

# MULTI-BAND GPS SIGNAL TRACKING IN A HIGH DYNAMIC MANEUVERING SITUATION

Stanislas Boutoille, Serge Reboul, Mohammed Benjelloun  
*Laboratoire d'Analyse des Systèmes du Littoral (EA 2600)*  
50 Rue Ferdinand Buisson, B.P 699, 62228, Calais Cedex, France

Keywords: GPS, detection, information fusion.

Abstract: In a GPS receiver, the goal of the signal tracking is to synchronize local generated code and carrier with the received signal. After a step of acquisition, the receiver tracks the shifting of the local code provoked by the movements of the receiver and satellites. In the future evolution of the GPS, the system will have several carrier frequencies, then it will be possible to have several tracking systems working simultaneously for a same satellite. We present in this article a detection method for the tracking of the future multi-band GPS signal. This method is applied to the localization of a vehicle which makes high dynamic maneuver. We define a MAP detection criterion to fuse the shifts discriminator detection achieved on multi carrier frequencies. This criterion is defined in the case when shifts are not necessary simultaneous and when there is a lack of information on one frequency provoked by the unlocking of the code tracking on one of the carrier. Indeed, there is a difference between the instants of shifts on the different carrier frequencies. This difference is due to the effect of ionospheric propagation. The experimentations achieved on synthetic GPS signals show the advantages of the method compared to the classical algorithm.

## 1 INTRODUCTION

In the future GPS system, the civil code will be available on several carrier frequencies. This is the case for NAVSTAR with the three frequencies L1, L2 and L5 and for Galileo. The CDMA codes on these carriers will have different frequencies and shapes (BOC modulation). In this context it is interesting to perform the tracking of the codes simultaneously in the receiver on the different carriers. Indeed, we can combine the tracking information and then increase the accuracy and robustness of the tracking process. The synchronisation between the received code and the generated code is realized by the measurement of the maximum of the correlation. It is the step of acquisition. Each satellite has a different ranging code and so we can deduce the visible satellites and the distance between the satellite and the receiver. After this phase, the receiver tracks the shifting of the local code provoked by the movements of the receiver and satellites. In the GPS future evolution, the system will have several carrier frequencies, then it will be possible to have several tracking systems working simultaneously for a single satellite. We present in this article a detection method for tracking fusion applied to the

future multi carrier frequencies signal GPS. The generalized likelihood ratio (GLR) (Willksy and Jones, 1976) is a statistical test based on the likelihood ratio. It was first applied to the detection of abrupt changes on a signal coming from a linear system. We found in the literature two different approaches of the detection fusion: the centralized detection and the distributed one. The distributed detection, very largely studied by (Rao, 2001) and (Varshney, 1996), considers the detection at the level of each sensor and then carries out a global decision by combination of the local decisions. The difficulties in this case lie in the definition of the thresholds at the level of each local detector. Most of the works on this subject are based on criterion to be optimized such as Bayes or Neyman-Pearson. Actually a great number of papers deals with the problem of correlated decisions in the fusion case (Chen and Ansari, 1998). The centralized detection system considers all the measurements to perform the decision. This system offers the best performances but the quantity of information to be processed by the fusion system can quickly become significant. One of the drawbacks of this approach is its sensitivity to the synchronisation of the data. Various works concern the weighting of the distributions in order to favour

some measurements compared to others during fusion. The logarithm opinion pool (Benediktsson and Sveinsson, 1997) is derived from the joint probabilities using the Bayes' rule. The best difficulty associated to the use of this kind of method is the weight selection. Different interferences can disturb the GPS signal measurements and then improve the detection during the tracking process. Indeed, the received signal can be unlocked and then the tracking process can not perform any more. This happens more frequently with codes of high frequency. The goal of this work is to realize the detection in the tracking system on different frequencies in order to overcome these perturbations and then to increase accuracy and robustness. We propose a weighted hybrid fusion method inspired of the logarithmic opinion pool method. The introduction of weight in the hybrid fusion system, proposed in (Boutoille et al., 2004), increases the robustness in presence of unlocking. Hybridizing centralized and distributed fusion system permits to deal with unsynchronized signal on the carriers. Indeed, the ionosphere crossing causes a group velocity delay of the waves according to frequencies (M. Grewal and Andrews, 2001). The paper is organized as following. Section 2 describes the GPS signal model. The system of weighed fusion is described in section 3 and the fusion method in section 4. In section 5 we present numerical experimentations on synthetic GPS signals.

## 2 MODEL OF GPS SIGNAL

### 2.1 Statistical model

The incoming GPS signal is demodulated and correlated with respectively a carrier and a code generated by the receiver. Let consider the expression of the in-phase and quadrature components after correlation and demodulation for each time of samples  $t_k$  (Dierendonck et al., 1992):

$$I_k = \sqrt{2C/N_0T} R_f(\tau_k) \cos(\phi_k) + n_{ik} \quad (1)$$

$$Q_k = \sqrt{2C/N_0T} R_f(\tau_k) \sin(\phi_k) + n_{qk} \quad (2)$$

With :

$T$  = predetection bandwidth where the correlation is done,

$\phi_k$  = residual phase tracking error at time  $t_k$ ,

$\tau_k$  = is the shift between the local and the received code CDMA ,

$n_k$  = the in-phase and quadrature noise samples,

$R_f$  = correlation between filtered signal and the non-filtered code generated,

$C/N_0$  = signal-to-noise ratio normalized to a 1 Hz bandwidth.

In the non-coherent case, the mean of the early-minus-late discriminator is given by :

$$E [D_{\tau_k}] = \bar{I}_E^2 + \bar{Q}_E^2 - \bar{I}_L^2 - \bar{Q}_L^2 \quad (3)$$

Where  $I_E$  and  $Q_E$  are the in-phase and quadrature component, correlated with a code which is generated slightly early.  $I_L$  and  $Q_L$  are the same components correlated with a code slightly late. We can calculate the discriminator's statistical parameters. So for the mean :

$$E [D_{\tau_k}] = 2C/N_0T \left[ R_f^2\left(\tau_k - \frac{T_c}{2}\right) - R_f^2\left(\tau_k + \frac{T_c}{2}\right) \right] \quad (4)$$

And the variance :

$$\sigma_{D_{\tau_k}}^2 = 8+8 C/N_0T \left[ R_f^2\left(\tau_k - \frac{T_c}{2}\right) + R_f^2\left(\tau_k + \frac{T_c}{2}\right) \right] \quad (5)$$

The code's properties make that the delay is characterized by changes of stationnarities on the discriminator measurements. Indeed, when the delay exceeds the value of the sampling period, there is a change in the mean and the variance. It is from the detection of this change that the code locally generated is readjusted with the received code. After a step of acquisition, the value of the delay  $\tau_k$  in the expression of the correlation is zero. This step of acquisition is followed by a step of tracking where the local code is shifted to stay locked with the receiving code. In this step we try to keep the discriminator value close to zero.

### 2.2 Problem position

The statistical parameters and the detection quality, are function of the correlation measurement  $R_f$ . The expression of the correlation  $R_f$  changes with the frequency of the code CDMA. Therefore, for a fixed  $\tau_k$ , the correlation's value is different. A higher code frequency will have a narrower peak of correlation and will allow a better detection of the shifts caused by the delay. Indeed, the correlation's evolution for fixed value of  $\tau$  increases with the code frequency. Unfortunately, the sensitiveness of the code to unlock increases also with the code frequency. In the case of higher frequency, the tracking is more accurated but less robust especially when the relatives speeds between the receiver and the satellites are high. The goal of this work is to fuse the information coming from the frequencies of a multi carrier GPS receiver. In this context, we want to improve accuracy and robustness of the tracking for codes with different frequencies.

### 3 LOGARITHM OPINION POOL

The posterior probability to have a sequence of ratures  $\underline{r}$  in a signal  $\underline{y}$  is written (Reboul and Benjelloun, 2004):

$$\sup_{\{\underline{r}, \underline{\theta}\}} P_r(\underline{R} = \underline{r}/\underline{Y} = \underline{y}; \underline{\theta}) \quad (6)$$

We consider two hypothesis,  $H_1$  and  $H_0$  respectively for the presence and the absence of change on a signal. We can then write the following rule of decision:

$$\begin{aligned} \sup_{\{\underline{r}^1\}} P_r(\underline{R}^1 = \underline{r}^1/\underline{Y} = \underline{y}; \underline{\theta}^1) \\ \underset{\substack{< H_0 \\ > H_1}}{>} \\ \sup_{\{\underline{r}^0\}} P_r(\underline{R}^0 = \underline{r}^0/\underline{Y} = \underline{y}; \underline{\theta}^0) \end{aligned} \quad (7)$$

with  $\underline{r}^1$  and  $\underline{r}^0$ , the changes sequences respectively associated to the hypothesis  $H_1$  and  $H_0$ . The parameters  $\underline{\theta}^1$  and  $\underline{\theta}^0$  corresponding respectively to the sequences  $\underline{r}^1$  and  $\underline{r}^0$  are assumed to be known c.f. equations(6,7).

In the case of J signals, the information combination with the logarithm opinion pool method, gives the following test of hypothesis:

$$\begin{aligned} \sup_{\{\underline{r}^1\}} (\prod_{j=1}^J P_r(\underline{R}^1 = \underline{r}^1/\underline{Y}_j = \underline{y}_j; \underline{\theta}_j^1)^{\beta_j}) \\ \underset{\substack{< H_0 \\ > H_1}}{>} \\ \sup_{\{\underline{r}^0\}} (\prod_{j=1}^J P_r(\underline{R}^0 = \underline{r}^0/\underline{Y}_j = \underline{y}_j; \underline{\theta}_j^0)^{\beta_j}), \end{aligned} \quad (8)$$

where  $\beta_j$  is the weighted coefficient and  $\underline{y}_j$  the signal  $j$ . With this formulation we can define different tests for different values of  $\beta_j$ . For example if one signal is unlocked his associated value  $\beta_j$  is set to zero.

## 4 SYSTEM OF WEIGHTED FUSION

### 4.1 Centralized fusion

Let define with the expression 8, the centralised fusion test. The Bayes' theorem gives us:

$$\sup_{\{\underline{r}^1\}} \{ \prod_{j=1}^J (h(\underline{y}_j/\underline{r}_j^1; \underline{\theta}_j^1) f(\underline{\theta}_j^1) \pi_j(\underline{r}^1))^{\beta_j} \} \\ \underset{\substack{< H_0 \\ > H_1}}{>} \quad (9)$$

$$\sup_{\{\underline{r}^0\}} \{ \prod_{j=1}^J (h(\underline{y}_j/\underline{r}_j^0; \underline{\theta}_j^0) f(\underline{\theta}_j^0) \pi_j(\underline{r}^0))^{\beta_j} \}$$

where:

$\pi_j(\underline{r}^i)$  = is the prior law of the change configuration  $\underline{r}^i$  is the change configuration associated to the hypothesis  $H_i$  for the signal  $j$ ,  $i \in \{0, 1\}$ ,  $f(\underline{\theta}_j)$  = is the prior law for the statistical parameters. It is supposed to be an uniform law on all the values of  $\underline{\theta}_j$ .

By taking the logarithm of the preceding expression and let  $\lambda_j = \frac{\pi_j(\underline{r}^0)}{\pi_j(\underline{r}^1)}$ , we can then write the decision criterion:

$$\begin{aligned} \sup_{\{\underline{r}^1, \underline{r}^0\}} \{ \sum_{j=1}^J \beta_j \cdot (\ln(h_j(\underline{y}_j/\underline{r}_j^1; \underline{\theta}_j^1)) - \\ \ln(h_j(\underline{y}_j/\underline{r}_j^0; \underline{\theta}_j^0))) \} \underset{> H_1}{\overset{< H_0}{>}} \sum_{j=1}^J \beta_j \cdot \ln(\lambda_j) \end{aligned} \quad (10)$$

where  $\underline{r}^i$  the change point's configuration associated to the hypothesis  $H^i$  is the same for the two signals.

### 4.2 Decentralized fusion

In the decentralized case, we consider different configurations of change for each signal. The estimate of these configurations is done independently on each signal. We have:

$$\begin{aligned} \sum_{j=1}^J \{ \sup_{\{\underline{r}_j^1, \underline{r}_j^0\}} (\beta_j \cdot (\ln(h_j(\underline{y}_j/\underline{r}_j^1; \underline{\theta}_j^1)) - \\ \ln(h_j(\underline{y}_j/\underline{r}_j^0; \underline{\theta}_j^0)))) \} \underset{> H_1}{\overset{< H_0}{>}} \sum_{j=1}^J \beta_j \cdot \ln(\lambda_j) \end{aligned} \quad (11)$$

This fusion's system is adapted in the case where the changes are not synchronized.

### 4.3 Hybrid fusion

In the hybrid fusion method we proposed, we combine the decisions of the centralized fusion and decentralized fusion systems (Boutoille et al., 2004). The thresholds of the centralized and distributed fusion methods and the rule of combination are chosen to maximize the Neyman Pearson criterion. To define the decision rule of each fusion method, we use an optimization element by element. Then we search the decision rules that maximize the Lagrangian L (Varshney, 1996), define in this case as :

$$L = P_D - \left( \prod_{j=1}^J \lambda_j^{\beta_j} \right) (P_F - \alpha) \quad (12)$$

Let consider here the case of N local detector and a global fusion detector that combines the local decisions.  $u_i = j$  is the decision of the hypothesis  $H_j$  by the detector  $i$ .  $u_0 = j$  is the global decision of the fusion system. We have :

$$P_f = P(u_0 = 1/H_0) = \sum_u P(u_0 = 1/u)P(u/H_0) \quad (13)$$

where  $u$  is all the possible combinations of detections. Then we have :

$$L = P_D - (\prod_{j=1}^J \lambda_j^{\beta_j})(P_F - \alpha) = \quad (14)$$

$$P_D - (\prod_{j=1}^J \lambda_j^{\beta_j})P_F + \alpha(\prod_{j=1}^J \lambda_j^{\beta_j})$$

with :

$$L = \sum_{u=1}^N P(u_0 = 1/u)P(u/H_1) - \quad (15)$$

$$(\prod_{j=1}^J \lambda_j^{\beta_j})(\sum_u P(u_0 = 1/u)P(u/H_0)) - \alpha$$

and :

$$L = (\prod_{j=1}^J \lambda_j^{\beta_j})\alpha + \sum_u P(u_0 = 1/u)[P(u/H_1) \quad (16)$$

$$- (\prod_{j=1}^J \lambda_j^{\beta_j})P(u/H_0)]$$

We use an optimization elements by elements, and we have:

$$L = (\prod_{j=1}^J \lambda_j^{\beta_j})\alpha + \sum_{u^k} P(u_0 = 1/u_k = 0, u^k). \quad (17)$$

$$[P(u_k = 0, u^k/H_1) - (\prod_{j=1}^J \lambda_j^{\beta_j})P(u_k = 0, u^k/H_0)]$$

$$+ \sum_{u^k} P(u_0 = 1/u_k = 1, u^k).[P(u_k = 1, u^k/H_1)$$

$$- (\prod_{j=1}^J \lambda_j^{\beta_j})P(u_k = 1, u^k/H_0)]$$

where,  $u^k = (u_1, \dots, u_{k-1}, u_{k+1}, \dots, u_N)^T$ . In our case the measurements are correlated then we have :

$$P(u_1, u_2, \dots, u_N/H_1) \quad (18)$$

$$= P(u_1/H_1) \prod_{j=2}^N P(u_j/u_1, u_2, \dots, u_{j-1}, H_1)$$

with  $N$  the number of sensors.

Therefore :

$$P(u_k = 0, u^k/H_1) \quad (19)$$

$$= P(u_1/H_1) \prod_{j=2, j \neq k}^N [P(u_j/u_1, \dots, u_{j-1}, H_1)$$

$$\cdot P(u_k = 0/u_1, \dots, u_{k-1}H_1)]$$

Let :

$$P'(u^k/H_1) \quad (20)$$

$$= P(u_1/H_1) \prod_{j=2, j \neq k}^N P(u_j/u_1, \dots, u_{j-1}, H_1)$$

$$\neq P(u^k/H_1)$$

Since  $u_k$  does not depend on  $H_i$ , we can write :

$$P(u_k = 1/u_1, \dots, u_{k-1}, H_i) \quad (21)$$

$$= \int_{y_k} P(u_k = 1/u_1, \dots, u_{k-1}, y_k)P(y_k/H_i)dy_k$$

We can write :

$$L = C^k \quad (22)$$

$$+ \int_{y_k} P(u_k = 1/u_1, \dots, u_{k-1}, y_k)[C_1^k P(y_k/H_1)$$

$$- (\prod_{j=1}^J \lambda_j^{\beta_j})C_0^k P(y_k/H_0)]dy_k$$

with :

$$C^k = (\prod_{j=1}^J \lambda_j^{\beta_j})\alpha \quad (23)$$

$$+ \sum_{u^k} P(u_0 = 1/u_k = 0, u^k)[P'(u^k/H_1)$$

$$- (\prod_{j=1}^J \lambda_j^{\beta_j})P'(u^k/H_0)]$$

and :

$$C_i^k = \sum_{u^k} [P(u_0 = 1/u_k = 1, u^k) \quad (24)$$

$$- P(u_0 = 1/u_k = 0, u^k)]P'(u^k/H_1)$$

$C^k$  is independent of the decision rule associated to the detector  $k$ , then  $L$  is maximum when the integral of the expression 22 is maximum. We have :

$$P(u_k = 1/u_1, \dots, u_{k-1}, y_k) = 0, \text{if} \quad (25)$$

$$C_1^k P(y_k/H_1) - (\prod_{j=1}^J \lambda_j^{\beta_j})C_0^k P(y_k/H_0) < 0$$

and :

$$P(u_k = 1/u_1, \dots, u_{k-1}, y_k) = 1, \text{if} \quad (26)$$

$$C_1^k P(y_k/H_1) - (\prod_{j=1}^J \lambda_j^{\beta_j})C_0^k P(y_k/H_0) > 0$$

The decision rule of the detector  $k$  is given by:

$$\frac{P(y_k/H_1)}{P(y_k/H_0)} \underset{H_1}{\overset{H_0}{>}} \lambda'_k \quad (27)$$

With :

$$\lambda'_k = (\prod_{j=1}^J \lambda_j^{\beta_j}) \frac{C_0^k}{C_1^k} \quad (28)$$

We consider here the case of our application, two detectors (the decisions are coming from the two fusion methods) and the fusion rules AND and OR. Let  $\lambda'_1$  be the threshold value for the centralized fusion and  $\lambda'_2$  the threshold value for the distributed fusion criterion :

$$\lambda'_1 = (\prod_{j=1}^J \lambda_j^{\beta_j}) \frac{P f_2}{P d_2} \quad (29)$$



and :

$$\lambda'_2 = \left( \prod_{j=1}^J \lambda_j^{\beta_j} \right) \frac{P f_1^1}{P d_1^1} \quad (30)$$

Where  $P d_1^1$  is the probability to detect with the centralized fusion method when we have detected with the decentralized fusion method. For hybrid fusion AND we have:

$$\lambda'_1 \frac{P f_1^1}{P d_1^1} = \lambda'_2 \frac{P f_2}{P d_2} \quad (31)$$

For the OR hybrid fusion:

$$\lambda'_1 \frac{1 - P f_1^1}{1 - P d_1^1} = \lambda'_2 \frac{1 - P f_2}{1 - P d_2} \quad (32)$$

We observe in our experimentations that we have better performances for the AND fusion rule than for the OR fusion rule. We will use the AND fusion rule for the experimentations.

## 5 APPLICATION TO GPS SIGNAL TRACKING

We present experimentations in order to measure the performances of the method in the presence of unlocks on the signals. The code on L5 has a higher frequency than on L1. This frequency will then be brought to unlock more easily

### 5.1 Experimentations

In these simulations, we realize a code tracking in a realistic dynamic context. We consider positions of a dynamic receiver, an 8 satellites constellation, and a mean HDOP (Horizontal Dilution Of Precision) of 1 during the experimentation. The receiver has a 100 m/s speed with a trajectory that changes during 2 s. We provide measurement every ms. The sampling rate is fixed at 20.46 MHz and the frequency of the L1 code is 1.023 MHz against 10.23MHz for L5. The power of the signals is fixed for simulation at 40 dB.Hz for each satellite. When an unlocking occurs during the tracking process, the value of correlation is close to 0 and we have  $E[D_{\tau_k}] = 0$  and  $\sigma_{d_{\tau_k}}^2 = 8$  for the statistical parameters of the discriminator. In order to define the weighted coefficients, we realize first a test which indicates the presence of unlocking. This test is carried out independently on the singles frequencies measurements and the thresholds are fixed to detect the unlocking with probability one. In this case, we can define the values of the weighting coefficients  $\beta_j$ . We affect the value  $\beta_1 = 1$  and  $\beta_2 = 0$  when we detect an unlocking for L5 and the tracking is only performed on L1. In the same way, we affect

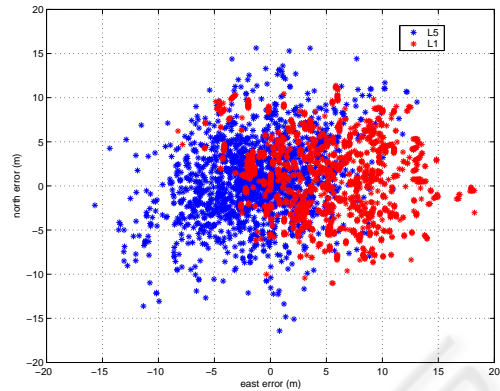


Figure 1: L1 and L5 error measurements.

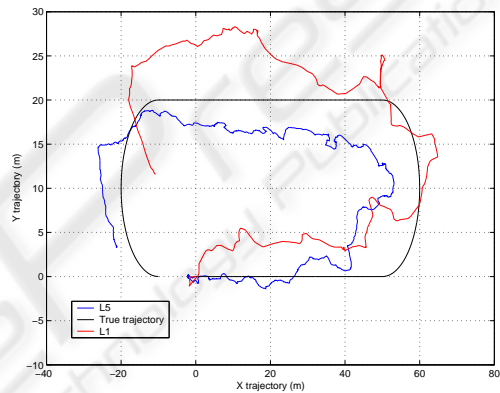


Figure 2: L1 and L5 trajectory measurements.

the value  $\beta_1 = 0$  and  $\beta_2 = 1$  when L1 is unlocked. For all the other cases, we fused the tracking of the two carriers by affecting  $\beta_1 = 0.5$  and  $\beta_2 = 0.5$ .

### 5.2 Results

We show figures 1 and 3 the position errors on X and Y for the receiver trajectory defined on figures 2 and 4. On figure 1, we display in blue the results obtained for the L5 code tracking and in red the results obtained for the L1 code tracking. The estimate trajectories displayed on figures 2 and 4 are obtained after filtering the GPS data with a navigation Kalman filter. We can see in this experimentation that the measurements on L5 give better performances than L1. But during the experimentation three satellites get unlocks on L5 and all the satellites stay locked on L1.

On figure 3, we display in blue the results obtained for the classical hybrid fusion tracking code and in red the results obtained for the weighted hybrid fusion. This experimentation shows that the tracking fusion method gives better performances than the single tracking process. The weighted hybrid fusion of-

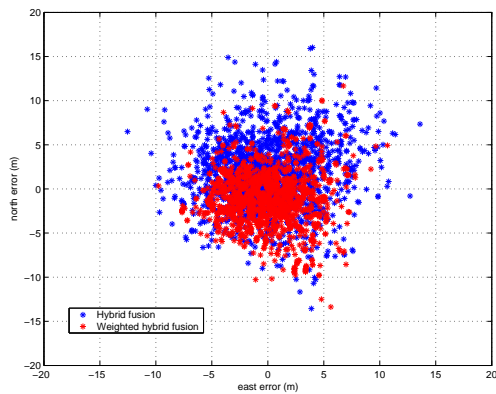


Figure 3: Hybrid fusion error measurements.

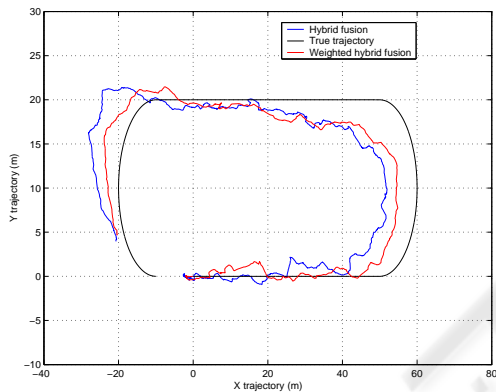


Figure 4: Hybrid fusion trajectory measurements.

fers best results than classical hybrid fusion and there is one satellite unlocked for the classical hybrid fusion while all the satellites are locked for weighted hybrid fusion. We report TABLE 1 measurements of the root mean squared error in meters on X and Y for each case as well as a measurement of the eccentricity. The eccentricity is the distances between the mean position and the real position. We can notice that the proposed method has better performances for this criterion.

Table 1: Root mean squared error in meters.

	X	Y	Eccentricity
L1	6.80	4.53	5.08
L5	4.43	4.55	1.36
Hybrid fusion	3.67	4.25	1.72
Weighted Hybrid fusion	2.88	3.15	1.04

## 6 CONCLUSION

The goal of this work is to track different GPS carrier frequencies in a high dynamic situation. We propose a weighted hybrid fusion method inspired of the logarithmic opinion pool method. The introduction of weight in the hybrid fusion system increases the robustness in presence of unlocks. Furthermore, hybridizing centralized and distributed fusion system will permit to deal with unsynchronized signal on the carriers. We show in the experimentations on synthetic GPS L1 and L5 signals, that our method of fusion increases tracking robustness and accuracy and avoided the unlocking for the higher frequency code. The perspectives of this work are about the generalization of the method for more than two carrier frequencies and its application to the future signals of GALILEO.

## REFERENCES

- Benediktsson, J. A. and Sveinsson, J. R. (1997). Hybrid consensus theoretic classification. *IEEE Transactions on Geoscience and Remote sensing*, 35(4):833–843.
- Boutoille, S., Reboul, S., and Benjelloun, M. (2004). Fusion of detections in a multi-carrier gps receiver. In *Proceedings of the Seventh International Conference on Information Fusion*, pages 85–90, Stockholm, Sweden.
- Chen, J. and Ansari, N. (1998). Adaptive fusion of correlated local decisions. *IEEE Transactions On Systems, Man., And Cybernetics - Part C : Applications And Reviews*, 28(2):276–281.
- Dierendonck, A. V., Fenton, P., and Ford, T. (1992). Theory and performance of narrow correlator spacing in a gps receiver. *journal of the Institute of Navigation, USA*, 39(3):265–283.
- M. Grewal, L. W. and Andrews, A. (2001). *Global Positioning System, Inertial Navigation and Integration*. Wiley Interscience, New Jersey.
- Rao, N. (2001). On fusers that perform better than best sensor. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 23(8):904–909.
- Reboul, S. and Benjelloun, M. (2004). Joint segmentation of a set of piecewise stationary processes. In *IEEE International Conference on Computational Cybernetics*, pages 191–195, Austria.
- Varshney, P. K. (1996). *Distributed detection and data fusion*. Springer.
- Willsky, A. and Jones, H. (1976). A generalized likelihood ratio approach to the detection and estimation of jumps in linear systems. *IEEE Transactions on Automatic Control*, 21:108–112.