Application of Fractal Codes in Recognition of Isolated Handwritten Farsi/Arabic Characters and Numerals

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Abstract. In this paper we proposed a new method for isolated handwritten Farsi/Arabic characters and numerals recognition using fractal codes. Fractal codes represent affine transformations which when iteratively applied to the range-domain pairs in an arbitrary initial image, the result is close to the given image. Each fractal code consists of six parameters such as corresponding domain coordinates for each range block, brightness offset and an affine transformation which are used as inputs for a multilayer perceptron neural network for learning and identifying an input. This method is robust to scale and frame size changes. 32 Farsi’s characters are categorized to 8 different classes in which the characters are very similar to each others. There are ten digits in Farsi/Arabic language and since two of them are not used in postal codes in Iran, therefore 8 more classes are needed for digits. According to experimental results, classification rates of 91.37% and 87.26% were obtained for digits and characters respectively on the test sets gathered from various people with different educational background and different ages.

1 Introduction

English, Chinese and Kanji isolated handwritten character recognition have long been a focus of study, but a little researches have been done on Farsi and Arabic. Some previous works on recognition of isolated characters, words and scripts of Farsi and Arabic language have used some structural features [1] [2] and moment features [3]. Fractal theory has been used in several areas of image processing and computer vision. In this method, similarity between different parts of an image is used for representing of an image by a set of contractive transforms on the space of images, for which the fixed point is closed to the original image. This concept has been used recently by some researchers for face recognition [4] [5]. In the case of character recognition, we deal with the image of characters, and by using features that are extracted from the original image, the process of classification can be more facilitated. In this study we used fractal codes as features in the recognition process and fed them as inputs to a multilayer perceptron neural network for learning and identifying characters. As fractal codes are so sensitive to translation, scaling and rotation, some
preprocessing has been required. Location invariancy and scale normalization is achieved by finding the bounding rectangle of each character and scaling it to a 64×64 pixel image. The learning and test sets were gathered from various people with different educational background and different ages. 32 Farsi’s characters are categorized to 8 different classes based on the similarity of the characters in each class. There are ten digits in Farsi/Arabic language that two of them are not used in the postal codes in Iran. So digits can be categorized in 8 different more classes. A multilayer perceptron (MLP) neural network is used as a classifier which includes one hidden layer, 64 input nodes and 8 output nodes. The Overall classification rate for 8 classes of characters and 8 classes of numerals were 87.26% and 91.37% respectively. This paper is organized as follows. Section 2 describes the basic concepts in fractal encoding consisting of preprocessing phase followed by feature extraction from fractal-coded images. The experimental results are presented in section 3. Finally the conclusion remarks are given in section 4.

2. Feature Extraction

2.1 Pre-Processing

For a given binary image containing a single character or numeral, three pre-processing tasks are needed to make the system invariant to scale changes, frame size changes and rotation. To remove any differences due to location of character or numeral within the image, the bounding rectangle box of each character or numeral is found. Then this bounding box is scaled to 64×64 pixel image, for scale normalization.

2.2 Overview of Fractal Image Coding

With the advent of information age, the need for mass information storage and retrieval grew. Different image compression methods have been focused for a long time to reduce this massive information, but a novel promising approach called fractal image coding has drawn much attention recently. The fundamental principles of fractal coding consist of the representation of any image by a contractive transform of which the fixed point is close to the original image. A transform W is said to be contractive if for any two points \( p_1, p_2 \) the distance between them satisfy equation (1) for some \( s<1 \).

\[
d(W(p_1), W(p_2)) < s \times d(p_1, p_2)
\]

Banach’s fixed point theorem guarantees that, within a complete metric space, the fixed point of such a transformation may be recovered by an iterated application, thereof to an arbitrary initial element of that space [6]. Fractal encoding is based on the concepts and mathematical results of iterated function systems (IFS). Fractal compression became a practical reality with the introduction by Jacquin of the partitioned IFS (PIFS), which differs from an IFS in that each of the individual mappings operates on a subset of the image, rather than on the entire image [7]. An image to be
encoded is partitioned into non-overlapping range blocks $R$ with the size $N \times N$ and overlapping domain blocks $D$ with the size $2N \times 2N$ as depicted in Fig.1. The task of fractal encoder is to find a $D$ block in the same image for every $R$ block such that transformation of this domain block $W(D)$ minimize the collage error in equation (2).

Distances are usually measured by mean square error (MSE), since optimization of the standard block mappings is simple under this measurement [7].

$$\text{Collage Error} = \min \| R_i - W(D_j) \|$$  \hspace{1cm} (2)

Suppose we are dealing with a $64 \times 64$ binary image in which each pixel can have one of 256 levels (ranging from black to white). Let $R_1, R_2, ..., R_{256}$ be $4 \times 4$ pixel non-overlapping sub-squares of the image, and let $D$ be the collection of all $8 \times 8$ pixel overlapping sub-squares of the image. The collection $D$ contains $57 \times 57 = 3249$ squares. For each $R$ block, search through all of $D$ blocks to find a $D_j \in D$ which minimizes equation (2). There are 8 ways to map one square onto another. The square can be rotated to 4 orientations or flipped and rotated into 4 other orientations which are depicted in Fig.2. Having 8 different affine transformations, it means comparing $8 \times 3249 = 25992$ squares with each of the 256 range squares.

As mentioned before, a $D_j$ block has 4 times as many pixels as an $R_i$, so we must either subsample (choose 1 from each $2 \times 2$ sub-square of $D_j$) or average the $2 \times 2$ sub-squares corresponding to each pixel of $R_i$ when we minimize equation (2) [8]. Minimizing equation (2) means two things. First it means finding a good choice
for $D_i$. Second, it means finding a good contrast and brightness setting $s_i$ and $o_i$ for $W_i$ in equation (3).

$$
\begin{bmatrix}
x \\
y \\
z 
\end{bmatrix}
= \begin{bmatrix}
a & b & 0 \\
c & d & 0 \\
0 & 0 & s_i 
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z 
\end{bmatrix}
+ \begin{bmatrix}
e_i \\
f_i \\
o_i 
\end{bmatrix}
$$

(3)

Assume two squares containing $n$ pixel intensities, $a_1, a_2, \ldots, a_n$ for $D_i$ and $b_1, b_2, \ldots, b_n$ for $R_i$. By minimizing equation (4), $s$ and $o$ can be obtained.

$$\text{R} = \sum_{i=1}^{n}(s.a_i + o - b_i)$$

(4)

This will give us a contrast and brightness setting that makes the affinely transformed $a_i$ values have the least squared distance from the $b_i$ values. The minimum of $R$ occurs when the partial derivation respect to $s$ and $o$ are zero, which occurs when

$$
\begin{align*}
\textbf{s} &= \frac{n^{2}\left(\sum_{i=1}^{n}a_i\right) - \left(\sum_{i=1}^{n}a_i\right)\left(\sum_{i=1}^{n}b_i\right)}{\left(n^{2}\sum_{i=1}^{n}a_i^2 - (\sum_{i=1}^{n}a_i)^2\right)} \\
\textbf{o} &= \frac{\sum_{i=1}^{n}b_i - s\sum_{i=1}^{n}a_i}{n^2}
\end{align*}
$$

(5)

In this case,

$$\text{R} = \frac{\sum_{i=1}^{n}b_i^2 + s(\sum_{i=1}^{n}a_i^2 - 2s\sum_{i=1}^{n}a_ib_i) + 2o(\sum_{i=1}^{n}a_i^2) + o(\sum_{i=1}^{n}a_i^2 - 2\sum_{i=1}^{n}b_i)}{n^2}$$

A choice of $D_i$, along with a corresponding $s_i$ and $o_i$, determines a map $W_i$. The type of image partitioning used for the range blocks can be so different. A wide variety of partitions have been investigated, the majority being composed of rectangular blocks. Different types of range block partitioning were described in [9]. In this research we used the simplest possible range partition consists of the fixed size square blocks, that is called fixed size square blocks (FSSB) partitioning.

2.3. Encoding Algorithm

The procedure for finding a fractal model for a given image is called encoding, compression, or searching for a fractal image representation. Encoding algorithm can be summarized as follow [10]:

1- Input the original binary image.
2- Partition the input image into $R$ blocks according to FSSB partitioning scheme.
3- Create a list of $D$ blocks.
4- Search for a fractal match. Given a $R_i$ region, loop over all possible $D$ blocks to find the best match using a given metric. This is the most time consuming step of the whole algorithm.
5. Select fractal elements. After finding the best match, fractal elements which are 6 real numbers are selected as follow:

a,b: (x,y) coordinates of the D block which is always pointed to the top-left corner of the D block square.

c,d: (x,y) coordinates of the corresponding R block

e: The number of the affine transformation that makes the best match. (a number between 1 and 8).

f: The intensity which is a number between 0 and 256.

2.4. Decoding Algorithm

The reverse process of generating an image from a fractal model is called decoding, decomposition, or displaying a fractal format image. In the case of character recognition, although it is not necessary to decode the fractal models that are obtained from previous section, we have done it to verify the validation of coding algorithm. Decoding process starts with an arbitrary initial image that is a uniform gray level picture with 128 intensities in our case. Then the decoding algorithm is iterated about 6 to 16 times. The results for different iterations and different R block sizes are depicted in Fig.3 and Fig.4. After each iteration, the average of error and peak signal to noise ratio (PSNR) are calculated according to equations (6) and (7). Table.1. shows these results.

\[
\text{average of error} = \frac{\sum_{i=1}^{M} \sum_{j=1}^{N} (J(i,j) - I(i,j))}{M \times N}
\]

\[
\text{PSNR} = 20 \times \log \left( \frac{255}{\text{average of error}} \right)
\]

Fig.3  decoding algorithm’s results for N=4
(a) original image  (b) arbitrary initial image
(c) decoded image after 1 iteration  (d) decoded image after 5 iteration

Fig.4  decoding algorithm’s results for N=8
(a) original image  (b) decoded image after 1 iteration
(c) decoded image after 5 iteration  (d) decoded image after 15 iteration
Table.1 Average of error and PSNR versus number of iteration

<table>
<thead>
<tr>
<th>Number Of Iteration</th>
<th>Average Of Error</th>
<th>PSNR (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.45</td>
<td>40.32</td>
</tr>
<tr>
<td>1</td>
<td>1.95</td>
<td>42.30</td>
</tr>
<tr>
<td>2</td>
<td>1.39</td>
<td>44.06</td>
</tr>
<tr>
<td>3</td>
<td>1.38</td>
<td>45.53</td>
</tr>
<tr>
<td>4</td>
<td>1.18</td>
<td>46.69</td>
</tr>
<tr>
<td>5</td>
<td>1.06</td>
<td>47.55</td>
</tr>
<tr>
<td>6</td>
<td>0.99</td>
<td>48.16</td>
</tr>
<tr>
<td>7</td>
<td>0.95</td>
<td>48.57</td>
</tr>
<tr>
<td>8</td>
<td>0.92</td>
<td>48.84</td>
</tr>
<tr>
<td>9</td>
<td>0.90</td>
<td>49.02</td>
</tr>
<tr>
<td>10</td>
<td>0.89</td>
<td>49.14</td>
</tr>
<tr>
<td>11</td>
<td>0.88</td>
<td>49.21</td>
</tr>
<tr>
<td>12</td>
<td>0.87</td>
<td>49.26</td>
</tr>
<tr>
<td>15</td>
<td>0.86</td>
<td>49.34</td>
</tr>
</tbody>
</table>

With the increase in R blocks size, the PSNR and average of error decreases and increases respectively, and also the decoding algorithm became faster. These results are shown in Table.2.

3. Experimental Results

In Farsi language, there are ten digits that are shown in Fig.5. Because of similarity between (0, 5) and (2, 3) especially in handwritten text, digits (0) and (2) are not used in postal codes in Iran. Thus, we have 8 different classes for digits.

Table.2. Encoding Results with different parameters

<table>
<thead>
<tr>
<th>N</th>
<th>Encoding Time (Sec)</th>
<th>Average Of Error</th>
<th>PSNR (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>6.78</td>
<td>0.884</td>
<td>49.19</td>
</tr>
<tr>
<td>8</td>
<td>3.81</td>
<td>0.927</td>
<td>48.76</td>
</tr>
<tr>
<td>16</td>
<td>1.86</td>
<td>1.01</td>
<td>48.00</td>
</tr>
</tbody>
</table>

Dots play an important role in Farsi characters. For example as shown in Fig. 6, there are four different characters that only differ in number of and the position of dots. To simplify, we neglect these dots and consider characters in their main form.
without the dots. We categorize Farsi characters into 8 different classes which are shown in Table.3.

![Fig.6. Four Farsi Characters with different dots and similar patterns](image)

**Table.3.** Final character classes

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ا</td>
<td>ف</td>
<td>ب</td>
<td>ج</td>
<td>د</td>
<td>س</td>
<td>ک</td>
<td>م</td>
</tr>
<tr>
<td></td>
<td>گ</td>
<td>ط</td>
<td>ق</td>
<td>پ</td>
<td>چ</td>
<td>س</td>
<td>ط</td>
<td>ل</td>
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<td>ز</td>
<td>ب</td>
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<td>ر</td>
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<td>ژ</td>
<td>غ</td>
<td>و</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Training and test sets, for characters and numerals, were gathered from more than 200 people with different educational background. Our database contains 480 samples per digit (total of 3840), and 190 samples per character (total of 6080). We have used 100 samples of each character (total of 3200) for training and 90 samples (total of 2880) for test. We also used 280 samples of each digit (total of 2240) for training and 200 samples (total of 1600) for test. By using an MLP neural network as a classifier, the recognition rate of 91.37% and 87.26% are achieved for digits and characters respectively. (see Table.4 and 5).

**Table.4.** Classification results for characters

<table>
<thead>
<tr>
<th></th>
<th>Correct%</th>
<th>Error%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Training Set</td>
<td>97.4%</td>
<td>2.6%</td>
</tr>
<tr>
<td></td>
<td>(3117/3200)</td>
<td>(83/3200)</td>
</tr>
<tr>
<td>Test Set</td>
<td>87.26%</td>
<td>12.74%</td>
</tr>
<tr>
<td></td>
<td>(2785/3200)</td>
<td>(415/3200)</td>
</tr>
</tbody>
</table>

**Table.5.** Classification results for digits

<table>
<thead>
<tr>
<th></th>
<th>Correct%</th>
<th>Error%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Training Set</td>
<td>98%</td>
<td>2%</td>
</tr>
<tr>
<td></td>
<td>(2196/2240)</td>
<td>(44/2240)</td>
</tr>
<tr>
<td>Test Set</td>
<td>91.37%</td>
<td>8.63%</td>
</tr>
<tr>
<td></td>
<td>(2047/2240)</td>
<td>(193/2240)</td>
</tr>
</tbody>
</table>
4. Conclusion

In this research we have used fractal codes as features for Farsi digits and characters. By using an MLP neural network as a classifier, fair recognition rates are obtained. As we are aware, this is the first research in OCR which uses fractal codes as features, so using other partitioning methods such as quadtree may lead to better results.

References