WALSH TRANSFORM AS METHOD OF MIMO SYSTEMS IDENTIFICATION

Andrzej Żak
Department of Hydroacoustic and Radiolocation, Naval University of Gdynia, 81-103 Gdynia, Smidowicza 69, Poland

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Abstract: The paper presents method of MIMO system identification using Walsh transform. Paper includes description of mathematical basis of Walsh Transform. At the end of paper the results of research of identification for example multi input multi output object were presented.

1 INTRODUCTION

Identification is very general notation which has customary meaning in common speech. This notation is variously defined dependently on problem or science field but almost always is related with creating the mathematical models of objects. By the object's model it is understood the presentation of interested, essential properties in convenient form. Therefore identification often means the process in which results, basing on input and output signals, arise the mathematical model of object admitted as the best according to the accepted criterions.

Procedure of identification can be, in the simplest way, presented as it is done on figure 1.

Models appear in many fields of human knowledge. They can be meet in physics, biology, astronomy, technics, economy, sociology, psychology and many others. For engineers necessity of possession of models especially in projection phase has no need to be contended. For scientist models can be basis for wide research. In all cases possession of models allow for preliminary testing of creating systems what permits to shortening the time necessary for projection phase and inculcate of system. It also prevents from accidental damage of real object.

Often modeling starts from apply some physical laws which take part in investigated process. But it is only possible when these phenomenons are quite simple. If the numerical values of all external and internal conditions of modeled object are known and physical knowledge about this object is full it is possible to calculate coefficients of all parameters. However these cases are rare what become from knowledge shortage about internal process and indefinableness carried in by environment. In other hand very often they are only interesting dependences between object's input and output signals accepted that inputs signals are control signals and measured output signals give information about object state. In this case object can treated as single or multi input and single or multi output object but of unknown structure. In this

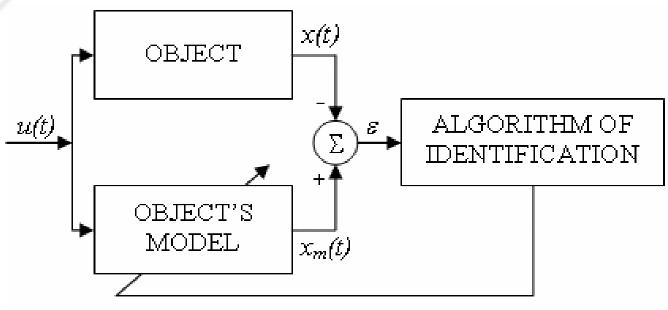

Figure 1: Schematic presentation of identification process

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time arise necessity to use identification or estimation of process parameters to calculate coefficients of models parameters. Values of this parameters must be choose in such way so at accepted mathematical structure will approximate the behaviour of object in the best way according to the accepted criterion. Because it is seldom possible to calculate the precision values of model's parameters, it means the coefficients of mathematical equations, the created model is only some simplicity of reality but very often such approximation is good enough.

2 WALSH TRANSFORM

Dynamical system can be considered as n-th dimensional linear object which is described by equation:

$$x(T_{k+1}) = Ax(T_k) + Bu(T_k) \tag{1}$$

where:

$x(T_k)$ – state vector;
$u(T_k)$ – control vector;
A – state matrix;
B – control matrix;
T_k – discreet time;

The main aim of identification is to calculate matrixes A and B knowing vector x and u.

Walsh function creates the family of binary, orthogonal function which belongs to periodical constant function. These functions are defined using Rademacher function which can be written as:

$$R_k(t) = \begin{cases} +1 & for\ \frac{i-1}{2^{k+1}} \le t < \frac{i}{2^{k+1}} \quad i-even \\ -1 & for\ \frac{i-1}{2^{k+1}} \le t < \frac{i}{2^{k+1}} \quad i-odd \end{cases} \tag{2}$$

where:

$k = 0, 1, 2, \ldots$
$i = 1, 2, \ldots, 2^{k+1}$

The relationship between Walsh function and Rademacher function is as follow:

$$\begin{cases} w_0(t) = 1 & for\ 0 \le t < 1 \\ w_n(t) = \prod_{i=0}^{n} (R_k(t))^{n_i} & for\ 0 \le t < 1 \end{cases} \tag{3}$$

where:

$n = 0, 1, 2, \ldots$
n_i - value of i-th position after conversion of n into binary system.

The state vector and control vector can be presented as orthogonal evolvement relative to Walsh base function:

$$\begin{aligned} x(T_k) &= Fw(k) \\ u(T_k) &= Hw(k) \end{aligned} \tag{4}$$

where:

$w(k)$ – k-th vector of discreet values of Walsh function;
F – matrix of coefficients of orthogonal evolvement of state vector;
H – matrix of coefficient of orthogonal evolvement of control vector;

After bilateral summation of equation (1) and substituted (4) we can receive the following equation:

$$\sum_{i=0}^{k} x(T_i) + x(T_{k+1}) - x(T_0) = \sum_{i=0}^{k} (Ax(T_i) + Bu(T_i)) \tag{5}$$

The beginning vector $x(T_0)$ can be written as orthogonal evolvement relative to Walsh function as:

$$x(T_0) = [x(T_0), 0, 0, \ldots, 0]w(k) \tag{6}$$

Using properties of Walsh transform that:

$$\sum_{i=0}^{k} w(i) = Sw(k) \tag{7}$$

and

$$w(k+1) = Zw(k) \tag{8}$$

where:

S – operating matrix for summarization of Walsh functions;
Z – operating matrix for shift of Walsh function;

the equation (5) can be written as:

$$Tw(k) = AFSw(k) + BHSw(k) \tag{9}$$

where:

$T = FS + FZ - x(T_0)$.

On this equation only matrixes A and B which are target of identification process are unknown and rest of matrixes and vectors are known. Getting $(n+r)$-th random Walsh function it is possible to calculate all coefficients of A and B matrixes what ends the identification process.

3 RESEARCH

Research was made on multi input multi output dynamical object. This object was remotely operated underwater vehicle type Ukwial which is equipment of research unit of Naval University of Gdynia. The precision mathematical model described by nonlinear equations of this object is know and was created during experimental measurements. The target of identification process was to present this object according to the equation (1). It can be done by calculating the coefficients of state matrix and control matrix. These matrixes were calculated using presented above method of Walsh transform.

First step was to collect the measurements of control vector and state vector. Next step was to basis on matrix of randomly chosen Walsh function and measured control vectors and state vectors calculate the orthogonal evolvement of all these vectors. Then the beginning control vector was also orthogonal evolvement relative to Walsh function. Next the matrix:

$$T = (FS + FZ - x(T_0))W'$$

was calculated. Now the matrix:

$$Q = \begin{bmatrix} FSW' \\ HSW' \end{bmatrix}$$

was created. Basis on this matrixes the matrix

$$R = TQ^{-1}$$

was calculated. The last step was to separate the coefficients of state matrix and control matrix. It can be done because the matrix R include state matrix in its first n-th rows where n is a length of state vector and include control matrix in rows from $(n+1)$ to $(n+r)$ where r is the length of control vector.

The basement for calculation was state vector of this object which can be written as:

$$[u, v, w, p, q, r]$$

where:

$[u, v, w]$ – surge, sway, heave velocity respectively;

$[p, q, r]$ – roll, pitch, yaw velocity respectively.

These parameters described object movement are possibilities to measure because this object is by standards equipped at log for linear speeds measurements, electric compass and electrolytic inclination-meter.

The control vector for this object can be written as follow:

$$[\tau_x, \tau_y, \tau_z]$$

where:

τ_x, τ_y, τ_z – set forces of propeller in x, y, z direction.

The criterion of opinion was the difference between state vector generated by object and calculated by identification procedure.

During research the control vector at start point has value $[0,0,0]$. After eleven seconds it takes value $[220,0,0]$. Next in twenty first seconds the control vector has become to $[220,220,0]$ and on the end after thirty one seconds it takes values of $[220,220,220]$.

Below on figures 2 to 7 the graphs of change of every component of state vector generated by object (solid, gray line) and calculated by identification process (doted, black line) were presented.

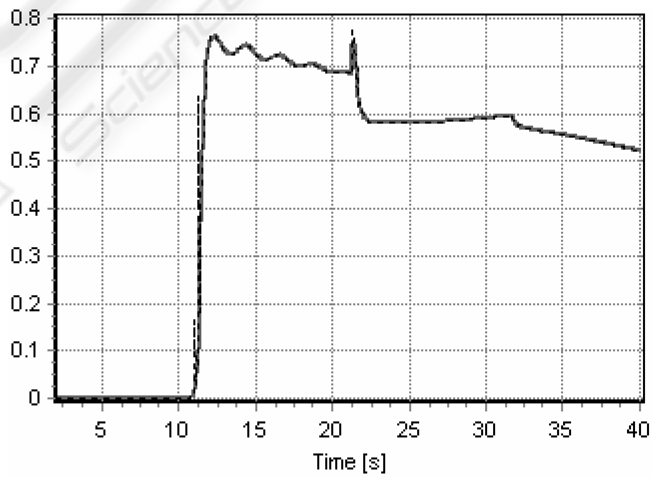

Figure 2: Changes of surge velocity generated by object and identified values

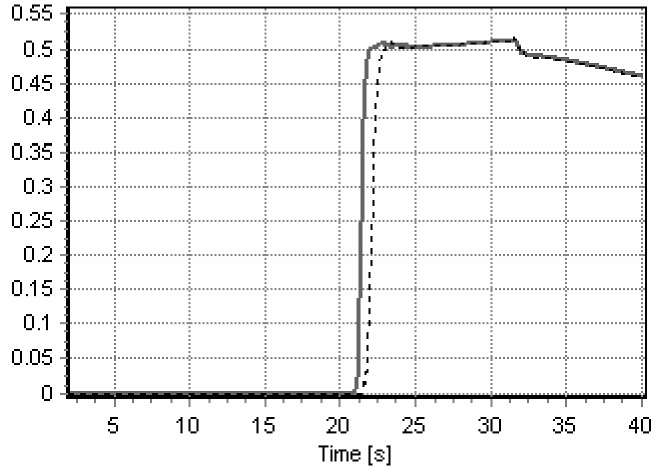

Figure 3: Changes of sway velocity generated by object and identified values.

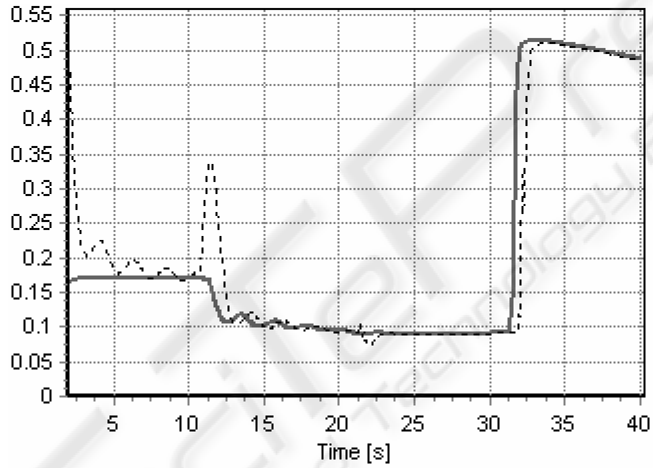

Figure 4: Changes of heave velocity generated by object and identified values

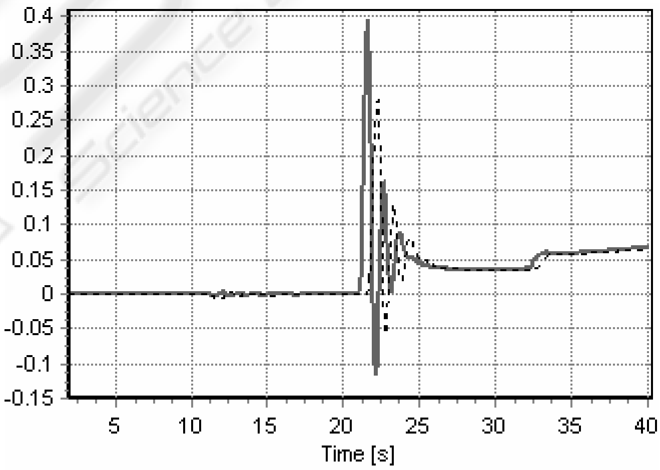

Figure 5: Changes of roll velocity generated by object and identified values

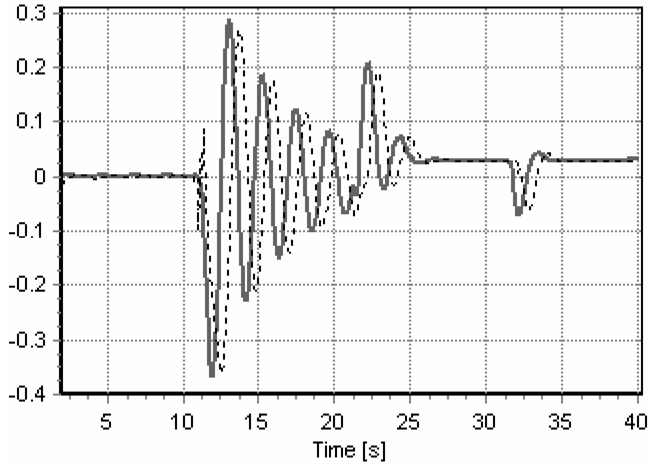

Figure 6: Changes of pitch velocity generated by object and identified values

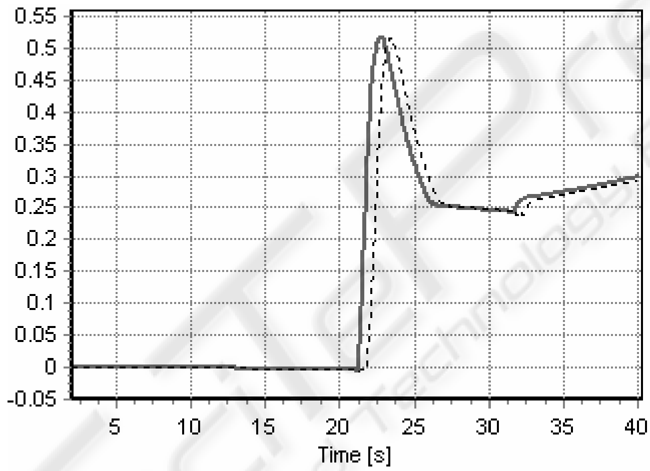

Figure 7: Changes of yaw velocity generated by object and identified values

4 CONCLUSION

As it is shown in research the Walsh transform is useful to identify dynamics of multi input, multi output object. Because of some limitation this method can be used to identify dynamics of objects which state vector is slow changed. In case of object with fast changed state vector presented method can’t calculate actual answer on time. This method is perceive as promise to practical use because of simplify of implementation on digital machine and flexibility of use.

Feature research will concentrated on optimal selection of Walsh transform length and on using this method to identify state vector of various dynamical object in control processes.

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