

# A NEW METHOD FOR WEIGHT UPDATING IN FUZZY COGNITIVE MAPS USING SYSTEM FEEDBACK

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**Keywords:** Fuzzy Cognitive Maps, Hebbian rule, State feedback, Weight Updating.

**Abstract:** Fuzzy Cognitive Maps (FCMs) have found many applications in social -financial -political problems. In this paper we propose a method of FCM operation, which can be used to represent and control any real system, including traditional electro-mechanical systems. In the proposed approach the FCM reaches its equilibrium point using direct feedback from the node values of the real system and the limitations imposed by the control objectives for the node values of the system. The experts' knowledge, which is represented in the weights of the nodes' interconnections, undergoes a continuous on-line adaptation based on feedback from the real system. An algorithm for weight updating is proposed, which is based on system feedback and which includes specially designed matrices that lead the FCM and consequently the real system associated with it in a balanced equilibrium state. The proposed methodology is tested by simulating the operation of a hydro-electric plant.

## 1 INTRODUCTION

Some problems of electrical and mechanical engineering are placed in the fuzzy part of science and they have been studied thoroughly enough the last years from a good many of scientists. A large number of different methods have occasionally been used in order to work out this kind of problems. The scientific community was placed under the obligation of giving solutions to problems the settlement of which seemed rather difficult the years before.

Fuzzy Cognitive Maps (FCM) can model dynamical complex systems that change with time following nonlinear laws (Kosko, 1992). FCMs use a symbolic representation for the description and modeling of the system. In order to illustrate different aspects in the behavior of the system, a fuzzy cognitive map is consisted of nodes with each node representing a characteristic of the system. These nodes interact with each other showing the dynamics of the system in study. An FCM integrates the accumulated experience and knowledge on the operation of the system, as a result of the method by which it is constructed, i.e., using human experts

who know the operation of system and its behavior.

Fuzzy cognitive maps have already been used to model behavioral systems in many different scientific areas. For example, in political science (Schneider, 1998), fuzzy cognitive maps were used to represent social scientific knowledge and describe decision-making methods (Kottas, 2003), (Zhang, 1989), (Georgopoulou, 2001). Kosko enhanced the power of cognitive maps considering fuzzy values for their nodes and fuzzy degrees of interrelationships between nodes (Kosko, 1992), (Kosko, 1997). After this pioneering work, fuzzy cognitive maps attracted the attention of scientists in many fields and they have been used in a variety of different scientific problems. Fuzzy cognitive maps have been used for planning and making decisions in the field of international relations and political developments (Kottas, 2003) and to model the behavior and reactions of virtual worlds. FCMs have been proposed as a generic system for decision analysis (Zhang, 1989), (Zhang, 1992) and as coordinator of distributed cooperative agents.

One open issue related to FCMs, is their operation in close cooperation with the real system they describe. This in turn implies that such an on-

line interaction with the real system might require changes in the weight interconnections, which reflect the experts' knowledge about the node interdependence. This knowledge might not be entirely correct or perhaps, the system has undergone to changes during its operation.

In this paper an FCM operation method is proposed, which is in close interaction with the system it represents. The FCM nodes are divided in control and reference nodes, where control nodes represent control variables of the system and reference nodes represent either variables with constant values or variables with desired (goal) values. In the proposed approach, the FCM reaches its equilibrium point using direct feedback from the node values of the real system and the limitations imposed by the reference nodes. The interconnections weights are on-line adjusted during this operation by using an extended Hebbian updating law, which uses the system feedback and employs two specially defined collateral matrices, which help the FCM to adjust its weights and reach an equilibrium point in a more realistic and balanced way.

The paper is organized as follows: Section 2 gives a short description of FCMs and their way of operation. Section 3 introduces the proposed combined operation of the FCM and the real system and presents the relevant Hebbian rule to update interconnections weights. The proposed weight updating method is extended in Section 3.1 to include the specially defined placement and calibration matrices. Section 4 gives a simulation study of a hydro-electric power plant, where a comparative study of the proposed method versus the traditional approach in reaching equilibrium points in FCM is made. The final conclusions are given in Section 5.

## 2 FUZZY COGNITIVE MAPS REPRESENTATION AND DEVELOPMENT

Fuzzy cognitive maps approach is a hybrid modeling methodology, exploiting characteristics of both fuzzy logic and neural networks theories and it may play an important role in the development of intelligent manufacturing systems. The utilization of existing knowledge and experience on the operation of complex systems is the core of this modeling approach. Experts develop fuzzy cognitive maps and they transform their knowledge in a dynamic cognitive map (Miao, 2001).

The graphical illustration of FCM is a signed directed graph with feedback, consisting of nodes and weighted interconnections. Nodes of the graph stand for the nodes that are used to describe the behavior of the system and they are connected by signed and weighted arcs representing the causal relationships that exist among nodes (Fig. 1). Each node represents a characteristic of the system. In general it stands for states, variables, events, actions, goals, values, trends of the system which is modeled as an FCM (Jang, 1995). Each node is characterized by a number  $A_i$ , which represents its value and it results from the transformation of the real value of the system's variable, for which this node stands, in the interval  $[0, 1]$ . It must be mentioned that all the values in the graph are fuzzy, and so weights of the interconnections belong to the interval  $[-1, 1]$ . With the graphical representation of the behavioral model of the system, it becomes clear which node of the system influences other nodes and in which degree.

The most essential part in modeling a system using FCMs, is the development of the fuzzy cognitive map itself, the determination of the nodes that best describe the system, the direction and the grade of causality between nodes. The selection of the different factors of the system, which must be presented in the map, will be the result of a close-up on system's operation behavior as been acquired by experts. Causality is another important part in the FCM design, it indicates whether a change in one variable causes change in another, and it must include the possible hidden causality that it could exist between several nodes. The most important element in describing the system is the determination of which node influences which other and in what degree. There are three possible types of causal relationships among nodes that express the type of influence from one node to the others. The weight of the interconnection between node  $C_i$  and node  $C_j$  denoted by  $W_{ij}$ , could be positive ( $W_{ij} > 0$ ) for positive causality or negative ( $W_{ij} < 0$ ) for negative causality or there is no relationship between node  $C_i$ , and node  $C_j$ , thus  $W_{ij} = 0$ . The causal knowledge of the dynamic behavior of the system is stored in the structure of the map and in the interconnections that summarize the correlation between cause and effect. The value of each node is influenced by the values of the connected nodes with the corresponding causal weights and by its previous value. So, the value  $A_j$  for each node  $C_j$  is calculated by the following rule, (Jang, 1995):

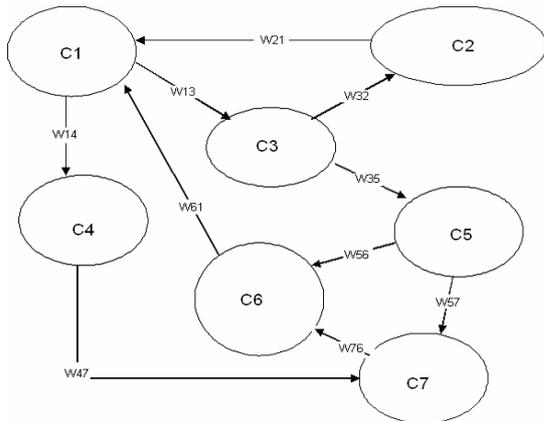


Figure 1: A simple fuzzy cognitive map

$$A_j^s = f \left( \sum_{i=1, i \neq j}^N A_i^{s-1} W_{ij} + A_j^{s-1} \right) \quad (1)$$

where  $A_j^s$ , is the value of node  $C_j$  at step  $s$ ,  $A_i^{s-1}$  is the value of node  $C_i$ , at step  $s-1$ ,  $A_j^{s-1}$  is the value of node  $C_j$  at step  $s-1$ , and  $W_{ij}$  is the weight of the interconnection between  $C_i$  and  $C_j$ , and  $f$  is a squashing function.

**Squashing functions:**

- 1)  $f = \tanh(x)$  maps the nodes values in  $[-1, 1]$
- 2)  $f = \frac{1}{1 + e^{-cx}}$  by using  $c=1$  we convert the nodes values in  $[0, 1]$ . It also called **sigmoid** function. The second function is the most common function which is used in FCM's.

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### 3 THE NEW METHOD FOR WEIGHT UPDATING

In this section we will analyze the proposed method of updating the interconnections weights of FCM taking into account feedback node values from the real system. Using the updated weights the FCM reaches a new equilibrium point by means of equation (1). Some of the new node values can be applied as control values to the real system. One commonly used technique for updating weights in FCMs is the Hebbian updating rule (Kosko, 1986 a,b), (Papageorgiou, 2004). In our approach the updating is made by using the conventional Hebbian

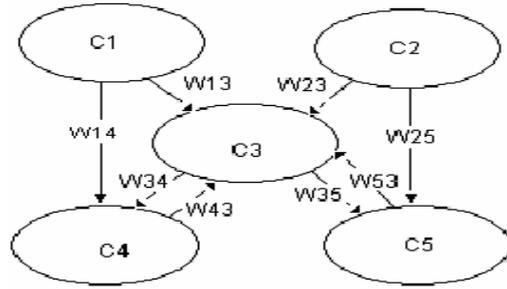


Figure 2: FCM with steady state nodes

rule, which however, uses measurements from the node values taken from the real system. This way the updating of the weights reflects real changes that have to be made in our knowledge about the system, which is represented by the interconnection weights. This situation is more apparent in cases where there exist steady value nodes, which, in the real system, are not affected by the values of the other nodes. In this case, if the FCM convergence equation (1) is left to operate with weight adjustments that do not take into account the steady node values fact, then the equilibrium point will give node values for the above mentioned nodes, which might be different than the steady values, which in turn implies an unrealistic point of operation for our system.

Let us, for example, analyze an FCM having one or more nodes with constant values. This means that no human action can intervene, in a mechanic way with this value. Suppose that in the FCM of Fig. 2 nodes C1 and C2 cannot change their values. The values of these nodes derive from the system that is examined. The table of interconnection weights for this system is:

$$W = \begin{bmatrix} 0 & 0 & W13 & W14 & 0 \\ 0 & 0 & W23 & 0 & W25 \\ 0 & 0 & 0 & W34 & 0 \\ 0 & 0 & W43 & 0 & 0 \\ 0 & 0 & W53 & 0 & 0 \end{bmatrix}$$

We see that columns 1 and 2 that concern nodes C1 and C2 are zero. When applying equation (1) for node value updating we have to consider the steady values of nodes C1 and C2 by using a companion adjusting equation. Thus, equation (1) is now replaced by the following two equations:

$$A_j^{s, FCM} = f \left( \sum_{i=1, i \neq j}^N A_i^{s-1, FCM} W_{ij} + A_j^{s-1, FCM} \right) \quad (2)$$

And for the steady state nodes the correction equation is:

$$A_j^{s,FCM} = A_j^{system} \quad (3)$$

where  $A_j^{system}$  is the node's value, derived from the real system. These values are either measured on-line or are known beforehand as the steady nodes values of the above example. In order to drive the FCM in a realistic representation of the system and its control actions we have to update the interconnection weights using these measured node values from the real system. Based on the updated weights, equations (1) and (2) will produce a new set of node values which represent the control actions applied to the real system. The procedure, which is depicted in Fig. 3, is repetitively applied during the operation of the system. The weights that are non zero are renewed according to the Hebbian rule:

$$p = A_j^{system} - \frac{1}{1 + e^{-\left( \sum_{i=1, i \neq j}^N A_i^{s,FCM} W_{ij} + A_j^{s,FCM} \right)}} \quad (4)$$

$$W_{ij}^k = W_{ij}^{k-1} + ap(1-p)A_i^{s,FCM} \quad (5)$$

where  $k$  is the number of iteration and  $a$  is the learning rate (usually =0.1).

The procedure described in Fig. 3 uses repetitively equations (2), (3), (4) and (5) to provide with an FCM, which totally corresponds and cooperates with the real system. The control nodes of the system (nodes C3, C4 and C5 of Fig. 2) are now taking values which take into account the steady node values (C1 and C2) and the weight interconnections updated values. In the next section we extend the weight updating equations to include two collateral matrices, the one been called placement matrix and the other calibration matrix. We will see that by including these two matrices in the weight updating equations the FCM results in more balanced and smooth variations of its node values.

### 3.1 The Extended Weight Updating Law

The motivation for developing this new extended updating law was to find a flexible and credible way to drive one or more elements (nodes) of a system in a desired position (value). The proposed extended method includes two auxiliary collateral matrices Q and R. Matrix Q incorporates experts' opinion about

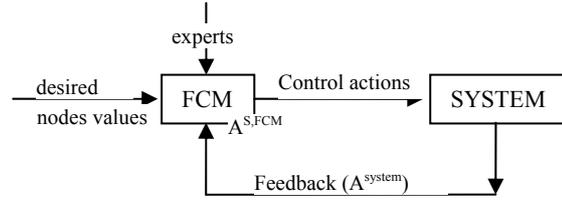


Figure 3: Control structure

the nodes that should be positively or negatively affected so that the driven node reaches the desired value, provided that the node interdependences are determined by weight matrix W. Matrix R contains elements that help FCM to converge to the desired node values by altering the connected to them nodes in a balanced way, avoiding saturation in the nodes having already large values. The two matrices Q and R can be included in the weight updating law with system feedback, described in the previous section, leading thus to a new FCM representing the system in amore desirable and realistic way.

The first matrix, called placement matrix, Q, has the same dimensions with matrix W. Each element  $Q_{ji}$  of the placement matrix Q can take one of the values  $\{-1, 0, 1\}$ , which reflect the way by which node  $C_i$  affects node  $C_j$  and determines the weights that should be updated in order to influence the change of node value  $C_j$ . A possible formation of matrix Q is the following: If one wants to drive node  $C_j$  from value  $A_j^{s-1}$  to a bigger value  $A_j^s$ , that is  $A_j^s > A_j^{s-1}$  then:

$$Q_{ji} = 1 \text{ if } W_{ij} > 0, Q_{ji} = -1 \text{ if } W_{ij} < 0, Q_{ji} = 0 \text{ if } W_{ij} = 0$$

In the opposite situations, when one wants to drive node  $C_j$  from value  $A_j^{s-1}$  to a smaller value  $A_j^s$ , that is  $A_j^s < A_j^{s-1}$  then:

$$Q_{ji} = 1 \text{ if } W_{ij} < 0, Q_{ji} = -1 \text{ if } W_{ij} > 0, Q_{ji} = 0 \text{ if } W_{ij} = 0$$

The use of this matrix will be clearer in section 4.

Incorporation of matrix Q in the weight updating equations is performed as follows:

$$p = A_j^{system} - \frac{1}{1 + e^{-\left( \sum_{i=1, i \neq j}^N A_i^{s,FCM} W_{ij} + A_j^{s,FCM} \right)}} \quad (6)$$

$$W_{ij}^k = W_{ij}^{k-1} + Q_{ij} * (ap(1-p)A_i^{s,FCM})$$

The calibration matrix R has the same dimension with matrix W. Each point  $R_{ij}$  of the calibration matrix R is computed by the following formula:

$$R_{ij} = n * \frac{\sum_{i=1}^{i=n} |W_{ij}|}{W_{ij}} \text{ if } W_{ij} \neq 0 \text{ and } R_{ij} = 0 \text{ if } W_{ij} = 0 \quad (7)$$

where n is the learning rate and is defined in the interval [0.01, 0.1].

It can be seen that for the computation of each element of R only the elements of each column of matrix W contribute. This is related to the fact that each column j of matrix W contains weight interconnection values from the nodes which affect node j. When matrix R is incorporated in the weight updating law, the new weights lead the FCM to a more balanced equilibrium point and prevent nodes, which already have large values, to saturate. At the same time matrix R also causes an enhancement to the values of nodes which have small values and which, of course affect the nodes to be changed.

Incorporation of matrix R in the weight updating equations is performed as follows:

$$p_j = A_j^{system} - \frac{1}{1 + e^{-\left( \sum_{i=1, i \neq j}^N A_i^{s,FCM} W_{ij} + A_j^{s,FCM} \right)}}$$

$$W_{ij}^k = W_{ij}^{k-1} + G * (ap(1 - p) A_i^{s,FCM}) \quad (8)$$

where :  $G = Q_{ij} * R_{ij} \quad (9)$

If  $C_j$  node must be in a desired value then  $A_j^{system} = A_j^{desired}$ , so that equation 4 for the nodes in a desired value becomes:

$$p_j = A_j^{desired} - \frac{1}{1 + e^{-\left( \sum_{i=1, i \neq j}^N A_i^{s,FCM} W_{ij} + A_j^{s,FCM} \right)}} \quad (10)$$

The complete algorithm which uses system feedback, the desired node values and the collateral matrices Q, R is shown in Fig. 4.

#### 4 SYSTEM SIMULATION STUDY

To demonstrate the method we choose a simple mechanical problem of a hydroelectric power station shown in Fig. 5. The FCM representation of the system is shown in Fig. 6. We want to regulate the flow in the two Hydro-generators (1 and 2). In order to achieve this we will use the proposed method, to control the system and to regulate the values of the report and control nodes.

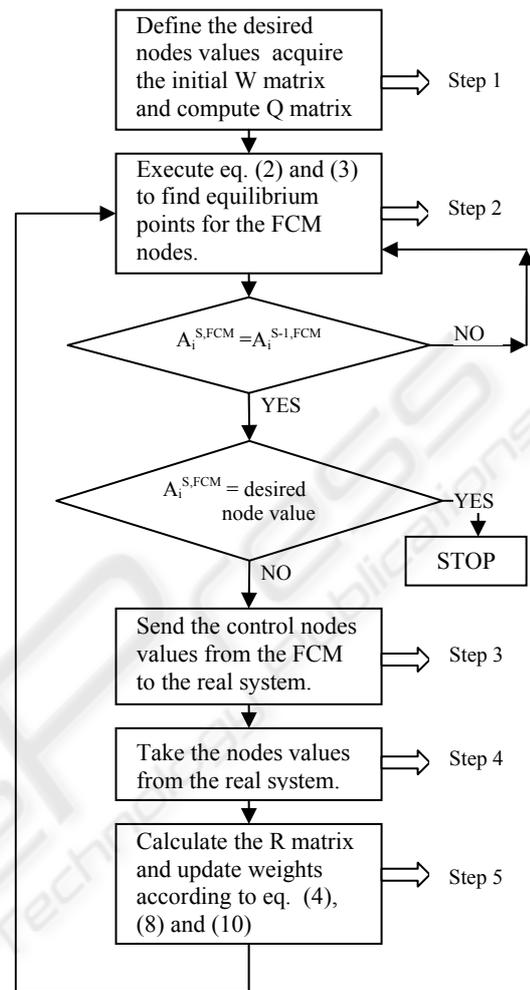


Figure 4: Schematic description of the proposed algorithm

The system has one steady value node [River - reference node1], three control nodes [Valve 2 - node 4, Valve 3 - node 6 and Valve 1 - node 2] and two simple operation nodes [Tank 1 - node 3, Tank 2 - node 5]. One or more of nodes 2, 3, 4, 5 and 6 values have to be regulated so that hydro-generators 1 and 2 can receive the desired water flow values.

Based on experts knowledge regarding the mechanics of the system a possible weight matrix W is the following:

$$W = \begin{bmatrix} 0 & -0.6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.76 & 0 & 0 & 0 \\ 0 & 0.81 & 0 & 0.38 & 0 & 0 \\ 0 & 0 & -0.6 & 0 & 0.8 & 0 \\ 0 & 0 & 0 & 0.7 & 0 & 0.6 \\ 0 & 0 & 0 & 0 & -0.42 & 0 \end{bmatrix}$$

which, after repetitively applying equations (2) and (3) will give the following equilibrium values for

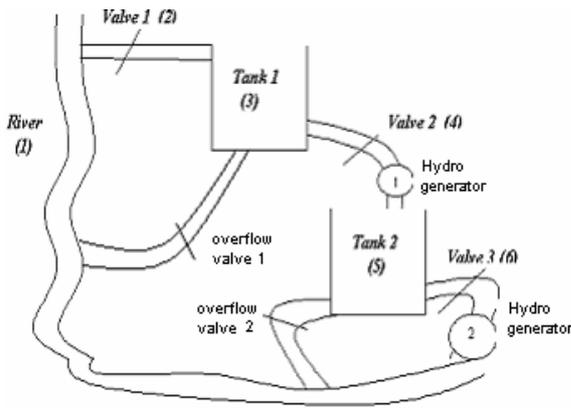


Figure 5: Hydroelectric station

the nodes of the FCM. It should be mentioned that in this case equation (3) applies only for the steady node 1 value, which in our example is 0.6.

$$A = [0.6 \quad 0.658 \quad 0.65 \quad 0.8 \quad 0.75 \quad 0.7]$$

Since we are not absolutely confident about the experts' opinion on matrix  $W$ , or we want to anticipate any physical changes occurred in the system during its operation we proceed in weight updating according to the procedure described in Fig. 3 using equations (2), (3), (4) and (5). It should be noted that, so far, the only desired node value in Fig. 3 is the steady node 1 (river). The improved weight matrix becomes:

$$W_{imp} = \begin{bmatrix} 0 & -0.9539 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.7592 & 0 & 0 & 0 \\ 0 & 0.6457 & 0 & 0.0729 & 0 & 0 \\ 0 & 0 & -0.598 & 0 & 0.7999 & 0 \\ 0 & 0 & 0 & 0.3519 & 0 & 0.2959 \\ 0 & 0 & 0 & 0 & -0.4201 & 0 \end{bmatrix}$$

which, after applying equations (2) and (3), gives the following FCM equilibrium node values.

$$A = [0.6 \quad 0.5592 \quad 0.6498 \quad 0.7402 \quad 0.7362 \quad 0.7183]$$

Case study 1

We now want to drive node 3 (tank 1) and node 5 (tank 2) to a specific value. We want to do that because the water height in these tanks will affect the water flow in Hydrogenerators 1 and 2, which in turn influences the produced power. In this approach we will use matrices  $Q$  and  $R$  and we will proceed following all the steps of Fig. 4.

Step 1

We assume that we desire the following values: node (3) = 0.652, node (5) = 0.7398, keeping always in mind that node 1 (river) has always a steady value

(0.6). Let also the initial weight matrix  $W$  equals matrix  $W_{imp}$  computed earlier.

We calculate matrix  $Q$  according to section 3.1.

$$Q = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} & 0 & 0 & 0 & 0 \\ 0 & 0 & \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} & 0 & 0 & 0 \\ 0 & 0 & 0 & \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} & 0 & 0 \\ 0 & 0 & 0 & 0 & \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} & 0 \\ 0 & 0 & 0 & 0 & 0 & \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{bmatrix}$$

The two sub-matrices of  $Q$ , enclosed by dotted lines, refer to nodes 3 and 5. The right sub-matrix refers to node 5 and declares that in order to drive node 5 in a specific value we have to update the elements of the  $W$  matrix which correspond to the points of the right dotted sub-matrix. The same rationale applies for the left sub-matrix, which now refers to node 3. The centre of each sub-matrix referring to the  $C_i$  node must be the element  $Q_{ii}$ . If we don't want to change  $C_i$  node then the corresponding sub-matrix is set to zero. For example if we want to drive only node 3 the  $Q$  matrix is:

$$Q = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} & 0 & 0 & 0 & 0 \\ 0 & 0 & \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} & 0 & 0 & 0 \\ 0 & 0 & 0 & \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} & 0 & 0 \\ 0 & 0 & 0 & 0 & \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} & 0 \\ 0 & 0 & 0 & 0 & 0 & \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{bmatrix}$$

Step 2

Now we execute step 2 of Fig. 4 to calculate equilibrium point for the FCM, which is:

$$A = [0.6 \quad 0.5592 \quad 0.6498 \quad 0.7402 \quad 0.7362 \quad 0.7183]$$

We must now correct node (3) and node (5) and drive them to 0.652 and 0.7398 respectively.

Step 3

We apply the FCM control nodes values to the real system

Step 4

We get the new node values from the real system. We assume that the real system instantly responds to the values imposed by step 3.

Step 5

Calculate calibration matrix  $R$  according to equation (7):

$$R = \begin{bmatrix} 0 & 0.1676 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.179 & 0 & 0 & 0 \\ 0 & 0.247 & 0 & 0.58 & 0 & 0 \\ 0 & 0 & 0.226 & 0 & 0.152 & 0 \\ 0 & 0 & 0 & 0.12 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0.29 & 0 \end{bmatrix}$$

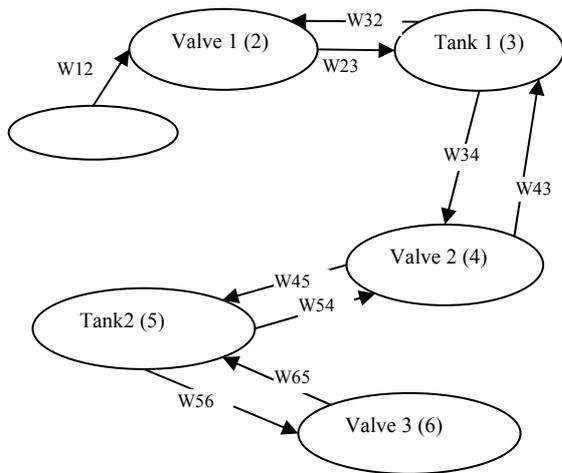


Figure 6: A Fuzzy cognitive map representing the hydroelectric factory of Fig. 5

Find the new W matrix that arises by using equations (4), (8) and (10). Go to step 2.

After 12 iterations we will find that the FCM accurately describes the operation of the real system. The final W matrix is:

$$W_{final} = \begin{bmatrix} 0 & -0.9539 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.7696 & 0 & 0 & 0 \\ 0 & 0.4148 & 0 & 0.0914 & 0 & 0 \\ 0 & 0 & -0.6138 & 0 & 0.8081 & 0 \\ 0 & 0 & 0 & 0.3818 & 0 & 0.3431 \\ 0 & 0 & 0 & 0 & -0.4121 & 0 \end{bmatrix}$$

and A vector is:

$$A = [0.6 \quad 0.5655 \quad 0.652 \quad 0.7485 \quad 0.7398 \quad 0.7273]$$

if we don't use matrices Q and R by executing the case study 1 from step 2 to step 5 we will conclude to the desired values for nodes 3 and 5 after 45 iterations. The other node values are however different since W matrix is in this case different than the one calculated above.

Case study 2

To make the use of the two matrices clearer we give the following example. Suppose we want to drive only node 3 (tank 1) in a specific value: node 3 = 0.76. We keep in mind that node 1 is a steady value node (0.6). Let also the initial weight matrix W equals matrix W<sub>final</sub> computed above.

Step 1

Calculate placement matrix Q according to Section 3.1:

$$Q = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Step 2

Now we execute step 2 of Fig. 4 to calculate equilibrium points for the FCM, which is:

$$A = [0.6 \quad 0.5655 \quad 0.652 \quad 0.7485 \quad 0.7398 \quad 0.7273]$$

We must now correct node (3) and drive it to 0.76.

Step 3

We apply the FCM control nodes values to the real system

Step 4

We get the new node values from the real system. We assume that the real system instantly responds to the values imposed by step 3.

Step 5

Calculate calibration matrix R according to equation (7):

$$R = \begin{bmatrix} 0 & 0.143 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.179 & 0 & 0 & 0 \\ 0 & 0.31 & 0 & 0.517 & 0 & 0 \\ 0 & 0 & 0.225 & 0 & 0.15 & 0 \\ 0 & 0 & 0 & 0.124 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0.296 & 0 \end{bmatrix}$$

Find the new W matrix that arises by using equations (4), (8) and (10). Go to step 2.

After 16 iterations we find that matrix W and vector A are:

$$W = \begin{bmatrix} 0 & -0.9539 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.7652 & 0 & 0 & 0 \\ 0 & 0.6 & 0 & -0.2642 & 0 & 0 \\ 0 & 0 & -0.5472 & 0 & 0.8081 & 0 \\ 0 & 0 & 0 & 0.3818 & 0 & 0.3431 \\ 0 & 0 & 0 & 0 & -0.4121 & 0 \end{bmatrix}$$

$$A = [0.6 \quad 0.6243 \quad 0.76 \quad 0.6808 \quad 0.7266 \quad 0.7273]$$

if we don't use matrices Q and R by executing the case study 2 from step 2 to step 5 we will conclude after 34 iterations to:

$$W = \begin{bmatrix} 0 & -0.5674 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.9236 & 0 & 0 & 0 \\ 0 & 0.8834 & 0 & -0.0423 & 0 & 0 \\ 0 & 0 & -0.4220 & 0 & 0.8962 & 0 \\ 0 & 0 & 0 & 0.2455 & 0 & 0.3431 \\ 0 & 0 & 0 & 0 & -0.3242 & 0 \end{bmatrix}$$

and

$$A = [0.6 \quad 0.7459 \quad 0.76 \quad 0.7019 \quad 0.76 \quad 0.7290]$$

It can be observed that, without matrices Q and R, the FCM drives the system to a different equilibrium point than the equilibrium reached using Q and R matrices. It is apparent that when the matrices are not used, in the new equilibrium points more node values are different than their initial values. On the contrary, when Q and R matrices are used only control nodes 2 and 4 are different than their initial equilibrium values. This fact is mainly due to matrix Q. In large systems which are difficult to change their operation we don't want main characteristics to be changed with no reason. Less changes we manage, in main characteristics (see valves), more flexible system we make. The effect of matrix R is made more apparent from the weight changes and the node value changes in the equilibrium points. It can be observed that by using matrix R the changes in the control node values are made in a more balanced way because in this case nodes 2 and 4, which affect node 3, change proportionally. In respect to the internal operation of the algorithm, this is connected to the fact that the weights are not allowed to reach their saturation values because their change is not allowed to be proportional to their previous value (see for example  $W_{32}$  and  $W_{34}$ ).

## 5 CONCLUSIONS

In this paper a new method for weight updating in FCMs using system feedback is proposed. So far, the existing approaches were using the simple method of weight updating without taking into account the feedback from the real system. The diversity of the proposed method lies in the fact that FCM reaches its equilibrium point using direct feedback from the node values of the real system and the limitations imposed by the reference nodes, which nodes represents either variables with constant values or variables with desired (goal) values. The weights are on-line adjusted during this operation by using an extended Hebbian updating law, which uses the system feedback and employs two specially defined collateral matrices, which help the FCM to adjust its weights and reach an equilibrium point in a more realistic and balanced way. Another benefit of using these matrices, which is drawn from experimental results, is the faster convergence of the weight updating algorithm.

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