model; the use of the weighting functions method is also described.

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Section 4 gives the results obtained in real time from the water level regulation. The final section of the paper presents the conclusion.

2 PROCESS DESCRIPTION

The process is schematically depicted in figure 1. The main goal is to control in closed loop the level in tank 1 by adjusting the liquid flow rate with the electric actuator pump. The sampling time period is fixed to 4s. While exploiting different step responses, we give in figure 2 the steady state characteristic. Then, the relation between the level and the flow rate is non linear. It’s well known that the capacity dominated process can be described by a first order linear differential equation about a desired operating level (William *et al.* 2000).

With the considered process, the manual draining vane can take different positions then the system can be modeled with an uncertainty first order model. In order to identify the model parameters, we have recorded two files of measures giving the evolution of the water level for a shape of crenel control. The obtained data, for the first and the second positions of the draining vane, are respectively represented in Figure 3 and Figure 4.

The presence of a numeric model is a necessary condition for the development of the predictive control, since it permits to calculate the predicted output on a finished horizon. Consider the single input single output process, which may be described by the CARIMA model as follows (Clarke *et al.* 1987):

$$A(q^{-1})\Delta y(k) = B(q^{-1})\Delta u(k)\,, \tag{1}$$

where $y(k)$ is the output signal and $u(k)$ is the input signal. The term $\Delta = 1 - q^{-1}$ corresponds to an integral action which permits the annulment of the permanent regime error. $A(q^{-1})$ and $B(q^{-1})$ are polynomials of degrees n_a and n_b in backward shift operator q^{-1}:

$$A(q^{-1}) = 1 + a_1 q^{-1} + \ldots + a_{na} q^{-n_a}\,, \tag{2}$$

$$B(q^{-1}) = b_1 q^{-1} + \ldots + b_{nb} q^{-n_b}\,. \tag{3}$$

For each operating region, a local CARIMA model is determined. The model parameters are identified, off-line, by using the (RLS) algorithm:

$$e(k) = y(k) - \hat{\theta}^T(k-1)\phi(k) \tag{4}$$

$$\hat{\theta}(k) = \hat{\theta}(k-1) + P(k)\phi(k)e(k) \tag{5}$$

$$P(k) = P(k-1) - \frac{P(k-1)\phi(k)\phi^T(k)P(k-1)}{1+\phi^T(k)P(k-1)\phi(k)} \tag{6}$$

where $e(k)$ is the prediction error; $\theta(k) = [a_1 \ldots a_{na}\; b_1 \ldots b_{nb}]^T$ is the parameter vector; $\phi(k) = [-y(k-1) \ldots -y(k-na)\; u(k-1) \ldots u(k-nb)]^T$ is the observation vector and $P(k)$ is the covariance matrix.

The input output data, considered in this work, belong to two different working points. These data given by figures 3 and 4 lead, respectively, to the following models:

$$\theta_1(k) = [-0.9864\;\; 0.0372]^T \text{ and}$$

$$\theta_2(k) = [-0.9803\;\; 0.0301]^T \tag{7}$$

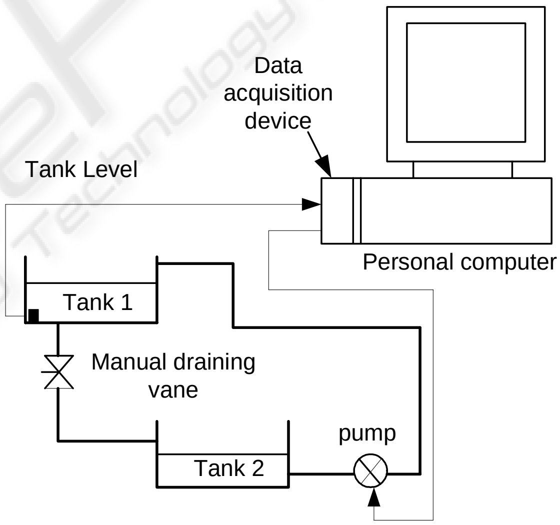

Figure 1: Laboratory process of level control

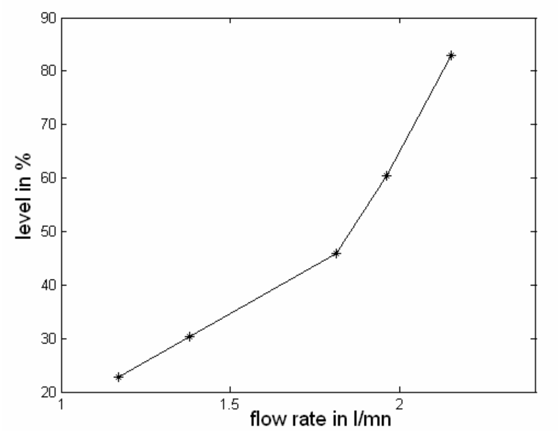

Figure 2: Non linear steady state characteristic

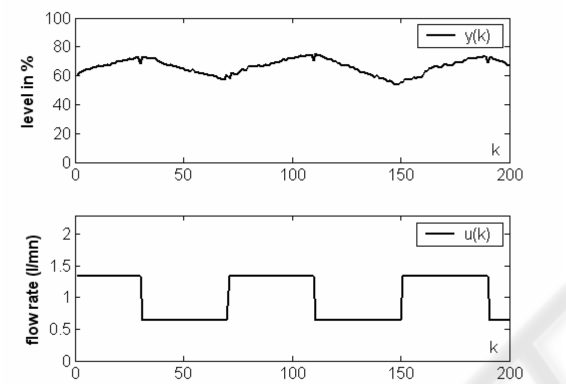

Figure 3: The water level *y(k)* and the flow rate *u(k)* (first position of the draining vane).

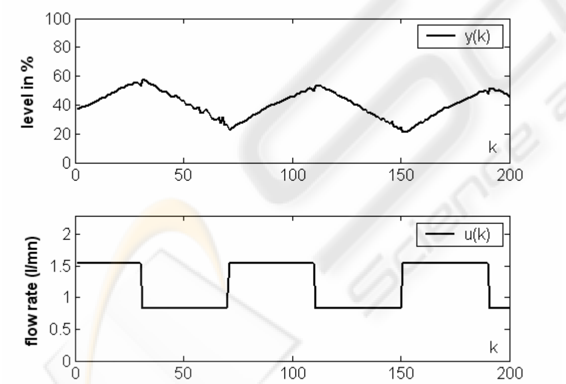

Figure 4: The water level *y(k)* and the flow rate *u(k)* (second position of the draining vane)

3 CONTROL AND DESIGN

For systems that present several modes of working, different models can be built which are specific to every mode of particular working of the system. We consider the set of models:

$$M = \{\theta_1(k), \ \ldots, \theta_n(k)\} \tag{8}$$

where n is the number of possible models. A multicriteria optimisation problem can be formulated as follows:

$$\min_{\Delta U}(J_1, \ldots, J_n) \tag{9}$$

where J_i is formulated by using the model $\theta_i(k)$.

3.1 Single criterion GPC

The objective of the generalized predictive control results in the minimization of the criterion under the following analytic relation (Clarke *et al.* 1987):

$$J = \frac{1}{2}\left(\sum_{j=1}^{N_2}\left(\hat{y}(k+j/k) - y_c(k+j)\right)^2 + \lambda \sum_{j=0}^{N_u-1}\left(\Delta u(k+j)\right)^2\right) \tag{10}$$

where N_2 is the prediction horizon, N_u is the control horizon, λ is the control increments weighting factor, $y_c(k)$ is the set point, $\hat{y}(k+j/k)$, $j \in [1, N_2]$, is the j step ahead predicted output and $\Delta u(k)$ is the control increment.

The minimization of the criterion requires the computation of the predicted output over the prediction horizon i.e.: $\hat{y}(k+j/k)$, $j \in [1, N_2]$. This can be achieved by the model of the process. The j step ahead predicted output is given by the following relation (Clarke *et al.* 1987, Ben Abdennour *et al.* 2001).

$$\hat{y}(k+j/k) = y_f(k+j/k) + y_a(k+j/k) \tag{11}$$

where $$y_f(k+j/k) = Q_j \Delta u(k+j-1)$$

and $$y_a(k+j/k) = R_j \Delta u(k-1) + G_j y(k)$$

where Q_j, R_j and G_j are polynomials solutions of Diophantine equations.

On a prediction horizon N_2 and on a control horizon N_u, it is possible to transcribe equation (11) under matrix shape:

$$\hat{Y} = Q\Delta U + H_1 Y + H_2 \Delta U_a \tag{12}$$

where $\hat{Y}$ is the vector of the predicted output, ΔU is the vector of the present and the future control increments. The vector Y is formed by the present value and the old values of the output, the vector ΔU_a is formed by the old increments of the control, matrices H_1, H_2 and Q are formed, respectively, by the coefficients of polynomials G_j, R_j and Q_j.

Based on these notations, we can write the criterion J in the following matrix form

$$J = [\hat{Y} - Y_c]^T [\hat{Y} - Y_c] + \lambda \Delta U^T \Delta U \tag{13}$$

where Y_c is the vector formed by the future set point sequence.

The optimal solution is obtained while annulling the gradient of J in relation to the vector of the increment control:

$$\Delta U = [Q^T Q + \lambda I_{N_u}]^{-1} Q^T (Y_c - Y_l) \tag{14}$$

where I_{Nu} is a unity matrix of dimension (N_u,N_u) and $Y_l = H_1 Y + H_2 \Delta U_a$.

The GPC is a receding control strategy, only the first element of the vector ΔU is used to compute the control to be applied to the process.

$$u(k) = u(k-1) + \Delta U(1) \tag{15}$$

3.2 Multi objective GPC

The objectives are often conflicting or competing. A powerful method for dealing with multiple objectives is the Pareto optimality concept. Multi objective problems usually have no unique solution, but a set of non dominated solutions, known as the Pareto optimal set (Xin *et al.* 2004). In the case of non convex objectives, genetic algorithms are used to solve the multi objective problems (Colette and Siarry 2002, Silva and Fleming 2002, Andrès-Toro *et al.* 2002). In this work, local models are linear, consequently, the criterion J_i is convex in the controller parameters and it can be efficiently solved by the weighting sum approach. The weighting functions method transforms the multi criteria problem to a single criterion one as follows (Colette and Siarry 2002).

$$J = \sum_{i=1}^{n} w_i J_i \tag{16}$$

where $$\sum_{i=1}^{n} w_i = 1 \text{ and } w_i \geq 0 . \tag{17}$$

The weighting sum approach consists to take all objectives in a single aggregating function. The modification of the w_i values that respect the constraint (17), leads to the Pareto optimal set. Since the optimization problem is convex, the solutions are uniformly repatriated on the Pareto surface. In this work, we have considered the optimal control, the value from the Pareto set that gives the minimum of the sum of all objectives.

The following algorithm is used to compute the optimal sequence of control:

1- Take w_1=0 and fix the step Δw_1.

2- Choose w_i, $i=2,\ldots,n$ that verify the relation (17),

3- Compute $\frac{\partial J_i}{\partial \Delta U}$ using (14) and $\frac{\partial J}{\partial \Delta U}$.

4- Compute the control sequence $\Delta U(k)$.

5- Increment w_1 ($w_1=w_1+\Delta w_1$), if $w_1<1$, return to step 2.

6- Take ΔU_{opt} that gives the minimum of the sum of all criteria.

7- Compute the control law as follows:

$$u(k) = u(k-1) + \Delta U_{opt}(1)$$

In this algorithm, the size of the Pareto optimal set depends on the choice of the step Δw_1. In this work, we have used Δw_1=0.1, then at each sampling time, we compute 10 solutions which form the Pareto set.

4 EXPERIMENT RESULTS

The multi objective predictive control scheme based on the CARIMA model was applied to level control in a laboratory process. In order to compare the behavior of the standard GPC and the MOGPC in presence of non-stationary process, we have fixed the draining vane of the process described in section 2, in the first position during 300 sampling period then we turn in the second position. The constraints imposed on the input signal are as follows:

$$0 \leq u(k) \leq 2.3 l / mn \tag{18}$$

A discrete PID regulator can be given by the following relation (Borne *et al.* 1993):

$$u(k) = u(k-1) + K_P (1 + \frac{T_e}{k_i} + \frac{k_d}{T_e}) \varepsilon(k) - (1 + 2\frac{k_d}{T_e}) \varepsilon(k-1) + \frac{k_d}{T_e} \varepsilon(k-2) \tag{19}$$

where K_p is the proportional gain; k_i is the integral constant; k_d is the derivate constant; T_e is the sampling time period; and $\varepsilon(k)$ is the error between the set point and the output signal. The PID parameters are computed based on the step response of the system and the Takahashi method (Borne *et al.* 1993).

The results obtained with the PID controller are shown in Figure 5. The control signal presents many fluctuations and the tracking error is not zero. In this work, we have considered fixed PID controller parameters. One can ameliorate the closed loop performances by using an adaptive PID controller to cope with the process dynamic changes. The results obtained with the GPC are shown in Figure 6. In this case, the controller is based on a nominal model which is obtained by:

$$\theta(k) = \frac{1}{2}(\theta_1(k) + \theta_2(k)) \qquad (20)$$

It's clear, from this figure, that the nominal model assures good performances in closed loop. The evolutions of the output/input and the set point signals, in the case of the MOGPC, are given in Figure 7. Obviously, we notice that performances in terms of the tracking error and the variance of the control signal are substantially ameliorated. Table 1 gives the variance of the control (V) for the three controllers.

$$V = \frac{1}{N}\sum_{k=1}^{N}\left(u(k)\right)^2 \qquad (21)$$

where N is the number of data measurement.

The variance obtained with the MOGPC is the lower, because the control signal obtained with this controller has few fluctuations compared to those obtained with the GPC and the PID controllers.

Table 1: Variance of the control

Controller	V
PID controller	1.6108
GPC (single criterion)	1.4546
GPC (multi objective strategy)	1.1248

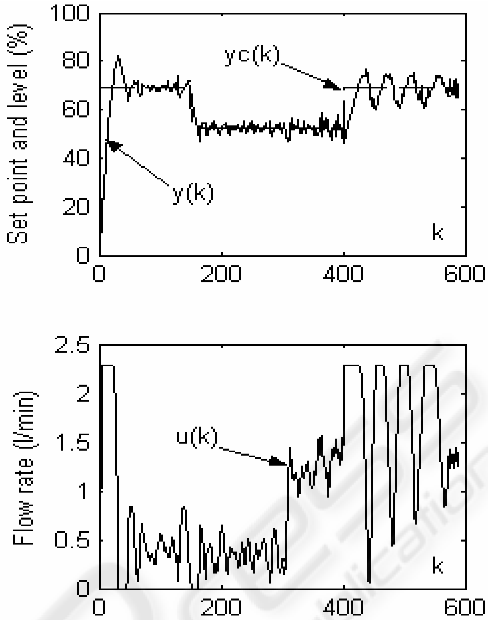

Figure 5: PID controller (k_p=0.5; k_i=3; k_d=0.1)

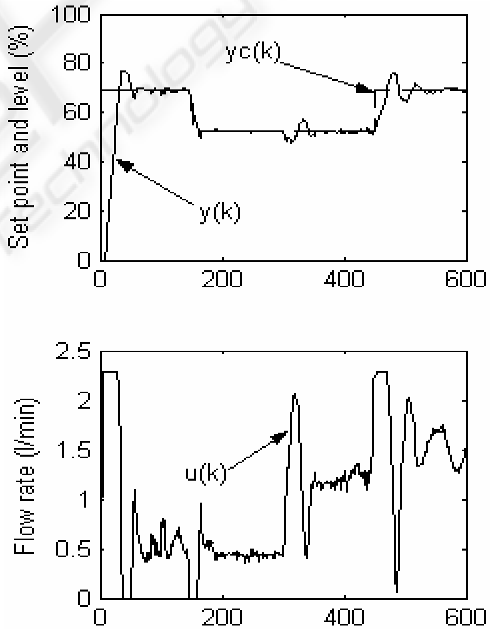

Figure 6: GPC (single quadratic criterion)
($N_2 = 7; N_u = 1;\ \lambda = 1$)

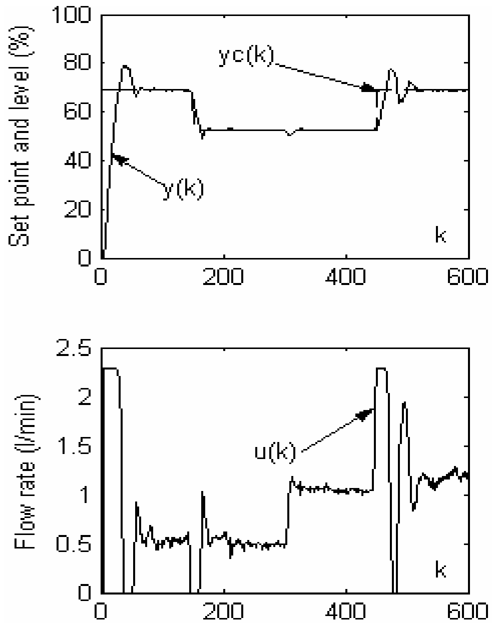

Figure 7: MOGPC (multi objective strategy)
(N_2 = 7; N_u = 1; $\lambda = 1$)

5 CONCLUSIONS

This paper has presented the multi objective predictive control. The process is characterized by a set of CARIMA model. Since considered models are linear, performance criteria are convex. Consequently, the weighted sum approach is used to compute the Pareto optimal set. An application of the studied strategy to a nonlinear model plant has been also presented.

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