COMPOSITIONAL ANALYSIS FOR REGULARITY, LIVENESS AND BOUNDEDNESS

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Abstract: By the linear algebraic representation of Petri nets, Desel introduced regularity property (Desel, 1992). Regularity implies a sufficient condition for a Petri net to be live and bounded. All the conditions checking the regularity of a Petri net are decidable in polynomial time in the size of a net (Desel and Esparza, 1995). This paper proves that regularity, liveness and boundedness can be preserved after applying many compositional operations to Petri nets. This means that, by applying these compositional operations, a designer can construct complex nets satisfying regularity, liveness and boundedness properties from simpler ones without forward analysis.

1 INTRODUCTION

As a graphical and mathematical tool, Petri nets provide a uniform environment for modeling, formal analysis, and design of discrete event systems. One of the major advantages of using Petri net models is that the same model can be used for the analysis of behavioural properties and performance evaluation, as well as for systematic construction of discrete-event simulators and controllers (Zhou and Venkatesh, 1999).

One net is regular if it satisfies the conditions of the Rank Theorem described in algebraic methods (Desel, 1992; Desel and Esparza, 1995). Regularity is a sufficient condition for ordinary nets to be live and bounded (Desel and Esparza, 1995). In general, a system is live if all its operations are eventually executable, starting not only from its initial state but also from any reachable state. A system is bounded if it has a finite number of states. In the terminology of Petri nets, liveness requires the firability of every transition starting from any reachable marking, boundedness implies that the number of tokens existing in every place will not exceed a certain limit.

For system designs specified in Petri nets, the major approaches for verification include reachability analysis, direct proving on the basis of definitions, mathematical programming, characterization and property-preserving transformation. This paper relates to the fourth approach, i.e., the property-preserving transformation. In this approach, the original net is assumed to satisfy some specific properties and the transformation is required to preserve these properties in the transformed net. The advantage of this approach is that the transformed net is automatically correct without the need of forward verification.

Transformations on Petri nets may be roughly classified into three groups, namely reduction, refinement and composition. In the literature, there has been much work related to transformations that preserve liveness and/or boundedness (Berthelot, 1986; Berthelot, 1987; Esparza, 1994; Koh and DiCesare, 1991; Suzuki and Murata, 1983; Valette, 1979; Souissi, 1991; Zhou, 1996; Huang, Jiao and Cheung, 2005). The preservation of regularity has not been considered. This paper introduces four kinds of composition operations in terms of places and transitions. They are: merging two places each coming from different net; merging two places coming from the same net; merging two transitions each coming from different net; merging two transitions coming from the same net. For each kind of the four composition operations, this paper proves that regularity, liveness and boundedness can be preserved automatically or under some simper conditions.

This paper is organized as follows: Section 2 presents some basics about Petri nets including algebraic characterizations. Four compositional operations in terms of places and transitions are

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introduced and the preservation of regularity is verified in Sections 3. Section 4 proves that liveness and boundedness can be preserved under these operations. In this section, an example is given to illustrate some results of this paper. Some concluding remarks are given in Section 5.

2 PRELIMINARIES OF PETRI NETS

This section outlines the definitions, terminology and properties as required in the paper.

A *net* is denoted by $N = (P, T, F)$, where P is a non-empty finite set of places, T is a non-empty finite set of transitions with $P \cap T = \varnothing$ and $F \subseteq (P \times T) \cup (T \times P)$ is a flow relation. The *pre-set* of x is defined as ${}^\bullet x = \{y \in P \cup T \mid (y, x) \in F\}$ and the *post-set* of x is defined as $x^\bullet = \{y \in P \cup T \mid (x, y) \in F\}$. Similarly, for any subset of $Y \subseteq P \cup T$, ${}^\bullet Y$ (resp., $Y^\bullet$) denotes the union of ${}^\bullet y$ (resp., $y^\bullet$) for all $y \in Y$. A net $N = (P, T, F)$ is said to be *pure* or *self-loop-free* iff ${}^\bullet x \cap x^\bullet = \varnothing$ $\forall x \in P \cup T$. We just discuss pure nets in this paper.

The *incidence matrix* V of a pure net N is a $|P| \times |T|$ matrix whose element v_{ij} at row p_i and column t_j is denoted as follows: $v_{ij} = 1$ if $p_i \in t_j^\bullet$; $v_{ij} = -1$ if $p_i \in {}^\bullet t_j$; and $v_{ij} = 0$ if $p_i \notin {}^\bullet t_j \cup t_j^\bullet$.

A *marking* of a net $N = (P, T, F)$ is a mapping $M: P \rightarrow \{0, 1, 2, \ldots\}$. A place p is said to be *marked* by M if $M(p) > 0$. A transition t is *enabled* or *firable* at a marking M if for every $p \in {}^\bullet t$, $M(p) \geq 1$. A transition t may be *fired* if it is enabled. Firing transition t results in changing the marking M to a new marking M', where M' *is* obtained by removing one token from each $p \in {}^\bullet t$ and by putting one token to every $p \in t^\bullet$. $R(N, M_0)$ denotes the set of all markings reachable from the *initial marking* M_0.

A transition t is said to be *live* in a Petri net (N, M_0) iff, for any $M \in R(N, M_0)$, there exists $M' \in R(N, M)$ such that t can be fired at M'. (N, M_0) is said to be *live* iff every transition of N is live. A place p is said to be *bounded* in (N, M_0) iff there exists a constant k such that $M(p) \leq k$ for all $M \in R(N, M_0)$. (N, M_0) is *bounded* iff every place of N is bounded.

For $x \in P \cup T$, the *cluster* of x, denoted as $[x]$, is the smallest subset of $P \cup T$ satisfying three conditions: (1) $x \in [x]$; (2) if $p \in P \cap [x]$ then $p^\bullet \subseteq [x]$; and (3) if $t \in T \cap [x]$ then ${}^\bullet t \subseteq [x]$. N is said to satisfy the *rank-and-cluster property* iff the rank of its incidence matrix is less than the number of its clusters by 1.

A net N is said to be *connected* iff every pair of nodes (x, y) satisfies $(x, y) \in (F \cup F^{-1})^*$. A net N is said to be *strongly connected* iff $(x, y) \in F^*$, i.e., there exists a directed path from every node x to every node y. A *P-invariant* (resp., *T-invariant*) of N is a non-negative integer $|P|$-vector α (resp., $|T|$-vector β) satisfying the equation $\alpha V = 0$ (resp., $V\beta^T = 0$), where V is the incidence matrix of N. A P-invariant α (resp., T-invariant β) of a net is called semi-positive if $\alpha \geq 0$ and $\alpha \neq 0$ (resp., $\beta \geq 0$ and $\beta \neq 0$). The support of a semi-positive P-invariant α, denoted by $\langle \alpha \rangle$, is the set of places p satisfying $\alpha(p) > 0$, and the support of a semi-positive T-invariant β, denoted by $\langle \beta \rangle$, is the set of transition satisfying $\beta(t) > 0$ (Desel and Esparza, 1995).

A net N is *regular* (Desel 1992) iff (1) N is connected, (2) N has a positive P-invariant, (3) N has a positive T-invariant, and (4) N satisfies the rank-and-cluster property.

In N, a non-empty set of places D is said to be a *siphon* (resp., *trap*) iff ${}^\bullet D \subseteq D^\bullet$ (resp., $D^\bullet \subseteq {}^\bullet D$). A siphon (resp., trap) is said to be *minimal* if it does not properly contain any other siphon (resp., trap).

For more details, please refer to (Recalde, Teruel and Silva, 1998; Silva, Teruel and Colom, 1998).

3 FOUR COMPOSITIONAL OPERATIONS AND THE PRESERVATION OF REGULARITY

This section considers four compositional operations in terms of places and transitions. Two of them are very natural and can be found in the literature. The other two operations are a little similar to those in (Berthelot, 1987). Suppose the original nets are regular. We will prove that, for two of the four compositional operations, the regularity can be automatically preserved. For the other two ones, some simple conditions will be provided under which the regularity can be preserved.

3.1 Merging a Pair Of Places From Two Nets

COMPOSITION-BY-PLACE (composition via merging a pair of places from two different nets): Consider two disconnected ordinary nets $N_1 = (P_1 \cup$

$\{p_1\}$, T_1, F_1) and $N_2 = (P_2 \cup \{p_2\}, T_2, F_2)$, where $T_1 \cap T_2 = \varnothing$ and $F_1 \cap F_2 = \varnothing$. Let N be composed from N_1 and N_2 by merging the pair of places p_1 and p_2 into p_{12}, that is, $N = (P, T, F)$, where $P = P_1 \cup P_2 \cup \{p_{12}\}$, $T = T_1 \cup T_2$ and $F = F_1 \cup F_2$.

Theorem 1: Let N, N_1 and N_2 be defined in COMPOSITION-BY-PLACE. Then, N is regular if N_1 and N_2 are regular.

Proof: In order to show that N is regular, we must prove that N is connected, has a positive P-invariant and a positive T-invariant, and satisfies: $Rank(N) = |C_N| - 1$. The incidence matrices of N, N_1 and N_2 have the forms:

$$V = \begin{array}{c} \\ P_1 \\ P_2 \\ p_{12} \end{array} \begin{array}{c} \begin{array}{cc} T_1 & T_2 \end{array} \\ \begin{pmatrix} V_{11} & 0 \\ 0 & V_{22} \\ V_{31} & V_{32} \end{pmatrix} \end{array}$$

$$V_1 = \begin{array}{c} \\ P_1 \\ p_1 \end{array} \begin{array}{c} T_1 \\ \begin{pmatrix} V_{11} \\ V_{31} \end{pmatrix} \end{array} \quad V_2 = \begin{array}{c} \\ P_2 \\ p_2 \end{array} \begin{array}{c} T_2 \\ \begin{pmatrix} V_{22} \\ V_{32} \end{pmatrix} \end{array}$$

(1) Since N_1 and N_2 are connected, it is obvious that N is also connected.

(2) Since N_1 and N_2 have positive P-invariants, there exist α_1 and α_2 such that $\alpha_1 > 0$, $\alpha_2 > 0$, $\alpha_1 V_1 = 0$ and $\alpha_2 V_2 = 0$. Let $\alpha = (\alpha_2(p_2)\alpha_{1|P1} \quad \alpha_1(p_1)\alpha_{2|P2} \quad \alpha_1(p_1)\alpha_2(p_2))$. Then $\alpha > 0$ and $\alpha V = (\alpha_2(p_2)\alpha_{1|P1}V_{11} + \alpha_1(p_1)\alpha_2(p_2)V_{31} \quad \alpha_1(p_1)\alpha_{2|P2}V_{22} + \alpha_1(p_1)\alpha_2(p_2)V_{32})$ $=(\alpha_2(p_2)(\alpha_{1|P1}V_{11}+\alpha_1(p_1)V_{31})$ $\alpha_1(p_1)(\alpha_{2|P2}V_{22}+\alpha_2(p_2)V_{32}))$ $=(\alpha_2(p_2)\alpha_1V_1 \quad \alpha_1(p_1)\alpha_2V_2)$ $= 0$. This means that α is a positive P-invariant of N.

(3) Since N_1 and N_2 have positive T-invariants, there exist β_1 and β_2 such that $\beta_1 > 0$, $\beta_2 > 0$, $V_1\beta_1^T = 0$ and $V_2\beta_2^T = 0$. This means that $V_{11}\beta_1^T = 0$, $V_{31}\beta_1^T = 0$, $V_{22}\beta_2^T = 0$ and $V_{32}\beta_2^T = 0$. Let $\beta = (\beta_1 \quad \beta_2)$, then $V\beta^T = (V_{11}\beta_1^T \quad V_{22}\beta_2^T \quad V_{31}\beta_1^T + V_{32}\beta_2^T)^T = 0$. That is, β is a T-invariant of N.

(4) Since $Rank(V_1) = |C(N_1)| - 1$ and $Rank(V_2) = |C(N_2)| - 1$ and $|C(N_1)| \le |P_1|$ and $|C(N_2)| \le |P_2|$, $Rank(V_1) < |P_1|$ and $Rank(V_2) < |P_2|$. Hence, the bottom row V_{3i} is a linear combination of the other rows of V_{ii} and $Rank(V_i) = Rank(V_{ii})$ for $i = 1$ and 2. This also implies that the bottom row of V is a linear combination of the other rows of V and $Rank(V) = Rank(V_{11}) + Rank(V_{22}) = (|C(N_1)| - 1) + (|C(N_2)| - 1)$. Since the only change in clustering after COMPOSITION-BY-PLACE is that the two clusters ($[p_1]$ in N_1) and ($[p_2]$ in N_2) are merged to one, we have $|C(N)| = |C(N_1)| + |C(N_2)| - 1$. Thus, $Rank(V) = (|C(N_1)| - 1) + (|C(N_2)| - 1) = |C(N)| - 1$. □

3.2 Merging Two Non-neighboring Places In A Net

MERGE-N-PLACE (merging two non-neighboring places in a net)**:** Let a net $N = (P_0 \cup \{p_1, p_2\}, T, F)$ be a net and p_1 and p_2 satisfy: $({}^\bullet p_1 \cup p_1{}^\bullet) \cap ({}^\bullet p_2 \cup p_2{}^\bullet) = \varnothing$. Let $N' = (P_0 \cup \{p_{12}\}, T, F')$ be obtained from N by merging the places p_1 and p_2 into a single place p_{12}, where $F' = F \cup \{(t, p_{12}) \mid (t, p_1) \in F$ or $(t, p_2) \in F\} \cup \{(p_{12}, t) \mid (p_1, t) \in F$ or $(p_2, t) \in F\} - \{(t, p_i) \mid (t, p_i) \in F$, where $i = 1, 2\} - \{(p_i, t) \mid (p_i, t) \in F$, where $i = 1, 2\}$.

Note that since this paper just considers pure and ordinary nets, we add some conditions to p_1 and p_2, i.e., just considering non-neighboring places to be merged.

Theorem 2 below proposes a simple condition under which the regularity can be preserved after applying MERGE-N-PLACE.

Theorem 2: Let N and N' be involved in MERGE-N-PLACE, where $N = (P_0 \cup \{p_1, p_2\}, T, F)$ and $N' = (P_0 \cup \{p_{12}\}, T', F')$. Then, N' is regular if the following conditions hold:

(1) N is regular.
(2) there exists a positive P-invariant of N α such that $\alpha(p_1) = \alpha(p_2)$.
(3) p_1 and p_2 belong to the same cluster, and
(4) there exists at least one P-invariant α such that $\langle \alpha \rangle \cap \{p_1, p_2\} = \{p_i\}$, where $i = 1$ or 2.

Proof: The incidence matrices of N and N' have the following forms:

$$V = \begin{array}{c} \\ P_0 \\ p_1 \\ p_2 \end{array} \begin{array}{c} T \\ \begin{pmatrix} V_0 \\ V_1 \\ V_2 \end{pmatrix} \end{array} \quad V' = \begin{array}{c} \\ P_0 \\ p_{12} \end{array} \begin{array}{c} T \\ \begin{pmatrix} V_0 \\ V_1 + V_2 \end{pmatrix} \end{array}$$

(1) Since N is regular, N is connected. It is obvious that N' is also connected.

(2) Since there exists a positive P-invariant of N α such that $\alpha(p_1) = \alpha(p_2)$, let $\alpha' = (\alpha_{|P0} \; \alpha(p_1))$ and $\alpha' V' = \alpha_{|P0}V_0 + \alpha(p_1)(V_1 + V_2) = \alpha_{|P0}V_0 + \alpha(p_1)V_1 + \alpha(p_2)V_2 = 0$. This means that α is a positive P-invariant of N'.

(3) Since N is regular, N has a positive T-invariant. Let β be a positive T-invariant of N, then $V\beta^T = 0$. It is obvious that $V'\beta^T = 0$. That is, β also is a T-invariant of N'.

(4) Since p_1 and p_2 belong to the same cluster, $|C(N')| = |C(N)|$. Suppose that there exists one P-invariant α such that $\langle\alpha\rangle \cap \{p_1, p_2\} = \{p_2\}$. Then, the corresponding row of p_2 in V can be expressed as

a linear combination of the other rows of P_0. Hence, $Rank(V') = Rank(V)$. This means that $Rank(V') = Rank(V) = |C(N)| - 1 = |C(N')| - 1$. □

It is obvious from the proof of Theorem 2 that when p_1 and p_2 belong to different cluster, N' must be not regular if N is regular and Condition (3) holds.

3.3 Merging A Pair Of Transitions From Two Nets

COMPOSITION-BY-TRANSITION (composition via merging a pair of transitions from two different nets)**:** Consider two disconnected ordinary nets $N_1 = (P_1, T_1 \cup \{t_1\}, F_1)$ and $N_2 = (P_2, T_2 \cup \{t_2\}, F_2)$, where $P_1 \cap P_2 = \varnothing$, $(T_1 \cup \{t_1\}) \cap (T_2 \cup \{t_2\}) = \varnothing$, $T_1 \cap \{t_1\} = \varnothing$, $T_2 \cap \{t_2\} = \varnothing$ and $F_1 \cap F_2 = \varnothing$. Let N be composed from N_1 and N_2 by merging the pair of transitions t_1 and t_2. That is, $N = (P, T, F)$, where $P = P_1 \cup P_2$, $T = T_1 \cup T_2 \cup \{t_{12}\}$ and $F = F_1 \cup F_2$.

Theorem 3: Let N, N_1 and N_2 be defined in COMPOSITION-BY-TRANSITION. Then, N is regular if N_1 and N_2 are regular.

Proof: Similar to that of Theorem 1.□

3.4 Merging Two Non-neighboring Transitions In A Net

MERGE-N-TRANSITION (merging two transitions in a net)**:** Let a net $N = (P, T_0 \cup \{t_1, t_2\}, F)$ be a net and satisfy the following conditions: $({}^\bullet t_1 \cup t_1{}^\bullet) \cap ({}^\bullet t_2 \cup t_2{}^\bullet) = \varnothing$. Let $N' = (P, T_0 \cup \{t_{12}\}, F')$ be obtained from N by merging the transitions t_1 and t_2 into a single place t_{12}, where $F' = F' = F \cup \{(t_{12}, p) \mid (t_1, p) \in F \text{ or } (t_2, p) \in F\} \cup \{(p, t_{12}) \mid (p, t_1) \in F \text{ or } (p, t_2) \in F\} - \{(t_i, p) \mid (t_i, p) \in F, \text{ where } i = 1, 2\} - \{(p, t_i) \mid (p, t_i) \in F, \text{ where } i = 1, 2\}$.

Note that since this paper just considers pure and ordinary nets, we add some conditions to t_1 and t_2, i.e., just considering non-neighboring transitions to be merged.

Theorem 4: Let N and N' be involved in MERGE-N-TRANSITION, where $N = (P, T_0 \cup \{t_1, t_2\}, F)$ and $N' = (P, T_0 \cup \{t_{12}\}, F')$. Then, N' is regular if the following conditions hold:

(1) N is regular,
(2) there exists a positive T-invariant β such that $\beta(t_1) = \beta(t_2)$,
(3) t_1 and t_2 belong to the same cluster, and
(4) there exists at least one T-invariant β such that $\langle \beta \rangle \cap \{t_1, t_2\} = \{t_i\}$, where $i = 1$ or 2.

Proof: Similar to that of Theorem 2. □

4 PRESERVING LIVENESS AND BOUNDEDNESS

Desel and Esparza have shown that the regularity guarantees the existence of a live and bounded marking. This section will decide if a given initial marking M_0 ensures liveness and boundedness of (N, M_0), where N is the net obtained by applying the four compositional operations defined in Section 3.

Lemma 1 below characterizes liveness and boundedness of regular nets.

Lemma 1: (Desel and Esparza, 1995) Let N be a regular net. Then, a marking M of N is live and bounded iff it marks all minimal siphons of N.

In order to check whether the liveness and boundedness of a marked regular net, it is important to know whether all siphons are marked after applying the four compositional operations. The following propositions state the relationship of siphons of nets of before and after transformation.

Proposition 1: Suppose that N, N_1 and N_2 are defined in COMPOSITION-BY-PLACE. Let D be a siphon of N. Then, $D_i = D \cap P_i$ is a siphon of N_i if $p_{12} \notin D$, otherwise $D_i = (D \cap P_i) \cup \{p_i\}$ is a siphon of N_i, where $i = 1$ and 2.

Proof: Since D is a siphon of N, ${}^\bullet D \subseteq D^\bullet$. Since $P_1 \cap P_2 = \varnothing$, $T_1 \cap T_2 = \varnothing$ and $D = (D \cap P_1) \cup (D \cap P_2) = D_1 \cup D_2$ if $p_{12} \notin D$, it is obvious that ${}^\bullet D_1 \subseteq D_1{}^\bullet$ and ${}^\bullet D_2 \subseteq D_2{}^\bullet$. This means that $D_i = D \cap P_i$ is a siphon of N_i if $p_{12} \notin D$. If $p_{12} \in D$, then ${}^\bullet p_{12}$ in $N = ({}^\bullet p_1 \text{ in } N_1) \cup ({}^\bullet p_2 \text{ in } N_2)$ and $p_{12}{}^\bullet$ in $N = (p_1{}^\bullet \text{ in } N_1) \cup (p_2{}^\bullet \text{ in } N_2)$. Since ${}^\bullet D \subseteq D^\bullet$ and $T_1 \cap T_2 = \varnothing$, ${}^\bullet((D \cap P_i) \cup \{p_i\}) \subseteq ((D \cap P_i) \cup \{p_i\})^\bullet$, this means that $(D \cap P_i) \cup \{p_i\}$ is a siphon of N_i, where $i = 1$ and 2. □

Proposition 2: Suppose that N and N' are defined in MERGE-N-PLACE. Let D' be a siphon of N'. Then, D' is a siphon of N if $p_{12} \notin D'$, otherwise $D = D' \cup \{p_1, p_2\} - \{p_{12}\}$ is a siphon of N.

Proof: Since D' is a siphon of N', ${}^\bullet D' \subseteq D'^\bullet$ in N'. It is obvious that ${}^\bullet D' \subseteq D'^\bullet$ in N if $p_{12} \notin D'$. This means that D' is a siphon of N. If $p_{12} \in D'$, then ${}^\bullet p_{12}$ in $N' = ({}^\bullet p_1 \cup {}^\bullet p_2)$ in N and $p_{12}{}^\bullet$ in $N' = (p_1{}^\bullet \cup p_2{}^\bullet)$ in N. Since ${}^\bullet D' \subseteq D'^\bullet$, ${}^\bullet(D' \cup \{p_1, p_2\} - \{p_{12}\}) \subseteq (D' \cup \{p_1, p_2\} - \{p_{12}\})^\bullet$, this means that $D' \cup \{p_1, p_2\} - \{p_{12}\}$ is a siphon of N. □

Proposition 3: Suppose that N, N_1 and N_2 are defined in COMPOSITION-BY-TRANSITION. Let D be a siphon of N. Then, $D_i = D \cap P_i$ is a siphon of N_i, where $i = 1$ or 2.

Proof: Since $P_1 \cap P_2 = \varnothing$, $T_1 \cap T_2 = \varnothing$ and $D = (D \cap P_1) \cup (D \cap P_2) = D_1 \cup D_2$, it is obvious that ${}^\bullet D_1 \subseteq D_1{}^\bullet$ and ${}^\bullet D_2 \subseteq D_2{}^\bullet$ because of ${}^\bullet D \subseteq D^\bullet$. □

Proposition 4: Suppose that N and N' are defined in MERGE-N-TRANSITION. Let D' be a siphon of N'. Then, D' is a siphon of N if $\{t_{12}\} \notin {}^{\bullet}D'$.

Proof: Since D' is a siphon of N', ${}^{\bullet}D' \subseteq D'^{\bullet}$ in N'. It is obvious that ${}^{\bullet}D' \subseteq D'^{\bullet}$ in N if $\{t_{12}\} \notin {}^{\bullet}D'$, i.e., D' is a siphon of N if $\{t_{12}\} \notin {}^{\bullet}D'$. □

Theorem 5: Suppose that N and N_i are defined in COMPOSITION-BY-PLACE. Let M_i be an initial marking of N_i and M be obtained from M_i such that $M(p) = M_i(p)$ if $p \in P_i - \{p_1, p_2\}$ and $M(p_{12}) = \text{Max}\{M_1(p_1), M_2(p_2)\}$. Then, (N, M) is live and bounded if N_i is regular and (N_i, M_i) is live and bounded, where $i = 1$ and 2.

Proof: Since N_1 and N_2 are regular, by Theorem 3.1, N is regular. Since (N_1, M_1) and (N_2, M_2) are live and bounded, according to Lemma 1, all siphons of N_1 and N_2 are marked. Let D be a siphon of N. By Proposition 1, if $p_{12} \notin D$, then $D_i = D \cap P_i$ is a siphon of N_i. Hence, D_i is marked. If $p_{12} \in D$, then $(D \cap P_i) \cup \{p_i\}$ is a siphon of N_i by Property 1 and thus $(D \cap P_i) \cup \{p_i\}$ is marked. In this case, if p_i is marked for $i = 1$ or 2, then p_{12} is marked, otherwise, $D \cap P_i$ is marked. This means that all siphons of N are marked. Thus, (M, N) is live and bounded according to Lemma 1. □

Theorem 6 Suppose that N and N' are defined in MERGE-N-PLACE, and M is an initial marking of N. Let M' be obtained from M such that $M'(p) = M(p)$ if $p \in P - \{p_1, p_2\}$ and $M'(p_{12}) = \text{Max}\{M(p_1), M(p_2)\}$. Then, (N', M') is live and bounded if the following conditions hold:

(1) N is regular,
(2) p_1 and p_2 belong to the same cluster,
(3) there exists a positive P-invariant of N α such that $\alpha(p_1) = \alpha(p_2)$,
(4) there exists at least one P-invariant α such that $\langle \alpha \rangle \cap \{p_1, p_2\} = \{p_i\}$ for $i = 1$ or 2, and
(5) (N, M) is live and bounded.

Proof: By Conditions (1)-(4) and Theorem 2, N' is regular. Condition (5) implies that all siphons are marked according to Lemma 1. Consider any siphon D' of N'. By Proposition 2, if $p_{12} \notin D'$, then $D = D'$ is a siphon of N. Hence, D is marked. If $p_{12} \in D'$, then $(D \cap P) \cup \{p_1, p_2\}$ is a siphon of N and thus is marked. This means that all siphons of N' are marked. Thus, (N, M) is live and bounded according to Lemma 1. □

The example below shows the application of some results obtained in Section 3 and this section.

Example 1: In Figure 1, both (N_1, M_1) and (N_2, M_2) are live and bounded marked graphs. Of course, N_1 and N_2 are regular. After applying COMPOSITION-BY-PLACE to them, p_1 and p_2 are merged into r_1 and (N, M) shown in Figure 2 is obtained. By Theorem 1 and Theorem 5, N is regular and (N, M) is live and bounded. After applying MERGE-N-PLACE to (N, M), p_3 and p_4 are merged into r_2 and (N', M') shown in Figure 3 is obtained. Since, in N, p_3 and p_4 belong to the same cluster and $\{p_3, p_{12}, p_{13}\}$ is a P-component, by Theorem 2 and Theorem 6, N' is also regular and (N', M') is live and bounded.

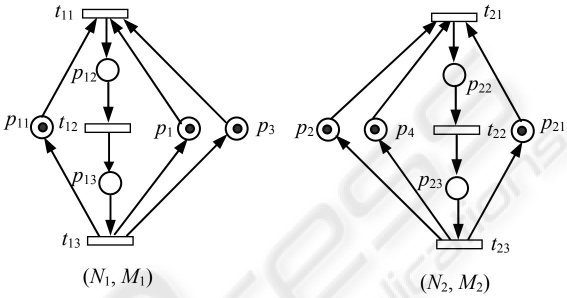

Figure 1: Two live and bounded Petri nets

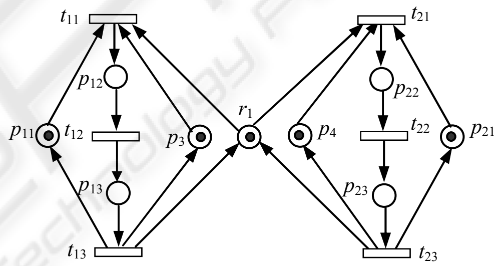

Figure 2: Petri net (N, M) obtained from Figure 1 by merging p_1 and p_2 into r_1.

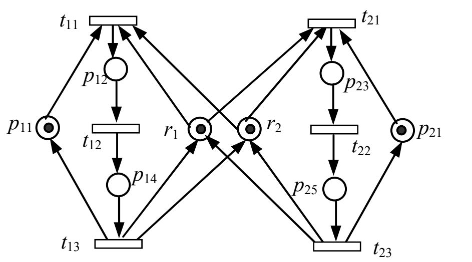

Figure 3: Petri net (N', M') obtained from Figure 2 by merging p_3 and p_4 into r_2.

Theorem 7: Suppose that N and N_i are defined in COMPOSITION-BY-TRANSITION. Let M_i be an initial marking of N_i and M be obtained from M_i such that $M(p) = M_i(p)$ if $p \in P_i$. Then, (N, M) is live and bounded if N_i is regular and (N_i, M_i) is live and bounded, where $i = 1$ and 2.

Proof: Similar to the proof of Theorem 5. □

Theorem 8: Suppose that N and N' are defined in MERGE-N-TRANSITION. Let M be an initial marking of N and M' be obtained from M such that $M'(p) = M(p)$ for any $p \in P$. Then, (N', M') is live and bounded if the following conditions hold:

(1) N is regular,
(2) t_1 and t_2 belong to the same cluster,
(3) there exists a positive T-invariant β such that $\beta(t_1) = \beta(t_2)$,
(4) there exists at least one T-invariant β such that $\langle \beta \rangle \cap \{t_1, t_2\} = \{t_i\}$ for $i = 1$ or 2, and
(5) (N, M) is live and bounded, and
(6) all input places of t_1 and t_2 are marked or every minimal siphon D' of N' with $t_{12} \in {}^\bullet D'$ is marked.

Proof: By Conditions (1)-(4) and Theorem 4, N' is regular. Condition (5) implies that all siphons are marked according to Lemma 1. Consider any siphon D' of N'. By Property 4, D' is a siphon of N if $\{t_{12}\} \notin {}^\bullet D'$ and thus D' is marked. By Condition (6), D' is marked if $t_{12} \in {}^\bullet D'$. This means that all siphons of N' are marked. Hence, (N, M) is live and bounded according to Lemma 1. □

5 CONCLUSION

This paper studied four compositional operations in terms of place and transition for pure and ordinary nets and showed that regularity, liveness and bounded-ness can be preserved automatically or under some simper conditions. These compositional operations are quite natural. COMPOSITION-BY-PLACE and COMPOSITION-BY-TRANSITION are usually be used to obtain more complex nets from some subnets. Liveness and boundedness preservations on the two operations for different subclasses of Petri nets have been studied under different conditions. Our results are based on the regularity preservation. MERGE-N-PLACE and MERGE-N-TRANSITION are two operations used in a net, a little similar to (Berthelot, 1987). Of course, these results that this paper are contributed are new and can accommodate the design of complex systems.

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REFERENCES

Berthelot, G., 1986. Checking properties of nets using transformations. In: Rozenberg, G.: LNCS Vol. 222: Advances in Petri nets 1985, pp. 19-40.

Berthelot, G., 1987. Transformations and decompositions of nets. In: Brauer, W.; Reisig, W.; Rozenberg, G., editors: LNCS Vol. 254: Petri Nets: Central Models and Their Properties, Advances in Petri Nets 1986, Part I, Proceedings of an Advanced Course, Bad Honnef, September 1986, pp. 359-376.

Desel, J., 1992. A proof of the rank theorem for extended free choice nets. In Jensen, K., editor: LNCS Vol. 616, Application and Theory of Petri Nets, Springer-Verlag, pp. 134-183.

Desel, J. and Esparza, J., 1995. Free choice Petri nets. London, Cambridge University Press.

Esparza, J. 1994. Reduction and synthesis of live and bounded free choice Petri nets, Information and Computation, Vol. 114, No. 1, pp. 50-87.

Huang, H., Jiao, L. and Cheung, T., 2005. Property-preserving subnet reductions for designing manufacturing systems with shared resources. Theoretical Computer Science, 332(1-3), pp.461-485.

Koh, I. and DiCesare, F., 1991. Modular transformation methods for generalized Petri nets and their applications in manufacturing automation, IEEE Transactions on Systems, Man and Cybernetics, 21, pp. 963-973.

Recalde, L., Teruel, E. and Silva, M., 1998. On linear algebraic techniques for liveness analysis of P/T systems, Journal of Circuits, Systems, and Computers, 8(1), pp.223-265.

Silva, M., Teruel, E. and Colom, J. M., 1998. Linear algebraic and linear programming techniques for the analysis of Place/Transition net systems, in Reisig, W. and Rozenberg, G., editors: LNCS Vol. 1491, Lectures on Petri Nets I: basic models, Springer-Verlag, pp.309-373.

Souissi, Y., 1991. On liveness preservation by composition of nets via a set of places, In: Rozenberg, G., editor: LNCS Vol. 524, Advances in Petri Nets, pp.277-295.

Suzuki, I. and Murata, T., 1983. A method for stepwise refinement and abstraction of Petri nets, Journal of Computer and System Sciences, 27, pp.51–76.

Valette, R., 1979. Analysis of Petri nets by stepwise refinements, Journal of Computer and System Sciences, 18, pp.35–46.

Zhou, M., 1996. Generalizing parallel and sequential mutual exclusions for Petri net synthesis of manufacturing systems, IEEE Int. Conf. On Emerging Technologies and Factory Automation, 1, pp.49-55.

Zhou, M. and Venkatesh, K., 1999. Modeling, simulation, and control of flexible manufacturing systems: A Petri net approach, World Scientific Publishing Co. Pte.Ltd.