ADAPTIVE SMITH PREDICTIVE CONTROL OF NON-LINEAR SYSTEMS USING NEURO-FUZZY HAMMERMSTEIN MODELS

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Abstract: This paper proposes an Adaptive Smith Predictor Controller (ASPC) based on Neuro-Fuzzy Hammerstein Models (NFHM) with on-line non-linear model parameters identification. The NFHM approach uses a zero-order Takagi-Sugeno fuzzy model to approximate the non-linear static function that is tuned off-line using gradient decent algorithm and to identify the linear dynamic function it is used the Recursive Least Square estimation with Covariance Matrix Reset (RLSCMR). This algorithm has the capability of follow fast and slow dynamic parameter changes. The proposed ASPC has special capabilities to control non-linear systems that have gain, time delay and dynamic changes through time. The implementation of the ASPC is made in two steps: first, off-line estimation of the non-linear static parameters that will be used to “get linear” the non-linearity of the system and second, on-line identification of the linear dynamic parameters updating direct and inverse models used in the ASPC. As an illustrative example, a gas water heater system is controlled with the ASPC. Finally, the control results are compared with the results obtained with the Smith Predictive Controller based in a Semi-Physical Model (SPMSCP).

1 INTRODUCTION

Industry control processes presents many challenging problems, including non-linear dynamic behaviour, uncertain time delay and time varying parameters. During the last decades, a very promising model based control solution used in industry processes with time delay is the predictive/Smith predictive model based controller. In this algorithm it is important to choose the right model representation of the linear/non-linear system. The model should be accurate and robust for all working points, with a simple mathematical representation and with a transparent representation that makes it interpretable. The most common non-linear modelling methods are: the NARX and NAARX models, neural-networks models fuzzy models and Hammerstein and Wiener models. When the knowledge of the control systems does not exist or the process is subject to changes in its dynamic characteristics it is important to use an adaptive control algorithm. There are two types of model based controllers: off-line tuned model based controllers (Abonyi el at., 2000), (Pottmann and Seborg 1997), (Vieira el at., 2003) and adaptive controllers with on-line parameters identification as (Abonyi el at., 1999) and (Fink el at., 2001). This paper presents a simple adaptive model based controller that uses the NFHM approach (Vieira el at., 2004b) with a modification that gives to the algorithm the capability of identify on-line the linear dynamic parameters of the linear part of the global

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model. Hammerstein model approach gives simple and interpretable models that facilitate the identification and its integration on control schemes. Based on this presupposes, it is proposed a simple solution for the adaptive control of non-linear systems that presents uncertain, time delay and time varying dynamic parameters. In time variant processes, the non-linear static functions are usually fixed through time (permitting tuning off line) and the changes are in its dynamic behaviour so it is necessary to identify the linear dynamic parameters on line (Fink et al., 2001).

Section 2 and 3, describes the Neuro-Fuzzy Hammerstein Model structure and the modified identification method. For the non-linear static function approximation it is used a zero-order Takagi-Sugeno fuzzy model tuned with gradient decent algorithm. With the inverse of this non-linear static function the non-linear system will be linear. For the linear variant dynamic function approximation it is used the on-line recursive least square parameter estimation with reset of covariance matrix algorithm. This algorithm has the capability of identify fast and slow dynamic parameter changes.

The challenger for non-linear on-line identification is to guarantee that all parameters of the varying dynamic model are correctly identifies even in the presence of a varying time delay and a noisy system. Section 4, describes the ASPC that is implemented in two steps: first, off-line estimation of the non-linear static parameters and second, on-line identification of the linear dynamic parameters updating direct and inverse models of the system used in the controller.

Section 5, shows the control results using the ASPC applied in to a domestic gas water heater system. The results are compared with the ones achieved with the Smith Predictive Controller based in a Semi-Physical Model (Vieira et al., 2004a).

Finally, in section 6, the conclusions and future works are pointed.

2 STRUCTURES OF THE NFHM

The NFHM consists of a series connection of a non-linear static function \( f(.) \) and a linear dynamic function \( G(s) \) as shown in Figure 1.

It is proposed, that the non-linear static function would be approximated by a zero-order Takagi-Sugeno fuzzy model. The fuzzy model function \( f(.) \) can be formulated as a set of \( r \) local constant functions \( z_i = d_{i_1}, \ldots, z_i = d_r \), where \( d_{i_1}, \ldots, d_r \) are constant parameters that are conjugated in the form of rules:

\[
R_{1..r} \; : \; \text{IF} \; u \; \text{is} \; A_{1..r} \; \text{THEN} \; z_{1..r} = d_{1..r} \tag{1}
\]

where \( A_{1..r} \) are the antecedent fuzzy sets for the input \( u \) and \( d_{1..r} \) are the consequent constant parameters. All fuzzy sets are bell shaped type membership functions see Figure 2.

From a given \( u \), the output of the fuzzy model \( z \) is inferred by computing the weight average of the rule consequents:

\[
z = \frac{\sum_{i=1}^{r} \beta A_i(u) d_i}{\sum_{i=1}^{r} \beta A_i(u)} \tag{2}
\]

where,

\[
\beta A_i(u) = \frac{1}{1 + \frac{\left| u - c_i \right|}{a_i b_i}} \tag{3}
\]

is the membership function of the input \( u \) relative to the fuzzy set \( A_i \) and \( a_i \), \( b_i \) and \( c_i \) are parameters for adjust the shape and centre of the fuzzy set \( i \).

The number of rules/number of fuzzy sets will depend only of the complexity of the real static non-linear function. After define the number of fuzzy set/rules the non-linear parameters \( a_i \), \( b_i \) and \( c_i \) for all fuzzy sets and the linear parameters \( d_i \) for \( i=1..r \) rules should calculate.

The second part of the structure of the NFHM is the definition of the linear dynamic function \( G(s) \).
This G(s) function is an n order linear system represented in discrete domain by equation 4

\[
y(k) = a_1 y(k-1) + a_2 y(k-2) + \ldots + a_n y(k-ny) \]

\[
+ b_0 z(k-nd) + \ldots + b_{nu} z(k-nd-nu)
\]

(4)

where \(a_1, a_2, \ldots, a_n\) and \(b_0, b_1, b_2, \ldots, b_{nu}\) are the numeric parameters of dynamic linear system. \(n+1\) and \(ny\) indicated the order of the regressors need for each variable and \(nd\) is the discrete time delay.

The global mathematical equation of the NFH global model is illustrated in equation 5.

\[
y(k) = a_1 y(k-1) + a_2 y(k-2) + \ldots + a_n y(k-ny) \]

\[
+ \sum_{i=1}^{nu} \beta_i \left( (u(k-nd))_i \right) \frac{1}{1} \]

\[
+ \ldots
\]

\[
+ b_{nu} \sum_{i=1}^{nu} \beta_i \left( (u(k-nd-nu))_i \right) \frac{1}{1}
\]

(5)

3 IDENTIFICATION OF THE NFHM

In time variant processes, the non-linear static functions are usually fixed through time so the parameters identification could be made off-line. Otherwise, the linear dynamic functions could present some changes in its dynamic behaviour so it is better to identify its parameters on line. For the identification of the non-linear static function the used training data should fulfill several requirements. The control signal \(u(k)\) applied to the system should be a step signal with a large number of steps in its universe \([U_{min}, \ldots, U_{max}]\). The large number of steps is very important to get the exact non-linearity of the system (number of steps depends on the non-linearity type function). Another important requirement is the time (number of samples) that the step control signal should be maintained with out any changes. This time should be long enough for the system achieving the stationary state (at least 5 time constants of the system that achieves 99.1% of the stationary state). The figure 3 illustrates a typical training control signal.

With this type of training signal it is possible to get, first, the stationary state data for training the non-linear static function, and second, the transitory data for the initialisation of the dynamic linear function.

![Figure 3: Typical training control signal.](image)

For training the non-linear static function there was used the two vectors with NS stationary state samples \(u_{ss}(k)\) and \(y_{ss}(k)\) obtained as shows figure 3

\[
u_{ss} = u_{ss}(1); u_{ss}(2); \ldots; u_{ss}(NS);\]

\[
y_{ss} = y_{ss}(1); y_{ss}(2); \ldots; y_{ss}(NS);
\]

(6)

For the initialisation of the dynamic linear function it is used all \((N)\) samples of all \(u\) (static and dynamic samples).

\[
u = u(1); u(2); \ldots; u(N);\]

\[
y = y(1); y(2); \ldots; y(N);
\]

(7)

The identification method imposes that all gain of the system is included in the non-linear static function and the dynamic linear function will have a unitary gain, at least in the off-line training phase.

The non-linear static function is approximated by a zero-order Takagi Sugeno fuzzy modelled. The tuning of the parameters used in the fuzzy model can be considered as a numerical optimisation procedure. Among the methods that have been implemented so far the gradient decent adaptation method permits accurate learning of all parameters of the fuzzy modelled. The fuzzy model is parameterised by the following parameters.

\[
\psi = \{aa, bb, cc, dd; i = 1..r\text{ rules/n\textsuperscript{o} fuzzy sets}\}
\]

(8)

The objective is to minimize the global prediction vector error between the model and the plant outputs. Therefore, the gradient decent method tends
to decrease the quadratic objective function based on
the vector error
\[
e = \frac{1}{2} (y_{mm} - y_{ss})^2 \text{ vectors of length } l = 1..N
\]
(9)
with \( z = y \) in stationary state, \( y_{mm} \) is the approximation output vector of the fuzzy model. The parameter set \( \Psi \), of the fuzzy model is changed via the following iterative (j) learning rule:
\[
\psi_{i}(j) - \psi_{i}(j-1) = \lambda \frac{\partial e(j)}{\partial \psi_{i}(j)} \]
(10)
where \( \lambda \) is the learning rate parameter, which controls the learning velocity of the algorithm. The number of iterations will depend on the decreasing of the total vector error \( \sum e \) using the learning vectors. When the algorithm achieves a predefined small value or a maximum number of iterations the iterative algorithm stops.

The partial derivatives of the model error \( e \) with respect to the parameters of the fuzzy model are given by:
\[
aa_{1}(j+1) = aa_{1}(j) + \Delta aa_{1}(j) = aa_{1}(j) - \lambda \frac{\partial e(j)}{\partial aa_{1}(j)}
\]
\[
\frac{\partial e(j)}{\partial aa_{1}(j)} = \sum_{i=1}^{N} \left( \frac{\partial e(j,1)}{\partial y_{mm}(j,1)} \frac{\partial y_{mm}(j,1)}{\partial R_{i}(j,1)} \frac{\partial R_{i}(j,1)}{\partial \beta A_{i}(j,1)} \frac{\partial \beta A_{i}(j,1)}{\partial aa_{1}(j,1)} \right)
\]
\[
\frac{\partial e(j)}{\partial bb_{1}(j)} = \ldots
\]
\[
\frac{\partial e(j)}{\partial cc_{1}(j)} = \ldots
\]
\[
\frac{\partial e(j)}{\partial d_{1}(j)} = d_{1}(j) - \lambda \sum_{i=1}^{N} \left( \frac{\partial e(j)}{\partial y_{mm}(j)} \frac{\partial y_{mm}(j)}{\partial d_{1}(j)} \right)
\]
(11)
In the initialisation, the antecedent membership functions and the consequent constant functions are equidistantly distributed over the input and output respective universes of discourse.

The second part of the learning algorithm is the definition of the linear dynamic function parameters. The first question that arises is the choice of the order/significant regressors of the modelled. To find the significant regressors of the system it could be

used à priori knowledge of the system or the pole-zero cancellation method.
To estimate the initial \( a_{1}, a_{2}, \ldots, a_{ny} \) and \( b_{0}, b_{1}, b_{2}, \ldots, b_{mu} \) vectors, it was used the Least Square algorithm.
The modification of the NFHM approach is exactly here in the identification of the linear dynamic parameters. In this method this parameters are calculated on-line with recursive least square algorithm with reset of covariance matrix (RLSCMR) as expressed in equation 12,
\[
z(k) = \left[ \begin{array}{c}
\lambda_{x}(k) \\
U(k) \\
n_{y}(k)
\end{array} \right]
\]
(12)
where \( \lambda_{x}(k) \) is the vector with the instant k estimated parameters, \( P(k) \) is the learning rate and \( P(k) \) is the covariance matrix. The Reset Covariance Matrix (RCM) algorithm is used for a fast convergence in the identified parameters. If the error \( e(k) \) is bigger than a pre-defined value the covariance matrix \( P \) is reset (starting values).

4 ADAPTIVE SMITH PREDICTOR CONTROL STRUCTURE

The Adaptive Smith Predictive Controller is based in the Internal Model Controller (IMC) architecture and is implemented in two phases. First is the offline estimation of the non-linear static parameters. Second is the on-line identification of the linear dynamic parameters updating the direct and inverse models of the system, as illustrated in figure 4. Offline, with the inversion of the non-linear static function, it is possible to transform the non-linear plant in to an approximate “linear” plant. Finally in closed loop control, iteration-by-iteration, the linear dynamic parameters are recalculated and updated.
The ASPC separates the time-delay of the plant from the model of the plant, so it is possible to predict the $y(k)$ $n$ steps earlier ($n =$ digital time-delay), compensating the negative time-delay effects in the control results. The incorrect prediction of the time delay may lead to aggressive control if the time-delay is under estimated or conservative control if the time-delay is over estimated (Tan et al., 2002).

5 ILLUSTRATIVE EXAMPLE: GAS WATER HEATER TEMPERATURE CONTROL

The global system has three main blocks: the gas water heater, a micro-controller board and a personal computer. The micro-controller board has three modules, all controlled by the flash-type microcontroller PHILIPS 89C51RD. The Sensors and Actuators module is used to read and actuate the inputs and outputs of the system. The Security module that is used for the supervision and control of the security conditions. The Communication module that is used for the acquisition/monitoring of the system data to the personal computer.

After a small description of the global system, it will be made a small description of the gas water heater system and its characteristics, for a detailed description see (Vieira et al. 2003) and it ends with the definition, identification and comparison of the proposed ASPC with the SPMSPC (Vieira et al. 2004a).

5.1 System Description

The gas water heater is a multiple input single output (MISO) system. The objective is to control the output water temperature, called hot water temperature (hwt). This variable depends of the cold water temperature (cwt), water flow (wf), gas flow (gf) (applied power) and the gas water heater dynamics. Considering that the cold water temperature is almost constant, the final objective is to control the delta water temperature ($\Delta t$) (difference between hot and cold water temperatures) reducing the number of inputs.

The gas water heater is physically composed by a gas burner, a permutation chamber, a ventilator, two gas valves and several sensors used for control and security as shown on figure 5. Operating range of the hwt is from 30ºC to 60ºC. Operating range of the cwt is from 5ºC to 25ºC. Finally, the operating range of the water flow is from 3.5 to 14.5 litters/minute.

One of the main characteristics of the gas water heater is its Maximum static Power ($\text{MaxP}$). The device used has 300 Kcal/min of maximum power. The MaxP depends on the physic characteristics of the permutation chamber and is given by the equation 13.

$$\text{MaxP} = \Delta t \left(\text{hwt} - \text{cwt}\right) \left[\text{Kcal/min}\right]$$

with

$$\text{gf} = \text{Max(gf)} \quad \Rightarrow \quad \Delta \text{max} = \frac{\text{MaxP}}{\text{wf}} \left[^{\circ}\text{C}\right] \quad (13)$$

The delta temperature is an unknown dynamic non-linear function $h$ that depends of the latest samples of the gas flow, delta temperature and water flow. See equation 14:

$$\Delta t(k) = h(gf(k - td),...,gf(k - td - nu), \Delta t(k - 1),...\Delta t(k - ny), wf(k - 1)) \quad (14)$$

Figure 6 shows the static gas water heater surface, where it is clear that there are two main variables that affect directly the delta temperature, which are the gas flow and the water flow, as
expected from equation 13. The relation between gas flow and the delta temperature presents a weak but important non-linearity for a specific water flow. However, the relation between water flow and the delta water temperature presents a strong non-linearity for a specific gas flow.

The gas water heater plant presents a variant time delay. The variation of the time delay function $td(t)$ depends mainly on the velocity of the water inside of the tubes in the permutation chamber. The time-delay approximation function $td(k)$ for the one second sampled system is illustrated in equation 15.

$$td(k) = \begin{cases} 
3 \text{ (sec) if } w(k) \geq 9.5 \text{ l/min} \\
4 \text{ (sec) if } 3.5 \leq w(k) < 9.5 \text{ l/min} \\
5 \text{ (sec) if } w(k) < 3.5 \text{ l/min} 
\end{cases} \quad (15)$$

**5.2 ASPC specific structure and on-line model identification**

From empirical knowledge, heating systems are usually first order systems plus a time delay, based in this knowledge the dynamic linear part of the NFHM will be considered a first order dynamic function. Therefore, the dynamic linear model function is expressed in equation 16.

$$\Delta t(k) = a_\Delta \Delta t(k-1) + b_\Delta \text{nlgf}(k - td)$$
$$\text{nlgf}(k - td) = \text{non-linear function of } gf(k - td) \quad (16)$$

The non-linear gas flow $\text{nlgf}(.)$ corresponds to the $f(.)$ non-linear function in the NFHM.

The ASPC is implemented in two steps: first, off-line estimation of the non-linear static parameters and second, on-line identification of the linear dynamic parameters updating the direct and inverse models of the plant, as illustrated in figure 7. The Time-Delay Approximation Function updates on-line the time delay approximation as expressed in equation 15 and illustrated in figure 7.

In this particular example the variable water flow, changes the time constant and the static gain of the system. Therefore, this static gain should be taking into account in the non-linear static function parameters calculation. The non-linear static parameters were calculated with a constant water flow of 9 l/m that gives a static gain of 0.873 (see training signal). The non-linear static function parameters calculated in (Vieira et al. 2004b) should be multiplied by the inverse of this particular static gain therefore this non-linear static parameters will be general for all water flow range.

First step is the non-linear static inverse function parameters identification off-line. It was used a zero-order TS fuzzy model implemented with three bell shaped fuzzy sets that impose three simple rules. With input universes of discourse normalized and using the training and test data sets used in the NFHM approach exposed in (Vieira et al., 2004b) the zero-order TS fuzzy model parameters are: $a_1=0.274$, $b_b=1.614$, $c_c=-0.023$, $d_d=0.1717$, $a_a=0.352$, $b_b=2.060$, $c_c=0.515$, $d_d=0.666$, $a_a=0.407$, $b_b=2.125$, $c_c=1.008$ and $d_d=1.249$.

After the non-linear static inverse function parameters calculus the initial linear parameters are calculated using the LS algorithm. The initial linear dynamic identification parameters found are $a_1=0.790$ and $b_0=0.210$.

Finally, iteration-by-iteration, the linear dynamic parameters are recalculated, updating the proposed ASPC based in NFHMs.
5.3 Comparative results using the ASPC and SPMSPC

For the comparison of the two controllers, ASPC and SPMSPC, the references hot water temperature and water flow variables were applied and the respective mean square errors (MSE) were calculated \( r(k-t_d)-y(k) \). It was used \( r(k-t_d) \) to avoid the error introduced by the time delay. The final results are expressed in table 1.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>ASPC</td>
<td>1.858</td>
</tr>
<tr>
<td>SPMSPC</td>
<td>1.953</td>
</tr>
</tbody>
</table>

As can be seen from the results both architectures ASPC and SPMSPC achieve good results. But the SPMSPC approach is not an adaptive model controller approach, therefore, it can be seen in Figure 9 that there is an error between the direct model and the real plant. The cause that is responsible for the difference between the model and the plant is called load. This load induced a worst control performance.

The ASPC approach is an adaptive model controller approach, therefore, it can be seen in Figure 8 that there is no load in the system. However, the control results are affected by the tuning time and variation of the linear parameters. In Figure 10 it can be seen that the linear dynamic parameters are similar in both controllers. The small differences observed became from the possible non-optimal parameters achieved with the genetic algorithms in the SPMSP and from the on-line adaptation of the linear dynamic parameters with a continues variation of the time delay that was approximated to the discrete time by equation 15.

6 CONCLUSIONS AND FUTURE WORK

This work presents a new model based Smith predictive adaptive controller using Hammerstein neuro-fuzzy model identification. It presents a new and simple method for the neuro-fuzzy Hammerstein model on-line identification and its generalisation. The NFHM approach uses a zero-order Takagi-Sugeno fuzzy model to approximate the non-linear static function that is tuned off-line using gradient decent algorithm and to identify the linear dynamic function it is used the Recursive Least Square on-line estimation with Covariance Matrix Reset (RLSCMR). The CMR algorithm is used for a faster convergence of the identified parameters because if the load presents big changes the parameters should
have a fast and stable change too, maintaining the robustness of the controller.

Finally, the proposed ASPC and SPMSPC control approaches were successful applied to an illustrative example: gas water heater system. The ASPC achieve better control results than the SPMSPC because, even when the load (water flow / maximum power) changes the dynamic of the system, the linear parameters will adapt then selves. The SPMSPC was optimise for a fixed maximum power so if the maximum power changes the control results will be worse than the ones achieved with the ASPC.

REFERENCES
